Journal of Discrete Mathematics and Its Applications 7 (3) (2022) 141–146

Journal of Discrete Mathematics and Its Applications

Available Online at: http://jdma.sru.ac.ir

On dominaton type invariants of regular dendrimer

Bünyamin Şahin[1*], Abdulgani Şahin[2]

[1]Bayburt University, Education Faculty, 69000, Bayburt, Turkey

[2]Ağrı İbrahim Çeçen University, Faculty of Science and Letters, Department of Mathematics, 04100, Ağrı, Turkey

Academic Editor: Modjtaba Ghorbani

Abstract. In this paper domination number, vertex-edge domination number and edge-vertex domination are calculated for regular dendrimers.

Keywords: domination, vertex-edge domination, edge-vertex domination
Mathematics Subject Classification (2010): 05C69.

1 Introduction

Let $G = (V, E)$ be a simple connected graph whose vertex set V and the edge set E. For the open neighborhood of a vertex v in a graph G, the notation $N_G(v)$ is used as $N_G(v) = \{u|(u, v) \in E(G)\}$ and the closed neighborhood of v is used as $N_G[v] = N_G(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$.

A subset $S \subseteq V$ is a dominating set, if every vertex in G either is element of S or is adjacent to at least one vertex in S. The domination number of a graph G is denoted with $\gamma(G)$ and it is equal to the minimum cardinality of a dominating set in G. Fundamental notions of domination theory are outlined in the book [1].

A vertex v *ve*-dominates an edge e which is incident to v, as well as every edge adjacent to e. A set $S \subseteq V$ is a *ve*-dominating set if every edges of a graph G are *ve*-dominated by at

*Corresponding author (*Email address*: bsahin@bayburt.edu.tr)
Received 1 September 2022; Revised 7 September 2022; Accepted 17 September 2022
First Publish Date: 1 October 2022

 ©Shahid Rajaee Teacher Training University

least one vertex of S [2,5]. The minimum cardinality of a *ve*-dominating set is named with *ve*-domination number and denoted with $\gamma_{ve}(G)$.

An edge e *ev*-dominates a vertex v which is a vertex of e, as well as every vertex adjacent to v [2,5]. A subset $D \subseteq E$ is a edge-vertex dominating set (in simply, *ev*-dominating set) of G, if every vertex of a graph G are *ev*-dominated by at least one edge of D. The minimum cardinality of a *ev*-dominating set is named with *ev*-domination number and denoted with $\gamma_{ev}(G)$.

We attain three domination type invariants for regular dendrimers.

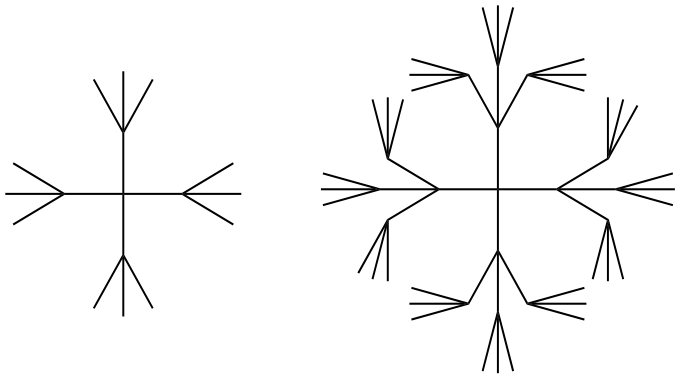

Figure 1. Dendrimers $T_{2,4}$ and $T_{3,4}$.

Dendrimers are highly branched trees [4]. A regular dendrimer $T_{k,d}$ is a tree with a central vertex v. Every non-pendant vertex of $T_{k,d}$ is of degree $d \geq 2$ and the radius is k, distance from v to each pendant vertex. Dendrimers $T_{2,4}$ and $T_{3,4}$ are demonstrated in Figure 1. Some properties of regular dendrimers are denoted in the following lemma [3].

