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Research Paper

Comparison of two methods for calculating ranking points using transitive triads

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Abstract. To date, the feasibility of constructing a complete graph invariant in polynomial time remains uncertain. Therefore, developing fast algorithms for checking non-isomorphism, including heuristic ones, is crucial. Successful implementation of these heuristics involves modifying existing graph invariants and creating new ones, both of which are still pertinent. Many existing invariants enable the distinction of a large number of graphs in real time. This paper introduces an invariant specifically for tournaments, a type of directed graph. Tournaments are interesting because the number of different tournaments, given a fixed order of vertices, matches the number of undirected graphs with the same fixed order. The proposed invariant considers all possible tournaments formed by subsets of vertices from the given digraph with the same set of arcs. For each subtournament, standard places are calculated and summed to determine the final vertex points, which constitute the new invariant. Our calculations reveal that the new invariant differs from the most natural tournament invariant, which assigns points to each participant. Initial computational experiments show that the smallest pair correlation between sequences representing these two invariants is observed at dimension 15.

Keywords. graph, directed graph, tournament, invariant, transitive triads.
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1 Introduction

Usually, a graph invariant is a value or an ordered set of values that somehow characterizes the structure of the given graph and does not depend on the way of ordering of the vertices. In the study of graph invariants, one notable example is the Graovac-Ghorbani (ABC_{GG}) index [1], a topological descriptor that offers improved predictive power compared to similar descriptors. There is a lot of research on tournaments [2, 3], and our focus here is on the study of tournaments [4].

In the previous paper "On an Invariant of Tournament Digraphs [5]," we proposed a potential invariant: the relative ranking points of participants remain consistent when calculated using two different methods (i.e., if two different ranking points are obtained using the first method, they remain different and maintain the same relative order when obtained using the second method). This paper will continue the exploration of this potential invariant, studying how the relative relationships between different ranking points change as the transitive triples in the same tournament digraph are sequentially altered, and how the correlation coefficient between the two sets of ranking points will vary.

2 Summary of "On an Invariant of Tournament Digraphs"

2.1 Introduction

Suppose there is a game and n players participate in it. The rules of the game are as follows:

- A two-on-two duel between the participating players;
- The result of each duel is only the winner, there is no draw;
- After each duel, the points of the winning player will increase by one point, and the points of the losing player will remain the same;
- Every player has to fight against all players except himself.

Ranking Points Rule

After the game is over, there are two methods to calculate the ranking points for the players:

1. **Standard Ranking Points Calculation:**
    - The player with the most points will get n ranking points;
    - The player who gets the second most points will get $n-1$ ranking points, and so on;
    - If there are two or more players with the same number of points, each player is given the average number of ranking points in their ranking range.

2. **Alternative Ranking Points Calculation:**
    - Think of the players in the entire game as a set, then any k players in it form a subset;
    - For each possible subset (except for a subset that contains only one player or does not contain any players), corresponding points are awarded according to the duel situation between the contestants in the subset (the duel situation comes from the complete competition), the points rules are the same as the previous points rules, and according to the newly obtained points, each participant is awarded the corresponding ranking points according to the previous rules for granting ranking points;
    - Add the ranking points obtained from each subset as new points to the corresponding players;
    - Finally, each player is given the ranking points obtained by processing the newly obtained points in the same way as the old ranking points.

2.2 Results and Discussion

We conducted computations on 1,000,000 different tournament graphs (with the number of participants not exceeding 15) to calculate ranking points using two mentioned methods. In these results, it was observed that the relative ranking points of participants remain consistent when calculated using two different methods. Furthermore, the correlation coefficient can be zero when the number of participants is odd. This is because there exist tournament graphs where each participant wins an equal number of matches when the number of participants is an odd number.

Furthermore, we propose a theory to demonstrate that the relative ranking points of participants remain consistent when calculated using two different methods, thus establishing it as an invariant. The theory is formulated as follows: Suppose that in any duel of n contestants, the person who gets the points K (where $K > 1$), on average, will have the new points at least 2^{n-2} points more than the new points of the person who gets the points $K - 1$.

Through these explorations, we believe that the consistency of relative ranking points of participants when calculated using two different methods represents a potential invariant. In this paper, we will further explore how this relationship changes when changing transitive triads and how the correlation between the ranking points calculated using the two mentioned methods changes, including identifying the minimum value.

