# SIMULTANEOUS ALLOCATION OF RELIABILITY & REDUNDANCY USING MINIMUM TOTAL COST OF OWNERSHIP APPROACH

**G. Kanagaraj***
Department of Mechanical Engineering, Thiagarajar College of Engineering, Madurai, Tamil Nadu, India, 625015

**N. Jawahar**
Depaertment of Mechanical Engineering and Dean (R&D) Thiagarajar College of Engineering, Madurai, Tamil Nadu, India, 625015

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## ABSTRACT
This paper addresses the mixed integer reliability redundancy allocation problems to determine simultaneous allocation of optimal reliability and redundancy level of components based on three objective goals. System engineering principles suggest that the best design is the design that maximizes the system operational effectiveness and at the same time minimizes the total cost of ownership (TCO). To evaluate the performance of the TCO allocation numerical experiments were conducted and compared with previous for the series system, the series-parallel system, the complex (bridge) system and the over speed protection system. From the results of the numerical investigation, reliability redundancy allocation based on minimum TCO will lead to a more reliable, economical design for the manufacturer as well as user compared with the initial cost optimum design and conventional reliability optimum design.

**KEYWORDS:** Reliability and redundancy allocation, Mixed integer non-linear programming, Total cost of ownership

## NOMENCLATURE

| | |
|---|---|
| b | Upper limit on resource |
| $C_s$ | The upper limit on the cost of the system |
| $c_i$ | The cost of each component in subsystem $i$ |
| $C_i$ | Cost function of subsystem $i$ |
| DTC | Down Time Cost |
| $g_i$ | The $i^{th}$ constraint function |
| $n$ | The number of subsystem or stages in the system |
| N | Number of failures over the mission time of $t$. |
| PC | Procurement Cost |
| **r** | $\equiv(r_1,r_2,r_3,\ldots r_n)$, the vector of the component reliabilities for the system |
| RC | Replacement Cost |
| $r_i$ | The reliability of each component in subsystem $i$ |

* To whom correspondence should be addressed. E-mail: gkmech@tce.edu

| | |
|---|---|
| $R_i$ | Reliability function of subsystem *i* |
| $R_s$ | The system reliability |
| t | Design life of the system |
| TCO | Total Cost of Ownership |
| $V_s$ | The upper limit on the sum of the subsystems products of volume & weight |
| $v_i$ | The volume of each component in subsystem *i* |
| $Ws$ | The upper limit on the weight of the system |
| $w_i$ | The weight of each component in subsystem *i* |
| **x** | $\equiv(x_1,x_2,x_3,\ldots x_n)$, the vector of the redundancy allocation for the system |
| $x_i$ | The number of components in subsystem *i* |
| $\beta i$ | Factor for replacement of components of subsystem *i* |
| γ | Annual down time cost |

## INTRODUCTION

In general, designers concentrate to reduce the procurement cost of the system or product to as low as possible to be more competitive in the market, because the price is a widely used determinant for the customer in the selection of product. This criterion often results in poor customer satisfaction. The reason is that the costs to be incurred in the years after purchase may be significant, exceeding initial procurement cost. Due to the highly volatile and competitive nature of the global market, customers (or) users today are closely examining the long term cost of ownership of a system instead of looking only at the lowest procurement cost. Hence, such costs should be included in any purchase decisions. The total cost of ownership (TCO) is a purchasing tool and philosophy to understand the true cost of buying a particular good or service [1]. Gartner's definition [2] of TCO states that TCO consists of the costs incurred throughout the lifecycle of an asset, including acquisition, deployment, operation, support and retirement. TCO models were initially developed by Gartner Research in 1987 and are now widely accepted. Carr and Ittner [3] present an overview of TCO approaches used by several organizations. Handfield and Pannesi [4] explored the concept of TCO specifically for components, using the product life cycle approach.

The system engineering principles suggest that the best design is the design that maximizes the system operational effectiveness and at the same time minimizes the TCO. The strength of TCO is to provide and understand the future costs that may not be apparent when an item is initially purchased [1-5]. TCO can be applied to the government (federal, state, local), industries (automobile, rail industry, semi conductor industry, aerospace, airlines, electronics, information etc.,) and individual business [6-9]. TCO provides many benefits that are documented in the literature [10] and confirmed by case studies. TCO drives the customer to look beyond the initial procurement cost for decision making, and provides a meaningful way to integrate reliability and maintenance strategies with product sales and service offerings [11]. Based on the above considerations, this paper addresses TCO based design strategy to evolve a product or system configuration that corresponds to a minimum TCO in order to provide maximum satisfaction to the customers.

One of the most important cost drivers in the TCO equation is product reliability and it will significantly impact maintenance costs, as well as fixed costs such as downtime [11]. The major cost elements of TCO are: Procurement Cost, Replacement Cost and Down Time Cost. Fig. (1) shows the relationship between the costs of TCO and system reliability.

Cost versus reliability curve for a system exhibits the following features:

- Procurement cost is a monotonic increasing function of reliability
- Replacement cost is a monotonically decreasing function of reliability
- Downtime cost is a monotonically decreasing function of reliability

Let $R_0$ be the reliability of the system that corresponds to minimum TCO. However, the functional or design specification of the system demands certain specified system reliability ($R_s$) which may be either less than $R_0$ or greater than $R_0$. If $R_s \leq R_0$ (Refer *R1* in Fig. (1)) then minimum TCO would correspond to $R_s$, and the optimal reliability becomes $R_s$. On the other hand $R_s \geq R_0$ (Refer *R2* in Fig. (1)), minimum TCO would correspond to *R2* and the optimal reliability becomes *R2*. The above discussions reveal that system reliability $R_s$ influences TCO. Besides, for a multistage series system having redundant components at all stages, the system reliability $R_s$ depends on stage reliability $R_i$ .

i.e., $R_s = f(R_i)$ (1)

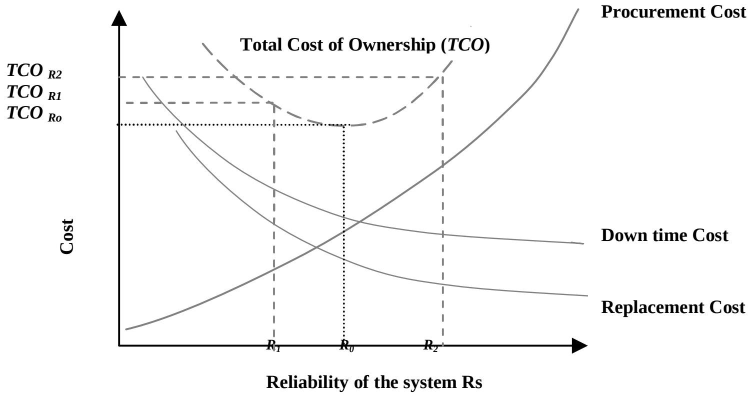


Fig.1. Relationship between reliability and cost elements of TCO.

