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Prediction of earing in deep drawing of anisotropic aluminum alloy sheet using BBC2003 yield criterion

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Abstract

This paper investigates the earing phenomenon in deep drawing of AA3105 aluminum alloy, experimentally and numerically. Earing defect is mainly attributed to the plastic anisotropy of sheet metal. In order to control such defect, predicting the evolution of ears in sheet metal forming analyses becomes indispensable. In this regard, the present study implements the advanced yield criterion BBC2003. Based on this yield function and the associated flow rule of plasticity, the constitutive model is derived. Accordingly, a user material VUMAT subroutine is developed and adopted in the commercial finite element software ABAQUS/Explicit. Several plane stress loading problems are designed, through which, the accuracy of the developed subroutine is verified. In addition, cylindrical cups of AA3105 aluminum alloy are fabricated using a deep drawing die. The earing defect was clearly observed on the recovered parts. Using the experimentally obtained constants of BBC2003 yield criterion for this alloy in VUMAT, deep drawing of the cylindrical cups was simulated. The results demonstrate that the earing profile can successfully be predicted using BBC2003 yield function.

Nomenclature

a	BBC2003 yield criterion constant
D	Diameter (mm)
DD	Diagonal direction
F	Force (N)
k	BBC2003 Yield function constant
M	BBC2003 Yield function constant
N	BBC2003 Yield function constant
P	BBC2003 Yield function constant
p	Pressure (MPa)
Q	BBC2003 Yield function constant
r	Anisotropy factor
RD	Rolling direction
S	BBC2003 Yield function constant
R	BBC2003 Yield function constant
T	BBC2003 Yield function constant
TD	Transvers direction
Y	Yield stress (MPa)
α	Stresses ratio
Γ	Sub-function of the yield criterion (MPa)
$\Delta\lambda$	Plastic multiplier
ε	Strain
$\bar{\varepsilon}$	Equivalent strain
θ	Angle (deg.)
Λ	Sub-function of the yield criterion (MPa)
μ	Friction coefficient
ν	Poisson's ratio
Φ	Yield function (MPa)
Ψ	Sub-function of the yield criterion (MPa)
σ	Stress (MPa)
$\bar{\sigma}$	Equivalent stress (MPa)
Superscripts	
p	Plastic
e	Elastic

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1. Introduction

In the recent years, computer simulation of sheet metal forming processes has been extensively used not only to predict the capacity of the forming machines but also to predict the formability and potential defects in the formed parts, as well as accurate tooling design. The accumulated experience in this field proves that the adopted constitutive model has a strong influence on the reliability of the obtained numerical results. In this regard, a significant amount of research work has been done for precise mathematical modelling of the mechanical behavior of the materials. In the case of sheet metals, plastic anisotropy has been the main subject of many of these researches [1]. Earing is one of the defects observed in deep drawing of cylindrical cups from rolled metal sheets. The difference in cup wall height which is referred to as earing. The major cause of this phenomenon is plastic anisotropy of sheet metals. Usually, the number of ears are four, nonetheless, the formation of six and eight ears has also been reported [2].

Accurate prediction of the ear evolution in forming anisotropic sheet metals demands implementation of advanced yield criteria. Chung and Shah [3] used Barlat1991 yield criterion to describe the mechanical behavior of a sheet metal in the simulation of free bulging and deep drawing of round cups. They incorporated this yield criterion into ABAQUS through developing a UMAT subroutine. Earing profiles and cup heights and thickness profiles were computed. A good agreement was found between finite element method (FEM) simulations and experiments. Yoon and coworkers [4] introduced a new yield criterion named Yld2004-18, which demanded 18 material constants. It was implemented into FEM simulation of deep drawing of aluminum alloy AA2090-T3 and a fictitious highly anisotropic material via user subroutine UMAT. The capability of the yield criterion in prediction of six and eight ears was approved. Moreover, they presented an analytical approach to calculate position and height of the ears based on anisotropy factor as well as blank and die dimensions. Gawad et al. [5] proposed a new multi-scale model in order to capture the texture-induced evolution of plastic anisotropy. They extended the BBC2008 yield criterion to allow anisotropy evolution and applied it for earing prediction. They concluded that in cup drawing of AA6016-T4 aluminum sheet, anisotropy evolution had only a minor effect on the macroscopic geometry of the cup.