3 Research Plan

In our previous study [5], we explored a potential invariant: the relative ranking points of participants remain consistent when calculated using two different methods. We validated this invariant by randomly generating 1,000,000 tournaments through a program and calculating the ranking points for each graph using these methods.

Now, we consider the role of transitive triads in this invariant. There are four ways to change a transitive triad:

1. Change the direction of the first edge, keeping other edges unchanged.
2. Change the direction of the second edge, keeping other edges unchanged.
3. Change the direction of the third edge, keeping other edges unchanged.
4. Change the direction of all edges.

Here, changing the direction of edges means altering the win-loss results of the participants' duels. The uniqueness of the fourth change method lies in the fact that the total number of wins for each participant remains unchanged (i.e., the ranking points calculated using the first method do not change), but the ranking points calculated using the second method may change. The reason is as follows:

When changing the direction of all edges in a transitive triad, even though the total number of wins for each participant remains unchanged, some subtournament may appear that are affected when calculating ranking points using the second method. A subtournament refers to a subgraph formed by selecting some nodes and their connecting edges from the entire Tournament. These subtournament may affect the ranking points calculated using the second method.

Specifically, during the calculation of the second ranking method, there exist subtournament that only contain the two participants involved in the fourth change. In these subtournament, the total number of wins for these participants changes, which alters their ranking points calculated by the first method. This change may not be compensated by other subtournament, as each ranking point and its corresponding number of wins are only correlated within the current subtournament graph, not equal. Therefore, this method may impact the overall ranking points.

Therefore, we plan to write a program that randomly generates numerous tournaments, identifies all transitive triads in each graph, and performs the four mentioned transformations on each triad. After each transformation, we will check if the invariant (the relative ranking points of participants remain consistent when calculated using two different methods) holds. At the same time, we will calculate the modified Kendall correlation coefficient. This correlation coefficient effectively reflects the relationship we need to study in this paper.

The calculation method for the modified Kendall correlation coefficient is as follows:

1. Pair all participants, resulting in $n(n+1)/2$ pairs, each referred to as a pair.
2. Classify these pairs into three categories:
   (a) First category: One participant's ranking points calculated by both methods are higher than the other participant's corresponding ranking points.

(b) Second category: One participant's ranking points calculated by the first method are higher than the other participant's ranking points, but the second method gives the opposite result.

(c) Third category: Both participants have the same ranking points calculated by either method.

Let n_1 be the number of pairs in the first category, n_2 the number of pairs in the second category, and n_3 the number of pairs in the third category. The formula for the modified Kendall correlation coefficient is:

$$\sigma = \frac{n_1 - n_2 + 0.5 \cdot n_3}{n(n+1)/2}$$

This correlation coefficient effectively reflects the relationship we need to study, because it comprehensively considers the consistency and differences between the different ranking methods. The number of pairs in the first category n_1 and the number of pairs in the third category n_3 reflect the consistency of the ranking results calculated by both methods, while the number of pairs in the second category n_2 reflects the opposite results of the two methods. Therefore, the coefficient can comprehensively reflect the relationship between the two calculation methods.

Evidently, if the relationship is invariant, the minimum value of this coefficient will be 0.5, as n_2 equals 0. Moreover, when the number of participants is odd and each participant wins an equal number of duels, there exists a type of tournament that will take this minimum value.

4 Research Method

To analyze the effect of the four transformations on transitive triads concerning the correlation coefficient and the potential invariant, we have enhanced the program used in the previous paper [5] by incorporating three essential functions: a function to identify transitive triads, a function to perform the four transformations, and a function to compute the Modified Kendall Correlation Coefficient. Below, we provide a concise explanation of these functions.