Suppose each stage *'i'* is built with $x_i$ number of components, then $R_i$ is a function of its component reliability $r_i$ and number of redundant components $'x_i'$:

i.e, $R_i = f(r_i, x_i)$ (2)

The arguments describe that the TCO of the system, thus becomes a function of $r_i$, $x_i$ ( for all *i*) which can be stated as:

$$TCO = f(r_i, x_i) \quad (3)$$

The cost elements of TCO are: Procurement Cost (PC), Replacement Cost (RC), and Down Time Cost (DTC) defined as a function of $r_i$ & $x_i$ as:

$$\textbf{TCO} = \sum_{i=1}^{n} C_i(x_i, r_i) + \sum_{i=1}^{n} \left( \ln\left( \frac{1}{R_i\ (x_i, r_i)} \right) \right) \beta_i\ c_i\ x_i + \left( 1 - \prod_{i=1}^{n} R_i\ (x_i, r_i) \right) t\gamma \quad (4)$$

The first term in the above TCO equation represents the total procurement cost of the system. The second term represents the replacement cost of failure components during the system life time. Failure of all components in any stage *i* leads to system failure. At that instance, all the components in the stage are replaced with new components. The cost of replacing the components in *'i'* is considered as $\beta_i * c_i$. Where $\beta_i$ is the factor to account for the increase in component cost and labor cost that is incurreds during replacements. Suppose the failure time of components at stage *i*, $1 \leq x_i \leq n$ , follows a negative exponential distribution with failure rate $\lambda_i$ and the system is required to operate for a specified time t. Then the reliability of stage *i* is $R_i = e^{\lambda_i t} = e^{-N}$ where N = number of failures over the mission time of *t*. Therefore the number of times, each component '*i*' with '$x_i$' number of components in parallel fails during the system life time '*t*' becomes:

$$\mathbf{N_i} = \left( ln\left( \frac{1}{(1-(1-r_i)^{x_i}\ )} \right) \right) \tag{5}$$

Hence the cost of replacement of failure component is stated as:

$$\mathbf{RC} = \sum_{i=1}^{n} \left( ln\left( \frac{1}{(1-(1-r_i)^{x_i}\ )} \right) \right) \ \beta_i\ c_i\ x_i \qquad\qquad \mathbf{2} \ge \beta_i \ge \mathbf{1} \tag{6}$$

The last term in TCO equation represents the downtime cost to the user since the system is not available for productive work during the failure and replacement time. Therefore, TCO of the system is influenced by the component reliability ($r_i$) and the redundancy level ($x_i$). The mathematical formulation of TCO pertains to the well-known reliability redundancy allocation problem (RRAP) belonging to the class of nonlinear mixed integer programming problems with separable constraints. The mathematical formulation becomes a mixed integer nonlinear programming problem (MINLP) in which the continuous variables represent the component reliabilities and the integer variables represent the level of redundancy. RRAP is the hardest problem in the reliability optimization field because the decision variables are mixed-integer and the system reliability function is nonlinear, non-separable, and non convex [12]. Reliability apportionment encompasses the problem of assigning the correct reliability to each subsystem in such a manner that the overall system reliability is equal to its goal. Once these subsystem reliabilities are established, the designer can select the materials, configurations, and types so that the overall reliability requirement can be achieved.

This paper attempts to evolve a system, based on minimum total cost of ownership approach in order to provide maximum satisfaction to the customers. The number of components $x_i$ and the component reliability $r_i$ of subsystem *i* are the decision variables to be determined for three different conflicting goals namely maximization of system reliability, minimization of system cost, and minimization of system total cost of ownership. The problem template is expressed as:

$$\begin{aligned} &Max\ \ or\ Min = f(\mathbf{r},\mathbf{x}) \\ &subject\ \ to \quad g(\mathbf{r},\mathbf{x}) \le \mathbf{b} \\ &0 \le r_i \le 1 \qquad x_i \in positive\ \ integer\ \ \mathbf{1} \le i \le n \end{aligned} \tag{7}$$

Where $r_i$ and $x_i$ are the reliability and the number of components in the $i^{th}$ subsystem respectively; *f (•)* is the objective function to be maximized or minimized; and *g (•)* is the constraint function and ***b*** is the upper limit on the resource; *n* is the number of subsystems.

The structure of this paper is organized as follows: In the first section the problem statement and corresponding mathematical equations are addressed for the three objective goals. The next section presents some numerical examples and results are presented to compare the performance of TCO based allocation with traditional allocation methods. Final section concludes with some important remarks.

## PROBLEM STATEMENT

This paper considers nonlinearly mixed integer reliability design problems in which both the number of redundancy components and the corresponding reliability of each component in each subsystem are to be decided simultaneously in three different objective goals namely maximization of system reliability, minimization of cost and minimization of total cost of ownership so as to meet the given resource constraints. The optimization problem may appear in the three forms expressed as follows.

**Case 1. Maximization of system reliability (R. based allocation)**

$$\begin{aligned}&Find\ \mathbf{r}\ \&\ \mathbf{x}\ which\ \ maximize\ R_s\\&Max\ \ R_s = f(\mathbf{r},\mathbf{x})\\&subject\ \ to\quad g(\mathbf{r},\mathbf{x}) \le \mathbf{b}\\&0 \le r_i \le 1 \qquad x_i \in positive\ \ integer\ \ \mathbf{1} \le i \le n\end{aligned} \tag{8}$$

**Case 2. Minimization of system cost (C based allocation)**

$$\begin{aligned}&Find\ \mathbf{r}\ \&\ \mathbf{x}\ which\ \ minimize\ C_s\\&Min\ \ C_s = f(\mathbf{r},\mathbf{x})\\&subject\ \ to\quad g(\mathbf{r},\mathbf{x}) \le \mathbf{b}\\&0 \le r_i \le 1 \qquad x_i \in positive\ \ integer\ \ \mathbf{1} \le i \le n\end{aligned} \tag{9}$$

**Case 3. Minimization of total cost of ownership (TCO based allocation)**

$$\begin{aligned}&Find\ \mathbf{r}\ \&\ \mathbf{x}\ which\ \ minimize\ TCO\\&Min\ \ TCO = f(\mathbf{r},\mathbf{x})\\&subject\ \ to\quad g(\mathbf{r},\mathbf{x}) \le \mathbf{b}\\&0 \le r_i \le 1 \qquad x_i \in positive\ \ integer\ \ \mathbf{1} \le i \le n\end{aligned} \tag{10}$$

Where **r** is the reliability vector ($r_1,r_2,\ldots..r_n$) of the system and **x** is the redundancy vector ($x_1,x_2,\ldots\ldots x_n$) of the system respectively; *f (•)* is the objective function to be maximized or minimized; and *g (•)* is the constraint function and *b* is the upper limit on the resource; *n* is the number of subsystems. The goal is to determine the number of the component ( $x_i$ ) and the component reliability ( $r_i$ ) in each subsystem to achieve optimal objective values.