In this article, based on the plane stress BBC2003 yield criterion and the associated flow rule, a VUMAT subroutine was developed and employed in the commercial finite element software ABAQUS/Explicit. The accuracy of the VUMAT was verified through plane stress analyses in which a single 3D shell element was subjected to tensile, shear and combined tensile and shear loading. In these verification analyses, BBC2003 criterion was set equivalent to the von Mises model by appropriate selection of the material parameters. Subsequently, deep drawing of round cups of anisotropic AA3105 aluminum alloy sheet was simulated using BBC2003 constitutive model. The developed VUMAT subroutine was validated by comparing the results of FEM simulation to the experimental counterparts.

2. BBC2003 yield criterion

BBC2003 yield criterion [6] is one of the most promising plane stress yield criteria available for orthotropic sheet materials. This yield function Φ is given as:

$$\Phi(\sigma_{ij}) = \bar{\sigma}(\sigma_{ij}) - Y = 0 \quad (1)$$

in which, σ_{ij} is the plane stress tensor, $Y > 0$ is an arbitrary reference yield stress and $\bar{\sigma} \geq 0$ is the equivalent stress, which is defined by

$$\bar{\sigma} = [a(\Gamma+\Psi)^{2k} + a(\Gamma-\Psi)^{2k} + (1-a)(2\Lambda)^{2k}]^{\frac{1}{2k}} \quad (2)$$

In Eq. (2), $k \in N \geq 1$ and $0 \leq a \leq 1$ are material parameters. Γ, Λ and Ψ are functions of plane stress tensor, which are expressed as follows:

$$\Gamma = \frac{\sigma_{11} + M\sigma_{22}}{2} \quad (3)$$

$$\Psi = \sqrt{\left(\frac{N\sigma_{11} - P\sigma_{22}}{2}\right)^2 + Q^2\sigma_{12}\sigma_{21}} \quad (4)$$

$$\Lambda = \sqrt{\left(\frac{R\sigma_{11} - S\sigma_{22}}{2}\right)^2 + T^2\sigma_{12}\sigma_{21}} \quad (5)$$

In Eqs. (2)-(5), *a, M, N, P, Q, R, S* and *T* are yield criterion constants. The integer exponent *k* is selected in accordance with the crystallographic structure of the metal sheet; *k* = 3 for BCC and *k* = 4 for FCC metals.

3. Development of VUMAT subroutine for BBC2003 yield criterion

In order to incorporate the BBC2003 yield criterion into ABAQUS/Explicit solver, the corresponding VUMAT subroutine needs to be developed. At the beginning of each time increment, total strain increment $\Delta\varepsilon_{ij}$, except strain increment in the thickness direction $\Delta\varepsilon_{33}$, are available from the computed increments of the displacement field. ABAQUS takes care of calculating total strain increment at each material point and passes it to VUMAT at time *t*. The stress state at the end of the time increment Δ*t*, must be determined using the following system of equations:

$$\Phi(\sigma_{ij}, Y) = 0 \quad (6)$$

$$\Delta\sigma_{ij} = C_{ijkl}\Delta\varepsilon_{kl}^{\mathrm{e}} = C_{ijkl}\left(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{\mathrm{p}}\right) \quad (7)$$

$$\Delta\varepsilon_{ij}^{\mathrm{p}} = \frac{\partial\Phi}{\partial\sigma_{ij}}\Delta\lambda \quad (8)$$

$$\Delta\bar{\varepsilon}^{\mathrm{p}} = \frac{\sigma_{kl}\frac{\partial\Phi}{\partial\sigma_{kl}}}{Y}\Delta\lambda \quad (9)$$

$$\Delta Y = \frac{\partial Y}{\partial\bar{\varepsilon}^{\mathrm{p}}}\Delta\bar{\varepsilon}^{\mathrm{p}} \quad (10)$$

In the above-mentioned equations, C_{ijkl} is the elastic stiffness tensor, $\Delta\lambda \geq 0$ is the plastic multiplier, and $\Delta\varepsilon_{ij}^{\mathrm{e}}$, $\Delta\varepsilon_{ij}^{\mathrm{p}}$ and $\Delta\bar{\varepsilon}^{\mathrm{p}}$ are the elastic, plastic and effective plastic strain increments, respectively.