4.1 Identifying Transitive Triads

The function for identifying transitive triads employs three nested loops. Each loop is responsible for finding one edge of the triad. A crucial detail in this process is starting the second and third loops at the current vertex value of the first loop plus one. This approach is necessary because transitive triads can exist in two forms: small $\rightarrow$ medium $\rightarrow$ large $\rightarrow$ small and small $\rightarrow$ large $\rightarrow$ medium $\rightarrow$ small. To identify all transitive triads without repetition, the starting values of the second and third loops are adjusted accordingly.

```
bool tour::FindTriangleCycle() {
    for (int i = triangleCycle[0]; i < nDim; i++) {
        for (int j = (i == triangleCycle[0] ? triangleCycle[1] : i + 1); j < nDim; j++) {
            if (i != j && matrix[i][j]) {
                for (int k = (i == triangleCycle[0] && j == triangleCycle[1] ? triangleCycle[2]+1 : i + 1); k < nDim; k++) {
                    if (k != i && k != j && matrix[j][k] && matrix[k][i]) {
                        bool skip = false;
                        int size = skipElements.size();
                        for (int l = 0; l < size; l++) {
                            if (skipElements[l] == i || skipElements[l] == j || skipElements[l] == k) {
                                skip = true;
                                break;
                            }
                        }
                        if (!skip) {
                            triangleCycle[0] = i;
                            triangleCycle[1] = j;
                            triangleCycle[2] = k;
                            return true;
                        }
                    }
                }
            }
        }
    }
    return false;
}
```

Figure 1. Function to Identify Transitive Triads

4.2 Performing the Four Transformations

The function for performing the four transformations includes a parameter `type` that determines the specific transformation to execute. The transformations are defined as follows:

1. Change the direction of the first edge, keeping other edges unchanged.
2. Change the direction of the second edge, keeping other edges unchanged.
3. Change the direction of the third edge, keeping other edges unchanged.
4. Change the direction of all edges.

```
void tour::ReverseTriangleCycle(int type) {
    int i = triangleCycle[0];
    int j = triangleCycle[1];
    int k = triangleCycle[2];
    if (type == 0 || type == 3) {
        matrix[j][i] = matrix[i][j];
        matrix[i][j] = !matrix[i][j];
    }
    if (type == 1 || type == 3) {
        matrix[k][j] = matrix[j][k];
        matrix[j][k] = !matrix[j][k];
    }
    if (type == 2 || type == 3) {
        matrix[i][k] = matrix[k][i];
        matrix[k][i] = !matrix[k][i];
    }
}
```

Figure 2. Function to Perform the Four Transformations

4.3 Computing the Modified Kendall Correlation Coefficient

The function to compute the Modified Kendall Correlation Coefficient uses dynamic programming to generate all pairwise combinations of participants. For each pair, the function calculates the product of the differences in ranking points obtained by the two methods and determines the sign of the product. The sign categorizes the pairs into three classes, which are then used to update the numerator of the correlation coefficient formula. The final coefficient is derived by dividing the numerator by the denominator.

```
void tour::cal_corr() {
    invariant = true;
    int number[2] = { 0, 0 };
    double sum = 0;
    CombiInit(2, number);
    for (;;) {
        double product = (rankingpoint1[number[0]]-rankingpoint1[number[1]])*(rankingpoint2[number[0]]-rankingpoint2[number[1]]);
        if (product > 0)sum += 1;
        else if (product < 0) {
            sum -= 1; invariant = false;
        }
        else sum += 0.5;
        if (!Combi_next(2, nDim - 1, number))break;
    }
    int tmp;
    tmp = nDim * (nDim - 1) / 2;
    corr = sum / tmp;
}
```

Figure 3. Function to Compute the Modified Kendall Correlation Coefficient

Using these functions, we subsequently analyzed transitive triads in 10,000 randomly generated tournaments, each containing between 11 and 17 participants.

5 Results and Summary

This section presents the results obtained from analyzing 10,000 tournaments. No examples violating the potential invariant (the relative ranking points of participants remain consistent when calculated using two different methods) were found. Additionally, examples with a correlation of 0.5 were found in tournaments of sizes 11, 13, 15, and 17. Examples with a correlation near 0.57 were also found in tournaments of sizes 12 and 14.

5.1 Examples with a Correlation of 0.5

Examples with a correlation of 0.5 were found in tournaments of sizes 11, 13, 15, and 17. These examples show that the Modified Kendall Correlation Coefficient can indeed reach 0.5 under certain conditions.

The following figures show tournaments of sizes 11, 13, 15, and 17, where the correlation coefficient reaches 0.5 after transforming specific transitive triads.