## NUMERICAL EXAMPLES AND DISCUSSION

To evaluate the performance of the proposed TCO based allocation approach from traditional reliability allocation approaches, four mixed integer nonlinear reliability design problems (P1~P4) are solved. These examples are the series system, series-parallel system, complex (bridge) system and overspeed protection system. All the above problems are solved separately in three cases. The mathematical formulations of the four reliability-redundancy problems are furnished below.

### P1. SERIES SYSTEM [Fig. (2(a))]

**Case 1. Maximization of system reliability (R based allocation)**

$$Max\ \ f(\mathbf{r},\mathbf{x}) = \prod_{i=1}^{n} R_i(x_i) \tag{11}$$

Subject to:

$$\sum_{i=1}^{n} v_i * x_i^{\ 2} \le V_s, \qquad \sum_{i=1}^{n} \alpha_i(-1000/\ln r_i)^{\beta_i}\,[x_i + \exp(x_i/4)] \le C_s, \qquad \sum_{i=1}^{n} w_i[x_i * \exp(x_i/4)] \le W_s$$

$0 \le r_i \le 1 \quad x_i \in \text{positive integer},\ 1 \le i \le n$

**Case 2. Minimization of system cost (C based allocation)**

$$Min\ \ C_s = \sum_{i=1}^{n} \alpha_i (-1000 / \ln r_i)^{\beta_i}\ [x_i + \exp(x_i / 4)] \tag{12}$$

Subject to

$$\prod_{i=1}^{n} R_i(x_i) \ge R_s, \qquad \sum_{i=1}^{n} v_i * x_i^2 \le V_s, \qquad \sum_{i=1}^{n} w_i [x_i * \exp(x_i / 4)] \le W_s$$

$0 \le r_i \le 1 \quad x_i \in \text{positive integer},\ 1 \le i \le n$

**Case 3. Minimization of total cost of ownership (TCO based allocation)**

$$Min\ \ TCO = \sum_{i=1}^{n} \alpha_i (-1000 / \ln r_i)^{\beta_i}\ [x_i + \exp(x_i / 4)] + \sum_{i=1}^{n} \left( \ln \left( \frac{1}{(1-(1-r_i)^{x_i})} \right) \right) \beta_i c_i x_i + \left( 1 - \prod_{i=1}^{n} (1-(1-r_i)^{x_i}) \right) t\gamma \tag{13}$$

Subject to:

$$\prod_{i=1}^{n} (1-(1-r_i)^{x_i}) \ge R_s, \qquad \sum_{i=1}^{n} v_i * x_i^2 \le V_s, \qquad \sum_{i=1}^{n} w_i [x_i * \exp(x_i / 4)] \le W_s$$

$0 \le r_i \le 1 \quad x_i \in \text{positive integer},\ 1 \le i \le n$

**P2. SERIES-PARALLEL SYSTEM [Fig. (2(b))]**

**Case 1. Maximization of system reliability (R based allocation)**

$$Max\ \ f(r, x) = 1 - (1 - R_1 R_2)(1 - (1 - R_3)(1 - R_4) R_5) \tag{14}$$

Subject to

$$\sum_{i=1}^{n} v_i * x_i^2 \le V_s,\ \sum_{i=1}^{n} \alpha_i (-1000 / \ln r_i)^{\beta_i}\ [x_i + exp(x_i / 4)] \le C_s,\ \sum_{i=1}^{n} w_i [x_i * exp(x_i / 4)] \le W_s$$

$0 \le r_i \le 1 \quad x_i \in \text{positive integer},\ 1 \le i \le n$

**Case 2. Minimization of system cost (C based allocation)**

$$Min\ \ C_s = \sum_{i=1}^{n} \alpha_i (-1000 / \ln r_i)^{\beta_i}\ [x_i + \exp(x_i / 4)] \tag{15}$$

Subject to

$$1 - (1 - R_1 R_2)(1 - (1 - R_3)(1 - R_4) R_5) \le R_s, \sum_{i=1}^{n} v_i * x_i^2 \le V_s, \sum_{i=1}^{n} w_i [x_i * exp(x_i / 4)] \le W_s$$

$0 \le r_i \le 1 \quad x_i \in \text{positive integer},\ 1 \le i \le n$

**Case 3. Minimization of total cost of ownership (TCO based allocation)**

$$Min\ \mathbf{TCO} = \sum_{i=1}^{n} c_i\,[\,x_i + exp(\,x_i\,/\,4\,)] + \sum_{i=1}^{n}\left(\mathbf{ln}\left(\frac{\mathbf{1}}{R_s}\right)\right)\beta_i\; c_i\; x_i \;+ (1 - R_s)t\gamma \tag{16}$$

Subject to:

$$1-(1-R_1R_2)(1-(1-R_3)(1-R_4)R_5) \le R_s,\ \sum_{i=1}^{n} v_i * x_i^{\,2} \le V_s,\ \sum_{i=1}^{n} w_i\,[\,x_i * exp(x_i\,/\,4)] \le W_s$$

$$0 \le r_i \le 1 \qquad x_i \in \text{positive integer},\ 1 \le\ i \le n$$

**P3. COMPLEX (BRIDGE) SYSTEM [Fig. (2(c))]**

**Case 1. Maximization of system reliability (R based allocation)**

$$\begin{aligned} &Max\ \ f(\mathbf{r},\mathbf{x}) = R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 \\ &- R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \end{aligned} \tag{17}$$

Subject to

$$\sum_{i=1}^{n} v_i * x_i^{\,2} \le V_s,\ \sum_{i=1}^{n} \alpha_i(\,-1000\,/\,ln\,r_i\,)^{\beta_i}\ [\,x_i + exp(\,x_i\,/\,4\,)] \le C_s,\ \sum_{i=1}^{n} w_i\,[\,x_i * exp(\,x_i\,/\,4\,)] \le W_s$$

$$0 \le r_i \le 1 \qquad x_i \in \text{positive integer},\ 1 \le\ i \le n$$

**Case 2. Minimization of system cost (C based allocation)**

$$Min\ \ \mathrm{C_s} = \sum_{i=1}^{n} \alpha_i(-1000/\ln r_i)^{\beta_i}\ [x_i + \exp(x_i\,/\,4)] \tag{18}$$