Standard explicit integration methods like classical forward-Euler (CFE) scheme, are fast but suffering inaccuracy in satisfying consistency condition,(6) at the end of each increment. On the contrary, implicit classical backward-Euler (CBE) integration scheme can precisely solve the consistency condition at the end of each increment, but demands the solution of a system of equation by iteration which increases the computational cost. Improved explicit integration techniques can be used to effectively combine the less expensive CFE and accurate CBE algorithms. For instance, Vrh et al. [7] and Halilovič et al. [8] introduced the next increment corrects error (NICE) explicit integration scheme. The application of NICE and higher order NICE method has shown that the fulfilment of the consistency condition during the integration is significantly enhanced in comparison to the CFE scheme. Therefore, in this study, due to its simplicity and accuracy, the NICE technique is used to develop the VUMAT subroutine. In NICE scheme, based on the Taylor power series expansion, the consistency condition,(6) is replaced by

$$\Phi + \mathrm{d}\Phi = 0 \quad (11)$$

Expansion of Eq. (11) in the incremental form yields:

$$\Phi + \frac{\partial\Phi}{\partial\sigma_{ij}}\Delta\sigma_{ij} + \frac{\partial\Phi}{\partial Y}\Delta Y = 0 \quad (12)$$

Now, $\Delta\lambda$ is calculated by replacing Eqs. (6)−(10) into Eq. (12) as:

$$\Delta\lambda = \frac{\Phi + \frac{\partial\Phi}{\partial\sigma_{ij}}C_{ijkl}\Delta\varepsilon_{kl}}{\frac{\partial\Phi}{\partial\sigma_{ij}}C_{ijkl}\frac{\partial\Phi}{\partial\sigma_{kl}} - \frac{\partial\Phi}{\partial Y}\frac{dY}{d\varepsilon_{eq}^{p}}\frac{\sigma_{kl}\frac{\partial\Phi}{\partial\sigma_{kl}}}{Y}} \quad (13)$$

As BBC2003 is a plane-stress yield criterion, the through-thickness strain increment $\Delta\varepsilon_{33}$ cannot be determined directly from flow rule (8). Consequently, for elastic and elastic-plastic deformation states this variable must be calculated separately. For elastic deformation, $\Delta\varepsilon_{33}$ is calculated as:

$$\varepsilon_{33} = \frac{-\upsilon}{1-\upsilon}(\varepsilon_{11} + \varepsilon_{22}) \quad (14)$$

In an elastic-plastic deformation increment, the consistency condition must be fulfilled and the normal stress must be zero. By imposing $\Delta\sigma_{33} = 0$ in Eq. (7), the through-thickness strain increment is obtained as:

$$\Delta\varepsilon_{33} = -\frac{C_{33kl}\Delta\varepsilon_{kl}^{*} - \left(\Phi + \frac{\partial\Phi}{\partial\sigma_{ij}}C_{ijkl}\Delta\varepsilon_{kl}^{*}\right)\Theta\beta}{(C_{3333} - \Theta^2\beta)} \quad (15)$$

where $\Delta\varepsilon_{ij}^{*} = 0$ if $i = j = 3$, otherwise, $\Delta\varepsilon_{ij}^{*} = \Delta\varepsilon_{ij}$. In addition, Θ and β are:

$$\beta = \left(\frac{\partial\Phi}{\partial\sigma_{ij}}C_{ijkl}\frac{\partial\Phi}{\partial\sigma_{kl}} - \frac{\partial\Phi}{\partial Y}\frac{dY}{d\bar{\varepsilon}^{\mathrm{p}}}\frac{\frac{\partial\Phi}{\partial\sigma_{ij}}\sigma_{ij}}{Y}\right)^{-1} \quad (16)$$