Lemma 1.1. *If $T_{k,d}$ is a tree with central vertex v, then*

i) The order of $T_{k,d}$ is $1 + \dfrac{d[(d-1)^k - 1]}{d-2}$.

ii) $T_{k,d}$ has d branches.

iii) Each branch of $T_{k,d}$ has $\dfrac{(d-1)^k - 1}{d-2}$ vertices.

iv) Each branch of $T_{k,d}$ has $(d-1)^{k-1}$ pendant vertices.

v) Each branch of $T_{k,d}$ has $\dfrac{(d-1)^{k-1} - 1}{d-2}$ nonpendant vertices.

vi) The number of vertices on radius k is $d(d-1)^{k-1}$.

2 Main results

We remind some well known properties of paths and cycles in the next lemma.

Lemma 2.1. *Let P_n path and C_n cycle with n vertices [5],*

i) $\gamma(P_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil$.

ii) $\gamma_{ve}(P_n) = \gamma_{ev}(P_n) = \lfloor \frac{n+2}{4} \rfloor$.

iii) $\gamma_{ve}(C_n) = \gamma_{ev}(C_n) = \lfloor \frac{n+3}{4} \rfloor$.

Theorem 2.2. *If $T_{k,d}$ be a regular dendrimer, then*

$$\gamma(T_{k,d}) = \begin{cases} 1 + \dfrac{(d-1)^k - d + 1}{d-2}, k & \text{is odd} \\ \dfrac{(d-1)^k - 1}{d-2}, & k \quad \text{is even} \end{cases}.$$

Proof. Let k is odd. In this case minimum cardinality dominating set of $T_{k,d}$ is consisted of central vertex v, vertices on radius $k = 2, 4, \ldots, k-1$. Summation of all vertices is by Lemma 1.1 (vi)

$$\gamma(T_{k,d}) = 1 + d(d-1) + d(d-1)^3 + \cdots + d(d-1)^{k-2},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\Big[1 + (d-1)^2 + \cdots + (d-1)^{k-3}\Big].$$

The second term of this equation is a geometric series such that $r = (d-1)^2$. So,

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-3}{2}+1} - 1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-1}{2}} - 1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{(d-1)^{k-1} - 1}{(d-1)^2 - 1},$$

$$\gamma(T_{k,d}) = 1 + \frac{(d-1)^k - d + 1}{d-2}.$$

Now let k is even. In this case minimum dominating set of $T_{k,d}$ is consisted of vertices on radius $k = 1, 3, \ldots, k-1$. Therefore,

$$\gamma(T_{k,d}) = d + d(d-1)^2 + d(d-1)^4 + \cdots + d(d-1)^{k-2},$$

$$\gamma(T_{k,d}) = d\Big[1 + (d-1)^2 + (d-1)^4 + \cdots + (d-1)^{k-2}\Big].$$

This equation is a geometric series such that $r = (d-1)^2$ and then,

$$\gamma(T_{k,d}) = d\frac{r^{\frac{k-2}{2}+1}-1}{r-1},$$

$$\gamma(T_{k,d}) = d\frac{r^{\frac{k}{2}}-1}{r-1},$$

$$\gamma(T_{k,d}) = d\frac{(d-1)^k-1}{(d-1)^2-1},$$

$$\gamma(T_{k,d}) = \frac{(d-1)^k-1}{d-2}.$$

□

Theorem 2.3. *If $T_{k,d}$ be a regular dendrimer, then*

$$\gamma_{ve}(T_{k,d}) = \begin{cases} 1+\dfrac{(d-1)^{k-1}-d+1}{d-2}, & k \quad \text{is even} \\ \dfrac{(d-1)^{k-1}-1}{d-2}, & k \quad \text{is odd} \end{cases}.$$

Proof. A vertex v *ve*-dominates an edge e which is incident to v, as well as every edge adjacent to e. This means a vertex *ve*-domianates every edge exist in maximum distance 2 from it. By this way we assume that k is even. In this case minimum *ve*-dominating set of $T_{k,d}$ is consisted of central vertex v, vertices on radius $k = 2,4,\ldots,k-2$. Thus,

$$\gamma_{ve}(T_{k,d}) = 1 + d(d-1) + d(d-1)^3 + \cdots + d(d-1)^{k-3},$$

$$\gamma_{ve}(T_{k,d}) = 1 + d(d-1)\left[1 + (d-1)^2 + \cdots + (d-1)^{k-4}\right].$$