Size	i	j	k	Type 1	Type 2	Type 3	Type 4
11	7	10	9	0.736364	0.500000	0.736364	0.672727
13	1	3	6	0.647436	0.647436	0.647436	0.500000
15	3	5	4	0.680952	0.500000	0.680952	0.628571
17	5	7	6	0.661765	0.500000	0.661765	0.613971

Table 1. Triangle Cycle and Modified Kendall Correlation Coefficient for Tournament. Here, i, j, and k represent the indices of the participants in the tournament. Type 1, Type 2, Type 3, and Type 4 indicate the Modified Kendall Correlation Coefficients under four different transformations.

```
11 0 The size of the tournaments; is it complete?
10 1 2 3 4 5 6 7 8 11 9 participant numbers#

6 5 5 5 5 5 5 5 5 5 4 old points#
11 6 6 6 6 6 6 6 6 6 1 old scores#

4561 3619 3551 3591 3577 3589 3567 3589 3567 3579 2623 new points#
11 10 2 9 5 7.5 3.5 7.5 3.5 6 1 new scores#

tournament matrix#
0 1 1 1 0 1 0 0 0 1 1
0 0 0 1 1 1 0 1 1 0 0
0 1 0 1 1 0 1 0 0 0 1
0 0 0 0 1 0 1 1 1 0 1
1 0 0 0 0 1 1 1 0 0 1
0 0 1 1 0 0 1 1 0 1 0
1 1 0 0 0 0 0 0 1 1 1
1 0 1 0 0 0 1 0 1 1 0
1 0 1 0 1 1 0 0 0 1 0
0 1 1 1 1 0 0 0 0 0 1
0 1 0 0 0 1 0 1 1 0 0

correlation#
0.672727
```

Figure 4. Tournament of size 11 where modifying specific transitive triads (in a specific manner) results in a correlation of 0.5.

```
13 0 The size of the tournaments; is it complete?
1 2 3 4 5 6 7 8 9 10 11 12 13 participant numbers#

6 6 6 6 6 6 6 6 6 6 6 6 6 old points#
7 7 7 7 7 7 7 7 7 7 7 7 7 old scores#

16383 16323 16426 16383 16383 16519 16340 16283 16371 16419 16359 16395 16395 new points#
7 2 12 7 7 13 3 1 5 11 4 9.5 9.5 new scores#

tournament matrix#
0 0 1 1 1 0 0 0 1 0 1 1 0
1 0 1 0 0 0 0 0 0 1 1 1 1
0 0 0 1 1 1 1 1 0 0 1 0 0
0 1 0 0 0 1 1 1 1 0 0 1 0
0 1 0 1 0 1 1 1 0 0 0 0 1
1 1 0 0 0 0 1 1 1 1 0 0 0
1 1 0 0 0 0 0 0 1 1 0 1 1
1 1 0 0 0 0 1 0 1 1 1 0 0
0 1 1 0 1 0 0 0 0 0 1 1 1
1 0 1 1 1 0 0 0 1 0 0 0 1
0 0 0 1 1 1 1 0 0 1 0 1 0
0 0 1 0 1 1 0 1 0 1 0 0 1
1 0 1 1 0 1 0 1 0 0 1 0 0

correlation#
0.5
```

Figure 5. Tournament of size 13 where modifying specific transitive triads (in a specific manner) results in a correlation of 0.5.

```
15 0 The size of the tournaments; is it complete?
5 1 2 3 6 7 8 9 10 11 12 13 14 15 4 participant numbers#

8 7 7 7 7 7 7 7 7 7 7 7 7 7 6 old points#
15 8 8 8 8 8 8 8 8 8 8 8 8 8 1 old scores#

91767 73631 73575 73679 73631 73803 73707 73331 73887 73887 73767 74019 73995 73727 55499 new points#
15 4.5 3 6 4.5 10 7 2 11.5 11.5 9 14 13 8 1 new scores#

tournament matrix#
0 0 0 0 1 0 1 0 1 0 1 1 1 1 1
1 0 0 1 1 1 1 0 0 0 0 0 1 0 1
1 1 0 1 0 0 0 1 1 0 0 0 0 1 1
1 0 0 0 0 0 1 1 1 0 1 0 1 1 0
0 0 1 1 0 0 0 1 1 1 0 1 0 0 1
1 0 1 1 1 0 0 1 0 0 1 0 0 1 0
0 0 1 0 1 1 0 0 1 1 1 1 0 0 0
1 1 0 0 0 0 1 0 1 1 0 1 0 0 1
0 1 0 0 0 1 0 0 0 1 1 1 1 1 0
1 1 1 1 0 1 0 0 0 0 0 0 1 0 1
0 1 1 0 1 0 0 1 0 1 0 1 0 0 1
0 1 1 1 0 1 0 0 0 1 0 0 0 1 1
0 0 1 0 1 1 1 1 0 0 1 1 0 0 0
0 1 0 0 1 0 1 1 0 1 1 0 1 0 0
0 0 0 1 0 1 1 0 1 0 0 0 1 1 0

correlation#
0.628571
```