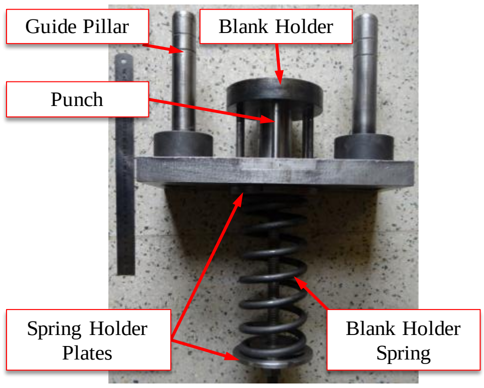
$$\Theta = \frac{\partial\Phi}{\partial\sigma_{ij}}C_{ij33} \quad (17)$$

In the VUMAT subroutine, based on the aforementioned equations, at the end of the increment for each material point, stresses are updated and the plastic strain and equivalent plastic strain are updated and stored as state variables.

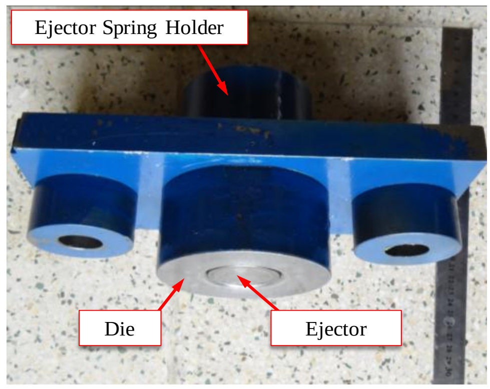
4. Deep drawing experiments

In order to perform the deep drawing experiments, a drawing die with a central blank holder spring was designed and manufactured (Fig. 1). Dimensions of the tooling and blank are given in Fig. 2. For successful die operation, it is essential to adjust the spring force limit using the following equation [9]

$$F_{\mathrm{Holder}} = \frac{\pi}{4}({D_0}^2 - {D_{\mathrm{N}}}^2)p \quad (18)$$



(a)



(b)

Fig. 1. Deep drawing die; (a) upper and (b) lower parts.

In Eq. (18), F_{Holder} is blank holder force, D_0 is the blank diameter, D_{N} is the effective diameter of the contact region between blank and holder and p is the holder pressure. According to for aluminum alloys $1.2 < p < 1.5$ MPa. In the experiments, blank diameter is D_0 = 86 mm, die diameter is 50 mm, punch diameter is 46.1 mm, die profile radius is 6 mm and punch profile radius is 3 mm. A helical spring, which could provide the required holder force, 4.6-5.8 kN, was selected for drawing experiments. Deep drawing experiments were conducted using a single stroke hydraulic press with a stroke speed of 0.1 mm/min. After deep drawing experiments, the wall height of the recovered cup was measured at different orientations; from the rolling direction (RD) to the transverse direction (TD).

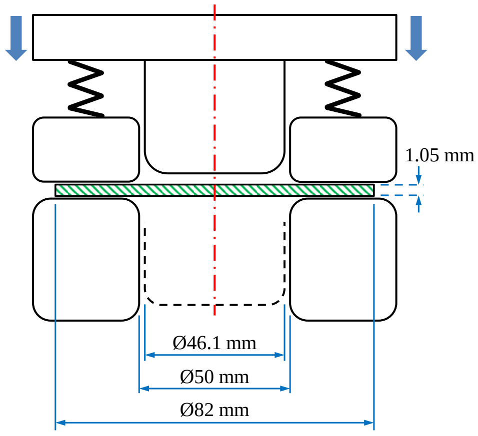


Fig. 2. Schematic view of the deep drawing setup.

