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On dominaton type invariants of regular dendrimer

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Abstract. In this paper domination number, vertex-edge domination number and edge-vertex domination are calculated for regular dendrimers.

Keywords. domination, vertex-edge domination, edge-vertex domination.

1 Introduction

Let $G = (V, E)$ be a simple connected graph whose vertex set V and the edge set E. For the open neighborhood of a vertex v in a graph G, the notation $N_G(v)$ is used as $N_G(v) = \{u|(u,v) \in E(G)\}$ and the closed neighborhood of v is used as $N_G[v] = N_G(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$.

A subset $S \subseteq V$ is a dominating set, if every vertex in G either is element of S or is adjacent to at least one vertex in S. The domination number of a graph G is denoted with $\gamma(G)$ and it is equal to the minimum cardinality of a dominating set in G. Fundamental notions of domination theory are outlined in the book [1].

A vertex v ve-dominates an edge e which is incident to v, as well as every edge adjacent to e. A set $S \subseteq V$ is a ve-dominating set if every edges of a graph G are ve-dominated by at least one vertex of S [2, 5]. The minimum cardinality of a ve-dominating set is named with ve-domination number and denoted with $\gamma_{ve}(G)$.

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An edge e ev-dominates a vertex v which is a vertex of e, as well as every vertex adjacent to v [2,5]. A subset $D \subseteq E$ is a edge-vertex dominating set (in simply, ev-dominating set) of G, if every vertex of a graph G are ev-dominated by at least one edge of D. The minimum cardinality of a ev-dominating set is named with ev-domination number and denoted with $\gamma_{ev}(G)$.

We attain three domination type invariants for regular dendrimers.

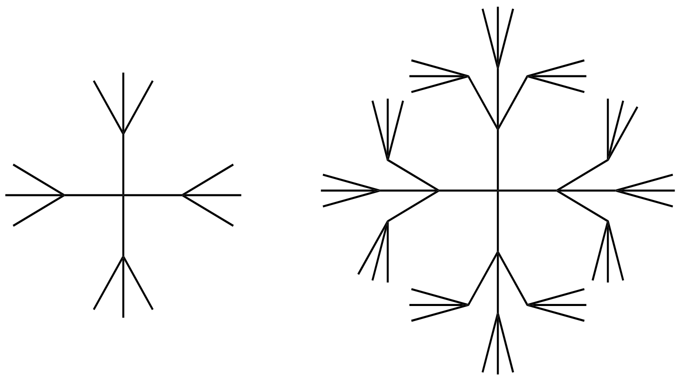

Figure 1. Dendrimers $T_{2,4}$ and $T_{3,4}$.

Dendrimers are highly branched trees [4]. A regular dendrimer $T_{k,d}$ is a tree with a central vertex v. Every non-pendant vertex of $T_{k,d}$ is of degree $d \geq 2$ and the radius is k, distance from v to each pendant vertex. Dendrimers $T_{2,4}$ and $T_{3,4}$ are demonstrated in Figure 1. Some properties of regular dendrimers are denoted in the following lemma [3].

Lemma 1.1. *If $T_{k,d}$ is a tree with central vertex v, then*

i) The order of $T_{k,d}$ is $1 + \dfrac{d[(d-1)^k - 1]}{d-2}$.

ii) $T_{k,d}$ has d branches.

iii) Each branch of $T_{k,d}$ has $\dfrac{(d-1)^k - 1}{d-2}$ vertices.

iv) Each branch of $T_{k,d}$ has $(d-1)^{k-1}$ pendant vertices.

v) Each branch of $T_{k,d}$ has $\dfrac{(d-1)^{k-1} - 1}{d-2}$ nonpendant vertices.

vi) The number of vertices on radius k is $d(d-1)^{k-1}$.

2 Main Results

We remind some well known properties of paths and cycles in the next lemma.

Lemma 2.1. *Let P_n path and C_n cycle with n vertices [5],*

i) $\gamma(P_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil$.

ii) $\gamma_{ve}(P_n) = \gamma_{ev}(P_n) = \lfloor \frac{n+2}{4} \rfloor$.

iii) $\gamma_{ve}(C_n) = \gamma_{ev}(C_n) = \lfloor \frac{n+3}{4} \rfloor$.

Theorem 2.2. *If $T_{k,d}$ be a regular dendrimer, then*

$$\gamma(T_{k,d}) = \begin{cases} 1 + \dfrac{(d-1)^k - d + 1}{d-2}, & k \quad \text{is odd} \\ \dfrac{(d-1)^k - 1}{d-2}, & k \quad \text{is even} \end{cases}.$$

Proof. Let k is odd. In this case minimum cardinality dominating set of $T_{k,d}$ is consisted of central vertex v, vertices on radius $k = 2,4,\ldots,k-1$. Summation of all vertices is by Lemma 1.1 (vi)

$$\gamma(T_{k,d}) = 1 + d(d-1) + d(d-1)^3 + \cdots + d(d-1)^{k-2},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\Big[1 + (d-1)^2 + \cdots + (d-1)^{k-3}\Big].$$