Figure 6. Tournament of size 15 where modifying specific transitive triads (in a specific manner) results in a correlation of 0.5.

```
17 0 The size of the tournaments; is it complete?
7 1 2 3 4 5 8 9 10 11 12 13 14 15 16 17 6 participant numbers#

9 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 7 old points#
17 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 1 old scores#

406271 328207 328319 329519 328031 327407 327903 327487 325663 326175 327535 329103 327559 327583 327599 325255 250927 new points#
17 13 14 16 12 5 11 6 3 4 7 15 8 9 10 2 1 new scores#

tournament matrix#
0 1 1 1 0 0 1 1 0 0 0 1 0 0 1 1 1
0 0 0 1 1 1 1 0 1 1 0 0 0 1 1 0 0
0 1 0 0 1 0 0 0 1 1 0 1 1 1 0 0 1
0 0 1 0 1 1 0 1 0 0 1 1 1 0 1 0 0
1 0 0 0 0 1 0 0 1 1 0 0 0 1 1 1 1
1 0 1 0 0 0 1 1 0 0 1 1 1 1 0 0 0
0 0 1 1 1 0 0 0 1 0 1 0 1 1 0 0 1
0 1 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0
1 0 0 1 0 1 0 0 0 1 1 0 0 0 1 1 1
1 0 0 1 0 1 1 1 0 0 0 0 0 1 0 1 1
1 1 1 0 1 0 0 0 0 1 0 0 1 0 1 1 0
0 1 0 0 1 0 1 1 1 1 1 0 1 0 0 0 0
1 1 0 0 1 0 0 1 1 1 0 0 0 1 0 1 0
1 0 0 1 0 0 0 0 1 0 1 1 0 0 1 1 1
0 0 1 0 0 1 1 1 0 1 0 1 1 0 0 0 1
0 1 1 1 0 1 1 0 0 0 0 1 0 0 1 0 1
0 1 0 1 0 1 0 1 0 0 1 1 1 0 0 0 0

correlation#
0.613971
```

Figure 7. Tournament of size 17 where modifying specific transitive triads (in a specific manner) results in a correlation of 0.5.

5.2 Examples with a Correlation Near 0.57

Examples with a correlation near 0.57 were found in Tournaments of sizes 12 and 14. Although these examples did not reach 0.5, they are very close.

The following figures show Tournaments of sizes 12 and 14, where the correlation coefficient reaches 0.57 after transforming specific transitive triads.

Size	i	j	k	Type 1	Type 2	Type 3	Type 4
12	1	2	7	0.780303	0.780303	0.583333	0.727273
14	1	5	12	0.747253	0.571429	0.747253	0.697802

Table 2. Triangle Cycle and Modified Kendall Correlation Coefficient for Tournament

```
12 0 The size of the tournaments; is it complete?
7 2 3 5 6 8 9 10 11 12 1 4 participant numbers#

7 6 6 6 6 6 6 6 6 6 5 0 old points#
12 7 7 7 7 7 7 7 7 7 2 1 old scores#

10147 8179 8217 8243 8171 8215 8163 8141 8221 8169 6235 2047 new points#
12 7 9 11 6 8 4 3 10 5 2 1 new scores#

tournament matrix#
0 0 0 1 0 1 1 1 0 1 1 1
1 0 0 0 1 1 0 0 1 1 0 1
1 1 0 0 0 1 0 1 0 1 0 1
0 1 1 0 0 0 1 0 1 0 1 1
1 0 1 1 0 0 0 0 1 0 1 1
0 0 0 1 1 0 1 1 0 1 0 1
0 1 1 0 1 0 0 0 1 0 1 1
0 1 0 1 1 0 1 0 0 1 0 1
1 0 1 0 0 1 0 1 0 0 1 1
0 0 0 1 1 0 1 0 1 0 1 1
0 1 1 0 0 1 0 1 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0

correlation#
0.727273
```