5. Simulation of deep drawing

Using the developed VUMAT, deep drawing of 1.05 mm thick AA3105 aluminum alloy sheet was simulated. The parameters of the BBC2003 yield criterion for this material were obtained by the authors [10] using an experimental-numerical approach. These parameters are presented in Table 1. Due to the orthotropic material properties of the aluminum sheet, only a quarter section of the cup with the corresponding symmetry boundary conditions is modeled. A total of 500 S4 shell elements were used for deep drawing analyses. The tooling components are modelled as analytical rigid bodies. To model the blank holder spring force, an axial connector with elastic properties corresponding to the “spring 1” was used. The contact condition is governed by the Coulomb friction model using an assumed coefficient of friction of μ = 0.1.

6. Results and discussion

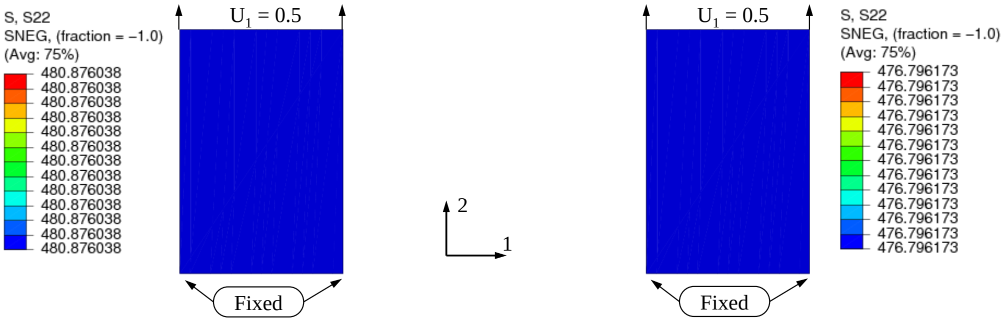
6. 1. Subroutine verification analyses

The accuracy of the developed VUMAT subroutine needs to be examined carefully before applying the code for forming simulations. A feasible verification approach would be to compare its results to a particular case with those obtained from well-established material models available in the ABAQUS library. An important feature of the BBC2003 criterion is that it recovers the standard von Mises criterion if $2a = k = T = S = R = Q = P = N = M = 1$. Using these values, it is possible to assess the accuracy of the developed VUMAT subroutine in comparison to the ABAQUS built-in Mises plasticity model. Therefore, several verification analyses were designed and conducted in which isotropic mechanical behavior is assumed for a single S4 shell element. In these analyses, a square shell element of a unit-length side was subjected to tensile, shear and combined tensile and shear loadings. The element was deformed such that large plastic strain was induced for each load case. Time increment was controlled directly and set at 5e-4 s for all verification analyses. The resulted stresses and plastic strains from developed VUMAT and ABAQUS built-in Mises model are compared at the end of loading step (Figs. (3-8)). From these figures, it is deduced that the results from BBC2003 criterion for isotropic material behavior are in close agreement with those of the ABAQUS Mises material model.
In order to compare the difference between two sets of results, stresses and plastic strains at each integration point of the element for above-mentioned loading cases have been tabulated in Table 2. The differences between two result sets are computed using the following equation:

$$\xi = \left|\frac{\mathrm{VUMAT} - \mathrm{ABQ}}{\mathrm{ABQ}}\right| \times 100 \qquad (19)$$

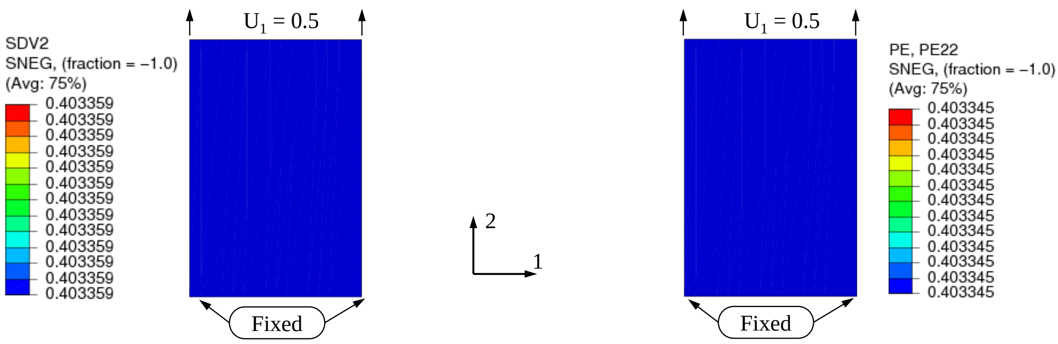
Table 1. BBC2003 yield criterion constants for aluminum alloy AA3105 [7].