The second term of this equation is a geometric series such that $r = (d-1)^2$. So,

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-3}{2}+1} - 1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-1}{2}} - 1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{(d-1)^{k-1} - 1}{(d-1)^2 - 1},$$

$$\gamma(T_{k,d}) = 1 + \frac{(d-1)^k - d + 1}{d-2}.$$

Now let k is even. In this case minimum dominating set of $T_{k,d}$ is consisted of vertices on radius $k = 1,3,\ldots,k-1$. Therefore,

$$\gamma(T_{k,d}) = d + d(d-1)^2 + d(d-1)^4 + \cdots + d(d-1)^{k-2},$$

$$\gamma(T_{k,d}) = d\Big[1 + (d-1)^2 + (d-1)^4 + \cdots + (d-1)^{k-2}\Big].$$

This equation is a geometric series such that $r = (d-1)^2$ and then,

$$\gamma(T_{k,d}) = d\frac{r^{\frac{k-2}{2}+1} - 1}{r-1},$$

$$\gamma(T_{k,d}) = d\frac{r^{\frac{k}{2}} - 1}{r - 1},$$

$$\gamma(T_{k,d}) = d\frac{(d-1)^k - 1}{(d-1)^2 - 1},$$

$$\gamma(T_{k,d}) = \frac{(d-1)^k - 1}{d-2}.$$

□

Theorem 2.3. *If $T_{k,d}$ be a regular dendrimer, then*

$$\gamma_{ve}(T_{k,d}) = \begin{cases} 1 + \dfrac{(d-1)^{k-1} - d + 1}{d-2}, k & \text{is even} \\ \dfrac{(d-1)^{k-1} - 1}{d-2}, & k \quad \text{is odd} \end{cases}.$$

Proof. A vertex v ve-dominates an edge e which is incident to v, as well as every edge adjacent to e. This means a vertex ve-domianates every edge exist in maximum distance 2 from it. By this way we assume that k is even. In this case minimum ve-dominating set of $T_{k,d}$ is consisted of central vertex v, vertices on radius $k = 2, 4, \ldots, k-2$. Thus,

$$\gamma_{ve}(T_{k,d}) = 1 + d(d-1) + d(d-1)^3 + \cdots + d(d-1)^{k-3},$$

$$\gamma_{ve}(T_{k,d}) = 1 + d(d-1)\Big[1 + (d-1)^2 + \cdots + (d-1)^{k-4}\Big].$$

The second term of this equation is a geometric series such that $r = (d-1)^2$. So,

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-4}{2}+1} - 1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-2}{2}} - 1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{(d-1)^{k-2} - 1}{(d-1)^2 - 1},$$

$$\gamma(T_{k,d}) = 1 + \frac{(d-1)^{k-1} - d + 1}{d-2}.$$

Now let k is odd. In this case ve-dominating set of $T_{k,d}$ is consisted of vertices on radius $k = 1, 3, \ldots, k-2$. Therefore,

$$\gamma_{ve}(T_{k,d}) = d + d(d-1)^2 + d(d-1)^4 + \cdots + d(d-1)^{k-3},$$

$$\gamma_{ve}(T_{k,d}) = d\Big[1 + (d-1)^2 + (d-1)^4 + \cdots + (d-1)^{k-3}\Big].$$

This equation is a geometric series such that $r = (d-1)^2$ and then,

$$\gamma_{ve}(T_{k,d}) = d\frac{r^{\frac{k-3}{2}+1} - 1}{r-1},$$

$$\gamma_{ve}(T_{k,d}) = d\frac{r^{\frac{k-1}{2}} - 1}{r-1},$$

$$\gamma_{ve}(T_{k,d}) = d\frac{(d-1)^{k-1} - 1}{(d-1)^2 - 1},$$

$$\gamma_{ve}(T_{k,d}) = \frac{(d-1)^{k-1} - 1}{d-2}.$$

□

Theorem 2.4. *If $T_{k,d}$ be a regular dendrimer, then*

$$\gamma_{ve}(T_{k,d}) = \gamma_{ev}(T_{k,d}).$$

Proof. We investigate $T_{1,d}$ firstly. The minimum cardinality *ev*-dominating set of $T_{1,d}$ is consisted of one of three edges which is incident the central vertex v. If we take $k = 3$ the minimum cardinality *ev*-dominating set of $T_{3,d}$ is consisted of edges lying between $k = 1$ and $k = 2$. The number of these edges are equal to the pendant vertices of $T_{2,d}$. If we continue like this,*ev*-domination number of $T_{k,d}$ is equal to the *ve*-domination number of $T_{k,d}$ when k is odd.

For the $T_{2,d}$ the minimum cardinality *ev*-dominating set of $T_{2,d}$ is consisted three edges incident the central vertex v. For the $T_{4,d}$ the minimum cardinality *ev*-dominating set is consisted of edges which are incident the v and the edges lying between $k = 2$ and $k = 3$. Number of the second type vertices is equal to the number of pendant vertices of $T_{3,d}$. If we continue like this, *ev*-domination number of $T_{k,d}$ is equal to the *ve*-domination number of $T_{k,d}$ when k even.

□

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