Figure 8. Tournament of size 12 where transforming certain transitive triads (in a specific manner) can achieve a correlation coefficient close to 0.57.

```
14 0 The size of the tournaments; is it complete?
5 1 2 3 4 7 8 9 10 11 13 14 12 6 participant numbers#

8 7 7 7 7 7 7 7 7 7 7 7 6 0 old points#
14 8 8 8 8 8 8 8 8 8 8 8 2 1 old scores#

45391 36767 36791 36967 36783 36879 36975 36967 36711 36607 36831 37071 28479 8191 new points#
14 5 7 10.5 6 9 12 10.5 4 3 8 13 2 1 new scores#

tournament matrix#
0 0 0 1 0 1 1 0 0 1 1 1 1 1
1 0 0 0 1 1 0 0 1 1 1 0 0 1
1 1 0 0 0 0 0 1 1 1 0 0 1 1
0 1 1 0 1 0 1 0 0 0 0 1 1 1
1 0 1 0 0 1 1 0 0 1 0 1 0 1
0 0 1 1 0 0 1 0 1 0 1 1 0 1
0 1 1 0 0 0 0 1 1 1 1 0 0 1
1 1 0 1 1 1 0 0 0 0 1 0 0 1
1 0 0 1 1 0 0 1 0 1 0 0 1 1
0 0 0 1 0 1 0 1 0 0 1 1 1 1
0 0 1 1 1 0 0 0 1 0 0 1 1 1
0 1 1 0 0 0 1 1 1 0 0 0 1 1
0 1 0 0 1 1 1 1 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0

correlation#
0.697802
```

Figure 9. Tournament of size 14 where transforming certain transitive triads (in a specific manner) can achieve a correlation coefficient close to 0.57.

5.3 Summary of Results

The analysis of 10,000 tournaments provides preliminary evidence that the potential invariant holds, as no counterexamples were observed. The observation of examples with a correlation of 0.5, particularly in graphs of sizes 11, 13, 15, and 17, suggests that the Modified Kendall Correlation Coefficient for ranking points calculated by the two methods can reach this value under specific conditions. Additionally, the presence of correlations close to 0.57 in tournaments of sizes 12 and 14 suggests that the minimum correlation for even-sized participant sets may be different and needs further research. While these findings are promising, further research is necessary to explore whether counterexamples to the invariant relationship exist across a wider range of conditions and sizes of tournaments, and to establish the robustness of this invariant.

6 Conclusions

In this paper, we continued to explore a potential invariant for tournament digraphs that was first introduced in [5]. we looked at the relative ranking points of players using two different methods and studied how changes in transitive triads within these tournaments affect the rankings and their correlations. After running extensive computer simulations with 10,000 randomly generated tournaments, we found that the proposed invariant seems to hold true, as no counterexamples were found. we used the Modified Kendall Correlation Coefficient to measure the relationship between the two ranking methods and found that tournaments with an odd number of players consistently had a minimum correlation of 0.5. For tournaments with an even number of players, the correlation was close to 0.57. These results support the proposed invariant. However, more research is needed to confirm this invariant across a wider range of tournament sizes and find any possible counterexamples. This study builds on previous research, providing deeper insights into our proposed invariant for tournament digraphs and expanding our understanding of this potential invariant.

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Data Availability Statement

Data is contained within the article.

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Conflicts of Interests

The authors declare that they have no conflicts of interest regarding the publication of this article.

References

[1] D. Pacheco, L. de Lima, C.S. Oliveira, On the Graovac–Ghorbani index for bicyclic graphs with no pendent vertices, MATCH Commun. Math. Comput. Chem. 86 (2021) 429–448.https://doi.org/10.48550/arXiv.2005.02141

[2] L. Beretta, F. M. Nardini, R. Trani, R. Venturini, An Optimal Algorithm for Finding Champions in Tournament Graphs, arXiv, 2018. https://doi.org/10.1109/tkde.2023.3267345

[3] J. Shen, L. Sheng, J. Wu, Searching for Sorted Sequences of Kings in Tournaments, SIAM J. Comput. 32 (2003) 1201-1209. https://doi.org/10.1137/S0097539702410053

[4] R. Gera, T. W. Haynes, S. T. Hedetniemi, Graph Theory: Favorite Conjectures and Open Problems - 2, Springer, 2018. https://doi.org/10.1007/978-3-319-97686-0

[5] B. F. Melnikov, B. Liu, On an Invariant of Tournament Digraphs, J. Appl. Math. Phys. 12 (2024) 2711–2722. https://doi.org/10.4236/jamp.2024.127162

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https://doi.org/ 10.22061/jdma.2024.11215.1093