k	*a*	*M*	*N*	*P*	*Q*	*R*	*S*	*T*
4	0.48826	1.01934	1.01059	0.98117	0.98196	0.97813	0.94598	0.97889



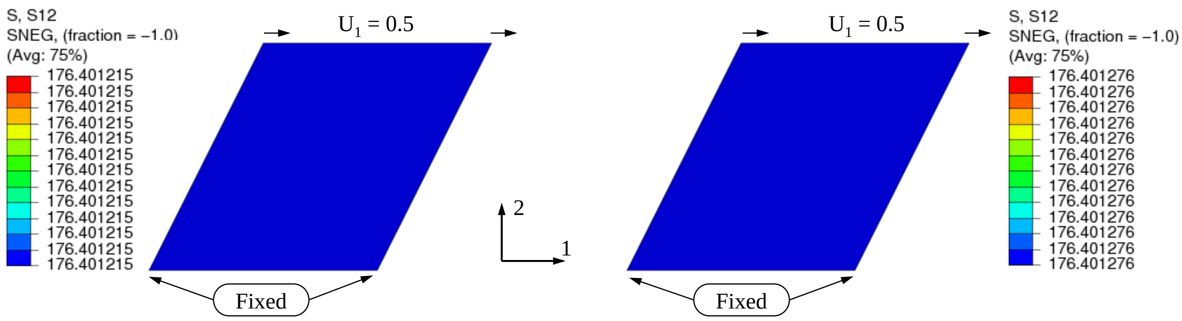
(a) (b)

Fig. 3. Tensile stress in tensile loading; (a) ABAQUS Mises plasticity compared to (b) VUMAT results.



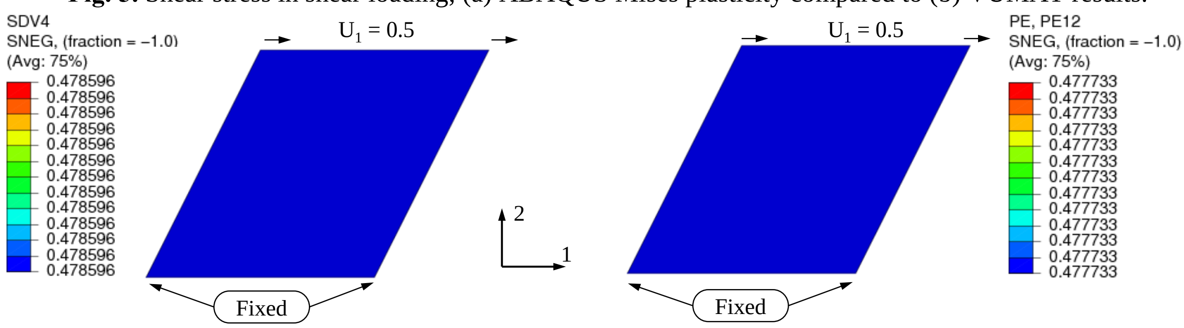
(a) (b)

Fig. 4. Plastic strain in tensile loading; (a) ABAQUS Mises plasticity compared to (b) VUMAT results.