Subject to

$$\begin{aligned} &R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 \\ &- R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \le R_s \end{aligned}$$

$$\sum_{i=1}^{n} v_i * x_i^{\,2} \le V_s, \qquad \sum_{i=1}^{n} w_i[x_i * \exp(x_i\,/\,4)] \le W_s$$

$$0 \le r_i \le 1 \qquad x_i \in \text{positive integer},\ 1 \le\ i \le n$$

**Case 3. Minimization of total cost of ownership (TCO based allocation)**

$$\mathrm{Min}\ \mathbf{TCO} = \sum_{i=1}^{n} c_i\,[\,x_i + exp(\,x_i\,/\,4\,)] + \sum_{i=1}^{n}\left(\mathbf{ln}\left(\frac{\mathbf{1}}{R_s}\right)\right)\beta_i\; c_i\; x_i \;+ (1 - R_s)t\gamma \tag{19}$$

Subject to

$$\begin{aligned} &R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 \\ &- R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \le R_s \end{aligned}$$

$$\sum_{i=1}^{n} v_i * x_i^{\,2} \le V_s, \qquad \sum_{i=1}^{n} w_i[x_i * \exp(x_i\,/\,4)] \le W_s$$

$$0 \le r_i \le 1 \qquad x_i \in \text{positive integer},\ 1 \le\ i \le n$$

**P4. OVER SPEED PROTECTION SYSTEM**

**Case 1. Maximization of system reliability (R based allocation)**

$$Max\ f(\mathbf{r},\mathbf{x}) = \prod_{i=1}^{n} [1-(1-r_i)^{x_i}] \tag{20}$$

Subject to

$$\sum_{i=1}^{n} v_i * x_i^2 \le V_s, \sum_{i=1}^{n} \alpha_i(-1000/ln(r_i))^{\beta_i}\ [x_i + exp(x_i/4)] \le C_s, \sum_{i=1}^{n} w_i [x_i * exp(x_i/4)] \le W_s$$

$$0.5 \le r_i \le 1-10^{-6},\ 1.0 \le x_i \le 10, \quad x_i \in positive\ integer, \quad r_i \in real\ number$$

**Case 2. Minimization of system cost (C based allocation)**

$$Min\ \mathrm{C_s} = \sum_{i=1}^{n} \alpha_i(-1000/\ln r_i)^{\beta_i}\ [x_i + \exp(x_i/4)] \tag{21}$$

Subject to

$$\prod_{i=1}^{n} [1-(1-r_i)^{x_i}] \le R_s,\ \sum_{i=1}^{n} v_i * x_i^2 \le V_s, \sum_{i=1}^{n} w_i [x_i * exp(x_i/4)] \le W_s$$

$$0.5 \le r_i \le 1-10^{-6},\ 1.0 \le x_i \le 10, \quad x_i \in positive\ integer, \quad r_i \in real\ number$$

**Case 3. Minimization of total cost of ownership (TCO based allocation)**

$$\text{Min TCO} = \sum_{i=1}^{n} \alpha_i(-1000/\ln r_i)^{\beta_i}\ [x_i + \exp(x_i/4)] + \sum_{i=1}^{n}\left(\ln\left(\frac{1}{(1-(1-r_i)^{x_i})}\right)\right)\beta_i\, c_i\, x_i + \left(1-\prod_{i=1}^{n}(1-(1-r_i)^{x_i})\right)t\gamma \tag{22}$$

Subject to

$$\prod_{i=1}^{n} [1-(1-r_i)^{x_i}] \le R_s,\ \sum_{i=1}^{n} v_i * x_i^2 \le V_s, \sum_{i=1}^{n} w_i [x_i * exp(x_i/4)] \le W_s$$

$$0.5 \le r_i \le 1-10^{-6},\ 1.0 \le x_i \le 10, \quad x_i \in positive\ integer, \quad r_i \in real\ number$$

The input parameters defining the specific instances of the four problems have the same values as Kuo et al. [13] Xu et al.[14], Hikita et al. [15], Dhingra [16], Yokota et al. [17] and Chen [18], and are show in Tables (1-3).

The numerical results are shown in Table (4) through (7), for the above problems based on three objective goals. These are reported and compared with solutions reported previously in the literature. Table (4) (P1) shows that solution of series system found by TCO based allocation is better than those reported in literature that consume excess cost initially. Table (5) shows that the solution of series-parallel problem found by minimum TCO approach is better than the solution found by Hikita et al. [15] and Hsieh et al. [19]. Compared with the solutions found by [18] and [20] in Table (6), the solutions found by the proposed approaches are relatively more significantly improved. In Table (7), the solution found by the proposed approach is much better than the previous best known solution by [22], [18] and [20].

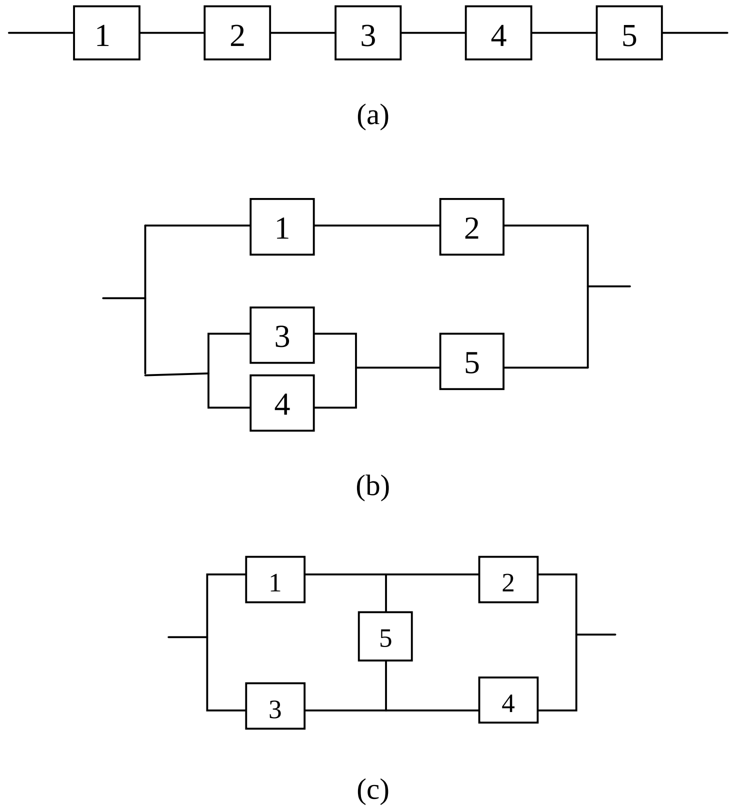


Fig. 2. (a) Series system, (b) series-parallel system and (c) complex (bridge) system.