The second term of this equation is a geometric series such that $r = (d-1)^2$. So,

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-4}{2}+1}-1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{r^{\frac{k-2}{2}}-1}{r-1},$$

$$\gamma(T_{k,d}) = 1 + d(d-1)\frac{(d-1)^{k-2}-1}{(d-1)^2-1},$$

$$\gamma(T_{k,d}) = 1 + \frac{(d-1)^{k-1}-d+1}{d-2}.$$

Now let k is odd. In this case *ve*-dominating set of $T_{k,d}$ is consisted of vertices on radius $k = 1,3,\ldots,k-2$. Therefore,

$$\gamma_{ve}(T_{k,d}) = d + d(d-1)^2 + d(d-1)^4 + \cdots + d(d-1)^{k-3},$$

$$\gamma_{ve}(T_{k,d}) = d\Big[1 + (d-1)^2 + (d-1)^4 + \cdots + (d-1)^{k-3}\Big].$$

This equation is a geometric series such that $r = (d-1)^2$ and then,

$$\gamma_{ve}(T_{k,d}) = d\frac{r^{\frac{k-3}{2}+1} - 1}{r-1},$$

$$\gamma_{ve}(T_{k,d}) = d\frac{r^{\frac{k-1}{2}} - 1}{r-1},$$

$$\gamma_{ve}(T_{k,d}) = d\frac{(d-1)^{k-1} - 1}{(d-1)^2 - 1},$$

$$\gamma_{ve}(T_{k,d}) = \frac{(d-1)^{k-1} - 1}{d-2}.$$

□

Theorem 2.4. *If $T_{k,d}$ be a regular dendrimer, then*

$$\gamma_{ve}(T_{k,d}) = \gamma_{ev}(T_{k,d}).$$

Proof. We investigate $T_{1,d}$ firstly. The minimum cardinality *ev*-dominating set of $T_{1,d}$ is consisted of one of three edges which is incident the central vertex v. If we take $k = 3$ the minimum cardinality *ev*-dominating set of $T_{3,d}$ is consisted of edges lying between $k = 1$ and $k = 2$. The number of these edges are equal to the pendant vertices of $T_{2,d}$. If we continue like this,*ev*-domination number of $T_{k,d}$ is equal to the *ve*-domination number of $T_{k,d}$ when k is odd.

For the $T_{2,d}$ the minimum cardinality *ev*-dominating set of $T_{2,d}$ is consisted three edges incident the central vertex v. For the $T_{4,d}$ the minimum cardinality *ev*-dominating set is consisted of edges which are incident the v and the edges lying between $k = 2$ and $k = 3$. Number of the second type vertices is equal to the number of pendant vertices of $T_{3,d}$. If we continue like this, *ev*-domination number of $T_{k,d}$ is equal to the *ve*-domination number of $T_{k,d}$ when k even.

□

References

[1] T. W. Haynes, S. T. Hedetniemi, P. J. Slater, Fundamentals of domination in graphs, Marcel Dekker, New York, 1998.

[2] J. R. Lewis, S. T. Hedetniemi, T. W. Haynes, G. H. Fricke, Vertex-edge domination, Util. Math. 81 (2010) 193-213.

[3] A. K. Nagar, S. Sriam, On eccentric connectivity index of eccentric graph of regular dendrimer, Math. Comput. Sci. 10 (2016) 229-237.

[4] G. R. Newkome, C. N. Moorefield, F. Vogtle, Dendrimers, dendrons: Concepts, Syntheses, Applications, Wiley-VCH, verlag GmbH and Co. KGaA, 2002.

[5] J. W. Peters, Theoretical, algorithmic results on domination and connectivity, Ph.D. thesis, Clemson University, 1986.

Citation: B. Sahin, A. Sahin, On dominaton type invariants of regular dendrimer, J. Disc. Math. Appl. 7(3) (2022) 141–146.

https://doi.org/10.22061/jdma.2022.842

COPYRIGHTS
©2023 The author(s). This is an open-access article distributed under the terms of the Creative Commons Attribution (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, as long as the original authors and source are cited. No permission is required from the authors or the publishers.