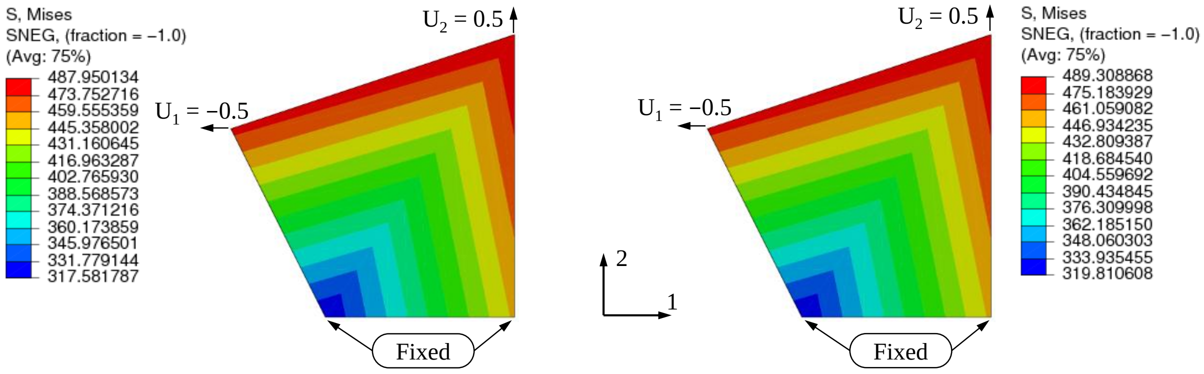
(a) (b)

Fig. 5. Shear stress in shear loading; (a) ABAQUS Mises plasticity compared to (b) VUMAT results.



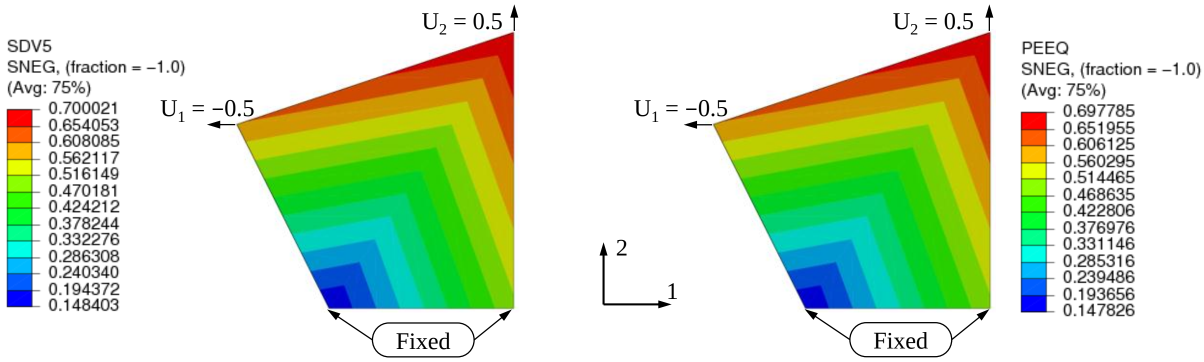
(a) (b)

Fig. 6. Plastic shear strain in shear loading; (a) ABAQUS Mises plasticity compared to (b) VUMAT results.



(a) (b)

Fig. 7. Mises stress in combined tensile and shear loading; (a) ABAQUS Mises plasticity compared to (b) VUMAT results.



(a) (b)

Fig. 8. Equivalent plastic strain in combined tensile and shear loading; (a) ABAQUS Mises plasticity compared to (b) VUMAT results.

Table 2. Single shell element loading results; VUMAT in comparison to ABAQUS Mises plasticity.

Loading type	Integration point number	Stress component				Plastic strain component			
			VUMAT	ABAQUS	ξ(%)		VUMAT	ABAQUS	ξ(%)
Tensile	1, 2, 3, 4	σ_{22}	480.876038	476.796173	0.855683	ε_{22}^{pl}	0.403359	0.403345	0.003471
Shear	1, 2, 3, 4	σ_{12}	176.401215	176.401276	$< 10^{-5}$	ε_{12}^{pl}	0.477696	0.477733	0.007745
Combined tensile and shear	1	σ_{Mises}	331.779144	333.935455	0.645727	$\bar{\varepsilon}$	0.194372	0.193656	0.369728
	2		416.963287	418.684540	0.41111		0.424212	0.422806	0.33254
	3		459.555359	461.059082	0.326305		0.516149	514465	0.32733
	4		473.752716	475.183929	0.301191		0.608085	0.606125	0.323366

The results of the developed VUMAT subroutine in the verification analyses are in close agreement with the ABAQUS CBE integration scheme. The slight difference between the results obtained from the VUMAT and ABAQUS Mises material model should be attributed to the difference in the integration schemes employed. It is possible to decrease these differences by selecting a smaller time increment. For example reducing the time increment by two order of magnitudes, in tensile loading analysis using VUMAT, reduces the relative error in σ_{22} to ξ = 0.011%. However, this higher accuracy is achieved at the expense of more computation cost.