For measuring the improvement, MPI (maximum possible improvement) can be used to measure the amount of improvement of the solutions found by the proposed approach to the previous best known solutions [20]. MPI is the fraction that the best feasible solution achieved of the maximum possible improvement, it is described as:

$$MPI(\%) = \frac{(R_{s\_TCO} - R_{s\_other})}{(1 - R_{s\_other})}$$

Where $R_{s\_TCO}$ represents the system reliability obtained by the proposed minimum TCO approach and $R_{s\_other}$ represents the system reliability obtained by other approaches in literature. By using the index, it is shown that the proposed TCO based allocation made more improvement in P2 ~ P4.

Based on the results of Table (5), five performance criteria for further comparison between this proposed method of allocation and other 20 combination approaches are defined as follows:

1) Optimality index (OI): the optimal system reliability value.
2) Reliability-cost ratio (RC): the ratio of system reliability to cost
3) Reliability-weight ratio (RW): the ratio of system reliability to the used weight
4) Reliability-product ratio (RP): the ratio of system reliability to the weight and volume of used product
5) Reliability-total cost of ownership (RTCO): the ratio of system reliability to the total cost of ownership.

Table 1. Data used in series system (P1) and complex system (P3).

| $i$ | $10^5\alpha_i$ | $\beta_i$ | $w_iv_i^2$ | $w_i$ | V | C | W |
|---|---|---|---|---|---|---|---|
| 1 | 2.330 | 1.5 | 1 | 7 | 110 | 175 | 200 |
| 2 | 1.450 | 1.5 | 2 | 8 | | | |
| 3 | 0.541 | 1.5 | 3 | 8 | | | |
| 4 | 8.050 | 1.5 | 4 | 6 | | | |
| 5 | 1.950 | 1.5 | 2 | 9 | | | |

Table 2. Data used in series - parallel system (P2).

| $i$ | $10^5\alpha_i$ | $\beta_i$ | $w_iv_i^2$ | $w_i$ | V | C | W |
|---|---|---|---|---|---|---|---|
| 1 | 2.500 | 1.5 | 2 | 3.5 | 180 | 175 | 100 |
| 2 | 1.450 | 1.5 | 4 | 4.0 | | | |
| 3 | 0.541 | 1.5 | 5 | 4.0 | | | |
| 4 | 0.541 | 1.5 | 8 | 3.5 | | | |
| 5 | 2.100 | 1.5 | 4 | 4.5 | | | |

Table 3. Data used in overspeed protection system (P4).

| $i$ | $10^5\alpha_i$ | $\beta_i$ | $w_iv_i^2$ | $w_i$ | V | C | W |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.5 | 1 | 6 | 250 | 400 | 500 |
| 2 | 2.3 | 1.5 | 2 | 6 | | | |
| 3 | 0.3 | 1.5 | 3 | 8 | | | |
| 4 | 2.3 | 1.5 | 2 | 7 | | | |

The comparison results based on the above five performance criteria are shown in Table 9. The results of Table (5) and the optimality index of Table (6) clearly show that the proposed method of allocation gets better system reliability performance and lower TCO than all the other methods by consuming 35.57 % of cost initially. The proposed method of allocation ranks 1 in system reliability and reliability-total cost of ownership ratio, 21 in the reliability-cost ratio, 3 in the reliability-product ratio, and 2 in the reliability-weight ratio, respectively. Numerical examples indicate that among the three proposed objectives, TCO based allocation provides acceptable values for four reliability redundancy allocation design problems considered in this paper.

Table 4. Comparison of solutions obtained with other algorithms for series system (P1).

| | **Hikita et al.[15]** | **Kuo et al. [13]** | **Xu et al. [14]** | **Hsieh et al. [19]** | **Chen [18]** | **Chen [20]** | **R based allocation** | **C based allocation** | **TCO based allocation** |
|---|---|---|---|---|---|---|---|---|---|
| **x** | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) | (3,2,2,3,3) |
| **r** | 0.777143 | 0.77960 | 0.77939 | 0.779427 | 0.779266 | 0.779435 | 0.7793989 | 0.7774428 | 0.8270626 |
| | 0.867514 | 0.80065 | 0.87183 | 0.869482 | 0.872513 | 0.871805 | 0.8718370 | 0.8703728 | 0.9063120 |
| | 0.896696 | 0.90227 | 0.90288 | 0.902674 | 0.902634 | 0.902824 | 0.9028854 | 0.9017688 | 0.9291183 |
| | 0.717739 | 0.71044 | 0.71139 | 0.714038 | 0.710648 | 0.711503 | 0.7114025 | 0.7088759 | 0.7732283 |
| | 0.793889 | 0.85947 | 0.78779 | 0.786896 | 0.788406 | 0.787720 | 0.7877995 | 0.7859155 | 0.8336877 |
| $R_s$ | 0.931363 | 0.92975 | 0.931677 | 0.931578 | 0.931678 | 0.931682 | 0.9316824 | 0.9300001 | 0.9652388 |
| MPI (%) | 49.4% | 50.5% | 49.1% | 49.1% | 49.1% | 49.11% | 49.1% | 50.3% | |
| Slacks of ($g_1$~ $g_3$) | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| | 0.00000 | 0.000010 | 0.013773 | 0.121454 | 0.001559 | 0.625102 | -0.000008 | 2.80809 | -96.65 |
| | 7.518918 | 10.57248 | 7.518918 | 7.518918 | 7.518918 | 7.518918 | 7.518918 | 7.518918 | 7.518918 |

Note: Slack is the unused resources; $MPI\ (\%) = (R_{s\_TCO} - R_{s\_other})/(1-R_{s\_other})$

Table 5. Comparison of solutions obtained with other algorithms for series –parallel system (P2).

| | **Hikita et.al.[15]** | **Hsieh et al. [19]** | **Chen [18]** | **Chen [20]** | **R based allocation** | **C based allocation** | **TCO based allocation** |
|---|---|---|---|---|---|---|---|
| **X** | (3,3,1,2,3) | (2,2,2,2,4) | (2,2,2,2,4) | (2,2,2,2,4) | (2,2,2,2,4) | (2,2,2,2,4) | (2,2,2,2,4) |
| **R** | 0.838193 | 0.785452 | 0.812485 | 0.819178 | 0.8195939 | 0.8201181 | 0.8201181 |
| | 0.855065 | 0.842998 | 0.843155 | 0.844602 | 0.8449464 | 0.8453718 | 0.8453718 |
| | 0.878859 | 0.885333 | 0.897385 | 0.895837 | 0.8955445 | 0.8957704 | 0.8957704 |
| | 0.911402 | 0.917958 | 0.894516 | 0.895151 | 0.8955078 | 0.8957704 | 0.8957704 |
| | 0.850355 | 0.870318 | 0.870590 | 0.868685 | 0.8684653 | 0.8687796 | 0.8687796 |
| $R_s$ | 0.99996875 | 0.99997418 | 0.99997658 | 0.99997665 | 0.9999766 | 0.999977 | 0.999977 |
| MPI (%) | 26.4% | 10.9% | 1.8% | 1.49% | 1.71% | 0 | |
| Slacks of ($g_1$~ $g_3$) | 53 | 40 | 40 | 40 | 40 | 40 | 40 |
| | 0.00000 | 1.194440 | 0.002629 | 0 | 0.0000273 | -0.718034 | -0.718034 |
| | 7.110849 | 1.609289 | 1.609289 | 1.609289 | 1.609289 | 1.609289 | 1.609289 |

Note: Slack is the unused resources; $MPI\ (\%) = (R_{s\_TCO} - R_{s\_other})/(1-R_{s\_other})$

Table 6. Comparison of solutions obtained with other algorithms for complex system (P3).

| | **Hikita et.al. [15]** | **Hsieh et al. [19]** | **Gen &Yun [21]** | **Chen [18]** | **Chen [20]** | **R based allocation** | **C based allocation** | **TCO based allocation** |
|---|---|---|---|---|---|---|---|---|
| **X** | (3,3,2,3,2) | (3,3,3,3,1) | (3,3,3,3,1) | (3,3,3,3,1) | (3,3,3,3,1) | (3,3,2,4,1) | (3,3,3,3,1) | (3,3,2,4,1) |
| **R** | 0.814483 | 0.814090 | 0.808258 | 0.812485 | 0.815878 | 0.8280840 | 0.7454456 | 0.8280864 |
| | 0.821383 | 0.864614 | 0.866742 | 0.867661 | 0.868265 | 0.8578051 | 0.8084230 | 0.8578048 |
| | 0.896151 | 0.890291 | 0.861513 | 0.861221 | 0.859217 | 0.9142417 | 0.7928609 | 0.9142407 |
| | 0.713091 | 0.701190 | 0.716608 | 0.713852 | 0.711529 | 0.6481475 | 0.5929169 | 0.6481462 |
| | 0.814091 | 0.734731 | 0.766894 | 0.756699 | 0.752922 | 0.7041650 | 0.6709976 | 0.7041618 |
| $R_s$ | 0.99978937 | 0.99987916 | 0.999889 | 0.9998892 | **0.9998893** | 0.9998896 | 0.9989999 | **0.9998896** |
| MPI (%) | 47.58% | 8.63% | 0.54% | 0.35% | 0.23% | 0 | 88.96% | |
| Slacks of ($g_1$~ $g_3$) | 18 | 18 | 18 | 18 | 18 | 5 | 18 | 5 |
| | 1.854075 | 0.376347 | | 0.001494 | 0 | -0.00002 | 79.5119 | 0 |
| | 4.264770 | 4.264770 | 4.264770 | 4.264770 | 4.264770 | 1.56047 | 4.2647 | 1.56047 |

Note: Slack is the unused resources; *MPI (%) =* $(R_{s\_TCO} - R_{s\_other})/(1-R_{s\_other})$

Table 7. Comparison of solutions obtained with other algorithms for overspeed system (P4).

| | **Dhingra[16]** | **Yokota et.al [17]** | **Ho Kim & Chang [22]** | **Chen [18]** | **Chen[20]** | **R based allocation** | **C based allocation** | **TCO based allocation** |
|---|---|---|---|---|---|---|---|---|
| **X** | (6,6,3,5) | (3,6,3,5) | (5,5,5,5) | (5,5,5,5) | (5,6,4,5) | (5,6,4,5) | (5,6,4,5) | (5,6,4,5) |
| **R** | 0.81604 | 0.965593 | 0.895644 | 0.903800 | 0.901899 | 0.9016148 | 0.9017563 | 0.9017563 |
| | 0.80309 | 0.760592 | 0.885878 | 0.874992 | 0.849636 | 0.8499211 | 0.8501080 | 0.8501050 |
| | 0.98364 | 0.972646 | 0.912184 | 0.919898 | 0.948071 | 0.9481414 | 0.9482296 | 0.9482296 |
| | 0.80373 | 0.804660 | 0.887785 | 0.890609 | 0.888268 | 0.8882229 | 0.8883835 | 0.8883835 |
| $R_s$ | 0.99961 | 0.999468 | 0.999945 | 0.999942 | **0.999955** | 0.9999547 | **0.999955** | **0.999955** |
| MPI (%) | 88.46% | 91.54% | 18.18% | 22.41% | 0 | 0.6% | 0 | |
| Slacks of ($g_1$~ $g_3$) | 65 | 92 | 50 | 50 | 55 | 55 | 55 | 55 |
| | 0.064 | -70.7336 | 0.9380 | 0.002152 | 0 | -0.000078 | -0.902779 | -0.902779 |
| | 4.348 | 127.5832 | 28.8037 | 28.803701 | 24.8019 | 24.8019 | 24.8019 | 24.8019 |

Note: Slack is the unused resources; *MPI (%) =* $(R_{s\_TCO} - R_{s\_other})/(1-R_{s\_other})$

Table 8. Comparison of results of example 1.