6. 2. Earing prediction

In this subsection, the developed VUMAT subroutine based on the BBC2003 yield criterion is validated. The deformed shape of the deep drawn cup from simulation and experiments are compared in Fig. 9. For clarifying the effectiveness of the BBC2003 advanced yield criterion in the prediction of earing, the same process was simulated using the ABAQUS built-in Hill-48 yield criterion. The details regarding the coefficients used for the Hill-48 can be found in [10]. Fig. 9 reveals the ability of the BBC2003 yield criterion in the prediction of the earing phenomenon. Both experiment and simulation predict the formation of 4 ears on the deep drawn cup.
The differences of cup wall height between RD and TD, obtained from simulations and experiment, are represented in Fig. 10. According to this figure, the predictions of the BBC2003 yield criterion are in good agreement with the experiment results. As shown, Hill-48 quadratic yield function is unable to predict the amplitude of ear correctly. In comparison to Hill-48, the BBC2003 results show an improvement of about 15% in the prediction of the maximum amplitude with respect to the Hill-48.

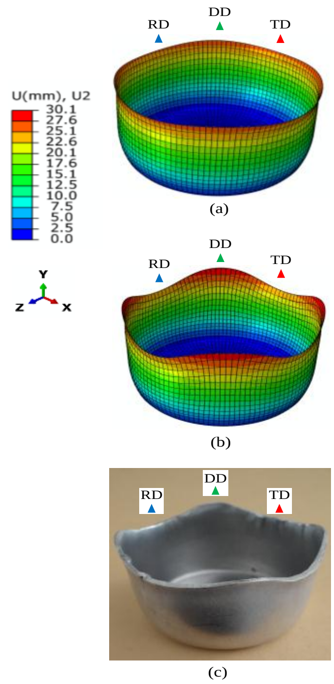


Fig. 9. Earing in deep drawing cups of aluminum alloy AA3105. Analysis prediction using (a) Hill-48 and (b) BBC2003 yield criteria in comparison to (c) experimental result.

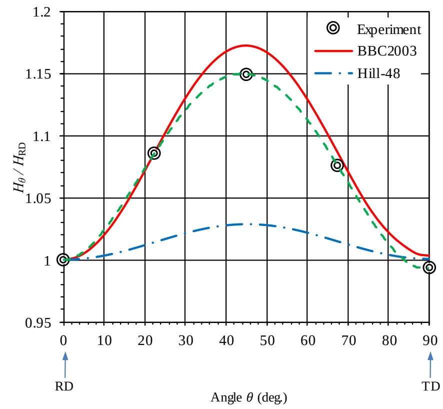


Fig. 10. Earing profile for AA3105 aluminum alloy obtained from experiment and finite element analyses.

7. Conclusions

In this paper, FEM simulation and prediction of earing in deep drawn cylindrical cups of anisotropic aluminum alloy sheet were performed. In this regard, the BBC2003 yield criterion was incorporated into ABAQUS/explicit solver through developing a VUMAT subroutine, considering the associated flow rule of plasticity. The constitutive equations were integrated numerically by applying NICE integration scheme which has been proved to be stable, accurate and computationally efficient. Both the constitutive model and the integration scheme were verified and implemented into ABAQUS/Explicit for further finite element simulations. In order to investigate the capability of this advanced yield criterion in predicting the occurrence of four ears in deep drawing of a round cup, the sheet metal deep drawing process was simulated. Based on the quite good agreement found between the simulated and experimentally obtained results, it can be concluded that the BBC2003 yield criterion is able to predict correctly the mechanical response of anisotropic sheet metals under complex loading conditions.

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