| **Algorithm / Approach** | **TCO** | **System reliability** | **Unused cost** | **Unused P** | **Unused W** |
|---|---|---|---|---|---|
| TCO based allocation | **445.5** | **0.9652388** | -96.65000 | 27 | 7.518918 |
| R based allocation | 516.0 | 0.9316824 | -0.000008 | 27 | 7.518918 |
| Chen. T [20] | 516.6 | 0.9316820 | 0.625102 | 27 | 7.518918 |
| Chen. T [18] | 516.6 | 0.9316780 | 0.001559 | 27 | 7.518918 |
| Gen .M & Yun. Y [21] | 516.6 | 0.9316760 | 0.003352 | 27 | 7.518918 |
| XKL [14] | 516.6 | 0.9316700 | 0.014000 | 27 | 7.519000 |
| Hsieh et al.[19] | 517.0 | 0.9315780 | 0.121454 | 27 | 7.518918 |
| Kim.H.G & Bae. [22] | 517.6 | 0.9314600 | 0.053194 | 27 | 7.518918 |
| G.S.Liu [23] | 518.0 | 0.9313940 | 0.004704 | 27 | 7.518918 |
| Hikita et al [15] | 518.2 | 0.9313630 | 0.000000 | 27 | 7.518918 |
| H-J (Hooke and Jeeves [24]) & K-I (Kohda and Inoue [28]) | 519.9 | 0.9310200 | 0.047000 | 27 | 7.519000 |
| C based allocation | 522.2 | 0.9300001 | 2.808090 | 27 | 7.518918 |
| LMBB (Kuo *et al.* [26]) | 526.2 | 0.9297500 | 0.000001 | 27 | 10.57200 |
| H-J & A-G-M (Tillman *et al.* [27]) | 600.3 | 0.9149400 | 0.033730 | 28 | 1.411800 |
| H-J (Hooke and Jeeves [24]) & S-V (Sharma and Venkateswaran [28]) | 621.6 | 0.9106800 | 0.014000 | 32 | 1.412000 |
| H-J (Hooke and Jeeves 1[24]) & G-A-G (Gopal *et al.* [29]) | 621.6 | 0.9106800 | 0.014000 | 32 | 1.412000 |
| G-A-G (Gopal *et al.* [30])& G-A-G (Gopal *et al.* [29]) | 652.7 | 0.9044600 | 0.025000 | 32 | 4.465000 |
| G-A-G (Gopal *et al.* [30]) & N-N (Nakagawa and Nakashima [31]) | 656.2 | 0.9037600 | 0.020000 | 27 | 7.519000 |
| G-A-G (Gopal *et al.* [30]) & K-I (Kohda and Inoue [25]) | 656.3 | 0.9037300 | 0.004000 | 27 | 4.465000 |
| H-J (Hooke and Jeeves [24]) & N-N (Nakagawa and Nakashima [31]) | 683.5 | 0.8982900 | 0.036000 | 47 | 25.84600 |
| G-A-G (Gopal *et al.* [30])& S-V Sharma and Venkateswaran [28]) | 683.9 | 0.8982100 | 0.013000 | 32 | 4.465000 |

Table 9. Detailed comparisons of results for example 1.

| **Algorithm / Approach** | **OI (Ranking)** | **RC (Ranking)** | **RW (Ranking)** | **RP (Ranking)** | **RTCO (Ranking)** |
|---|---|---|---|---|---|
| TCO based allocation | **0.9652388(1)** | 0.003553244(21) | 0.01162938(3) | 0.005014720(2) | **0.002167(1)** |
| R based allocation | 0.9316824**(2)** | 0.005342982(8) | 0.01122508(7) | 0.004840382(5) | 0.001806(2) |
| Chen. T [20] | 0.9316820**(3)** | 0.005323899(2) | 0.01122509(6) | 0.004840384(4) | 0.001804(3) |
| Chen. T [18] | 0.9316780**(4)** | 0.005323922(7) | 0.01122504(8) | 0.004840361(6) | 0.001803(4) |
| Gen .M & Yun. Y [21] | 0.9316760**(5)** | 0.005323965(6) | 0.01122501(9) | 0.004840351(7) | 0.001803(4) |
| XKL(Xu *et al.* [14]) | 0.9316700**(6)** | 0.005324255(4) | 0.01122494(10) | 0.004840322(8) | 0.001803(4) |
| Hsieh et al. [19] | 0.9315780**(7)** | 0.005327000(3) | 0.01122383(11) | 0.004839842(9) | 0.001802(5) |
| Him. H.G. & Bae. C [22] | 0.9314600**(8)** | 0.005324247(5) | 0.01122241(12) | 0.004839229(10) | 0.001799(6) |
| G.S.Liu [23] | 0.9313940**(9)** | 0.005322394(9) | 0.01122161(13) | 0.004838886(12) | 0.001798(7) |
| Hikita et al. [15] | 0.9313630**(10)** | 0.005322074(10) | 0.01122124(14) | 0.004838725(13) | 0.001797(8) |
| H-J (Hooke and Jeeves [24]) & K-I (Kohda and Inoue [25]) | 0.9310200**(11)** | 0.005321544(11) | 0.01121711(15) | 0.004836945(14) | 0.001791(9) |
| C based allocation | 0.9300001**(12)** | **0.005400951(1)** | 0.01120482(16) | 0.004831644(15) | 0.001781(10) |
| LMBB (Kuo *et al.* [26]) | 0.9297500**(13)** | 0.005312857(12) | 0.01120181(17) | 0.004908197(3) | 0.001767(11) |
| H-J & A-G-M (Tillman *et al.* [27]) | 0.9149400**(14)** | 0.005229236(13) | 0.01115780(18) | 0.004607222(19) | 0.001524(12) |
| H-J (Hooke and Jeeves [24]) & S-V (Sharma & Venkateswaran [28]) | 0.9106800**(15)** | 0.005204302(14) | 0.01167538(2) | 0.004585776(21) | 0.001465(13) |
| H-J (Hooke and Jeeves [24]) & G-A-G (Gopal *et al.* [29]) | 0.9106800**(15)** | 0.005204302(15) | 0.01167538(2) | 0.004585776(22) | 0.001465(13) |
| G-A-G (Gopal *et al.*[30] ) & G-A-G (Gopal *et al.* [29]) | 0.9044600**(16)** | 0.005169081(16) | 0.01159564(4) | 0.004625566(17) | 0.001386(14) |
| G-A-G (Gopal *et al.* [30]) & N-N (Nakagawa and Nakashima [31]) | 0.9037600**(17)** | 0.005164933(17) | 0.01088867(19) | 0.004695321(16) | 0.001377(15) |
| G-A-G (Gopal *et al.* [30]) & K-I (Kohda and Inoue [25]) | 0.9037300**(18)** | 0.005164289(18) | 0.01088831(20) | 0.004621832(18) | 0.001377(15) |
| H-J (Hooke and Jeeves [24])& N-N (Nakagawa and Nakashima[31]) | 0.8982900**(19)** | 0.005134142(19) | **0.01425857(1)** | **0.005158021(1)** | 0.001314(16) |
| G-A-G (Gopal *et al.* [30]) & S-V Sharma and Venkateswaran [28]) | 0.8982100**(20)** | 0.005133010(20) | 0.01151551(5) | 0.004593602(20) | 0.001313(17) |

## CONCLUSIONS

This paper considered minimum total cost of ownership approach for reliability-redundancy problems (RRAP) to determine simultaneous allocation of optimal reliability and redundancy level of components. To evaluate the performance of the proposed approach, numerical experiments were conducted and compared with previous experiments for the series system, the series-parallel system, the complex (bridge) system and the overspeed protection systems. Moreover, the solutions found by the proposed approach can dominate any other methods for the four example problems discussed in the literature. The performance of the proposed method of allocation has been compared with other 20 reliability-redundancy allocation methods. This clearly showed that the proposed approach obtained better system reliability and lowest ownership cost by consuming excess cost initially compared to the other methods, and also performed well in terms of three other criteria. This approach is more suitable, not only for designing a system for high reliability applications but for minimum cost of ownership to the user.

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**REFERENCES**
[1] L. M. Ellram "The total cost of ownership: an analysis approach for purchasing", *International Journal of Physical Distribution & Logistics Management*, Vol. 25, No. 8, pp. 4-23, (1995) .
[2] W. B. Gartner "A framework for describing the phenomenon of new venture creation" Academy of Management Review, Vol. 10, pp 696-706, (1985).
[3] L. P. Carr and C. D. Ittner "Measuring the Cost of Ownership", *Journal of Cost Management*, Vol. 6, No. 3, pp. 7-13. (1992).
[4] R. Handfield and R. Pannesi "Managing component life cycles in dynamic technological environments" *Journal of Purchasing and Materials Management,* Spring pp. 20-27, (1994).
[5] L. M Ellram. and A.B. Maltz "The use of total cost of ownership concepts to model the outsourcing decision", *International Journal of Logistics Management*, Vol. 6, No. 2, pp. 4-23, (1995).
[6] P. N. Pasqua "Power supply procurement, defining the value of your supplier, Total cost of ownership" *Applied Power Electronics Conference and Exposition, 1996, APEC'96 Conference Proceedings,* Vol.1, pp. 47-54, (1996).
[7] E. F. Hitt, Battelle and O. H. Columbus, "Total ownership cost use in management", *Digital Avionics Systems Conference, Proceedings., 17th DASC. The AIAA/IEEE/SAE*, Vol. 1, A32-1-5, (1998).
[8] Sohn and Moon. "How important is reliability in a Total cost of ownership analysis of Clusters" *Techwise Research, Inc November 1999 Version 1.1 ClusterTCO@TechWiseResearch.com.*
[9] S. Castellani and A. Grasso O'Neill J., Tolmie P. "Total cost of ownership: issues around reducing cost of support in a manufacturing organization case", *E-Commerce Technology Workshops,* pp. 122–130, (2005).
[10] L. M. Ellram "A frame work of total cost of ownership", *International Journal of Logistics Management,* Vol. 4, No. 2, pp. 49-60, (1993).
[11] E. R. Carrubba "Integrating life-cycle cost and cost-of-ownership in the commercial sector" *Reliability and Maintainability Symposium, 1992. Proceedings.* Annual, 21-23 Jan., pp 101-108, (1992).
[12] C. Ha and W. Kuo "Multi-Path Approach for Reliability-Redundancy allocation using a scaling method", *Journal of Heuristics,* Vol. 11, No. 3, pp. 201-217, (2005).
[13] W. Kuo, C. L. Hwang and F. A. Tillman "A note on heuristic methods in optimal system reliability", *IEEE Transactions on Reliability,* R 27, pp.320-324, (1978).
[14] Z. Xu, W. Kuo and H. H. Lin "Optimization limits in improving system reliability", *IEEE Transactions on Reliability,* Vol. R-39, pp.51-60, (1990).
[15] M. Hikita, Y. Nakagawa and H. Narihisa "Reliability optimization of systems by a surrogate-constraints algorithm", *IEEE Transactions on Reliability,* Vol. 41,No. 3, Pp. 473-80,(1992).
[16] A. K. Dhingra "Optimal apportionment of reliability & redundancy in series systems under multiple objectives", *IEEE Transactions on Reliability,* Vol. 41, No. 4, Pp.576-582, (1992).
[17] T. Yokota, M. Gen and Y. X. Li "Genetic algorithm for nonlinear mixed-integer programming problems and its application", *Computers and Industrial Engineering,* Vol. 30, No. 4, pp. 905-917, (1996).

[18] T. C. Chen "IAs based approach for reliability redundancy allocation problems", *Applied Mathematics and Computation*, Vol. 182, pp. 1556-1567, (2006).
[19] Y. C. Hsieh, T. C Chen and D. L Bricker "Genetic algorithms for reliability design problems", *Microelectronics Reliability*, Vol. 38, No. 10, pp. 1599-1605, (1998).
[20] T. C. Chen "Penalty Guided PSO for reliability design problems", *PRICAI 2006, LNAI 4099*, pp. 777-786, (2006).
[21] M. Gen and Y. Yun "Soft computing approach for reliability optimization", *Reliability Engineering & System Safety*, Vol. 91, pp.1008-1026, (2006).
[22] K. Ho-Gyun and B. Chang-Ok "Reliability-redundancy optimization using simulated annealing algorithms", *Journal of Quality in Maintenance Engineering*, Vol. 12, No. 4, pp. 354-363, (2006).
[23] G. S. Liu "A combination method for reliability-redundancy optimization", *Engineering Optimization*, Vol. 38, No. 4, pp. 485-499, (2006).
[24] R. Hooke and T. A. Jeeves "Direct search solution of numerical and statistical problems", *Journal of the Association of Computing Machinery*, Vol. 8, No. 2, pp. 212-229, (1961).
[25] T. Kohda and K. Inoue "A reliability optimization method for complex system with the criterion local optimality", *IEEE Transactions on Reliability*, Vol. R-31, Apr, pp. 109-111, (1982).
[26] W. Kuo, H. H. Lin, Z. Xu and W. Zhang "Reliability optimization with the Lagrange-multiplier and branch-and-bound technique", *IEEE Transactions on Reliability*, Vol. 36, No. 5, pp.624-630, (1987).
[27] F. A Tillman, C. L Hwang and W. Kuo "Determining component reliability and redundancy for optimum system reliability, *IEEE Transaction on Reliability*, R 26, No. 3, pp.162-165, (1977).
[28] J. Sharma and K. V. Venkateswaran "A direct method for maximizing the system reliability". *IEEE Transactions on Reliability* R-20: pp 256–9, (1971).
[29] K. Gopal and K. K. Aggarwal "A new method for solving reliability optimization problem, *IEEE Transaction on Reliability*, R 28, pp.36-38, (1978).
[30] K. Gopal, K. K. Aggarwal and J.S. Gupta "A new method for solving reliability optimization problem", *IEEE Transactions on Reliability*, Vol. R-29, Jan, pp 36-38, (1980).
[31] Y. Nakagawa and K. Nakashima "A heuristic method for determining optimal reliability allocation", *IEEE Transactions on Reliability*, Vol. 26, No. 3, pp 156–161, (1977).