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Research Paper

Orbit entropy versus the symmetry index

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Abstract. The aim of this paper is to compute the novel counting polynomial, orbit polynomial, the symmetry index and the orbit-entropy of several classes of networks.

Keywords. Small-world networks, symmetry index, orbit entropy

Mathematics Subject Classification (2010): 05C82, 91D30.

1 Introduction

For the first time, the meaning of graph entropy measures was introduced in the study of biological and chemical systems, with Rashevsky [39], see also [31–34] for the main contri-

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butions. In [34] the graph information content such as the entropy measure was interpreted. Since then, several kinds of graph entropy measures have been defined to focus on the structural characteristics of graphs [8–16, 20, 43].

In Section 2, the concepts of the entropy and symmetry index of a graph are given, and these measures are calculated for certain classes of graphs. Section 3 contains the main results. It is proved that there are several classes of graphs whose symmetry index is greater than or equal to the orbit-entropy measure, while many other classes have a greater orbit entropy.

2 Entropy measure, symmetry index, and orbit polynomial

In this paper, the automorphism group of the graph $G = (V(G), E(G))$ is denoted by $Aut(G)$. Two vertices $u, v \in V(G)$ are said to be similar, if there is an automorphism $\alpha \in Aut(G)$ such that $\alpha(u) = v$ or $\alpha(v) = u$. The set of all similar vertices to given vertex u is called the orbit of u under the action of $Aut(G)$ on the set of vertices. We denote the orbit of u by $\mathcal{O}_u$. It is clear that the collection of all orbits of G is a partition for the vertex set $V(G)$. If the automorphism group of G acts transitively on $V(G)$, then it is concluded that G is vertex-transitive. Equivalently, a vertex-transitive graph has only one orbit.

Let G be a graph on n vertices. A classical graph entropy measure defined by Mowshowitz [31] as

$$I_a(G) = -\sum_{i=1}^{k} \frac{|\mathcal{O}_i|}{n} \log(\frac{|\mathcal{O}_i|}{n}),$$

where $\mathcal{O}_i$ $(1 \leq i \leq k)$ are orbits of G. In above definition, the basis of logarithm function is 2.

Measuring the graph information content is due to Rashevsky's idea [39] who defined the concept of entropy of a graph. The symmetry index is another graph measure associated to the size of orbits of a graph defined by Mowshowitz and Dehmer in [30] as follows:

$$\begin{aligned} S(G) &= (\log(n) - I_a(G)) + \log|Aut(G)| \\ &= \frac{1}{n}(\sum_{i=1}^{k} |\mathcal{O}_i| \log|\mathcal{O}_i|) + \log|Aut(G)|. \end{aligned}$$

Finally, the orbit polynomial [6, 21, 22] is defined as $O_G(x) = \sum_{i=1}^{k} x^{|\mathcal{O}_i|}$, and the modified orbit polynomial is $O_G^\star(x) = 1 - O_G(x)$.

Consider $A = Aut(G)$. For a vertex v in $V(G)$, the stabilizer subgroup of A at vertex v is defined as $A_v = \{\alpha \in Aut(G) : \alpha(v) = v\}$.

Theorem 2.1. *[17] (**Orbit-stabilizer Theorem**) Let A be a permutation group acting on a set Ω and u be an arbitrary point in the set Ω. Then $|A| = |A_u||u^A|$.*

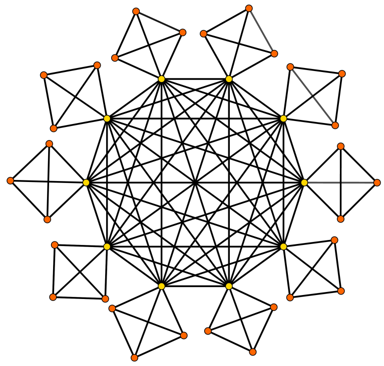

Figure 1. The graph $\mathcal{K}_n^r$, where $n = 10$ and $r = 4$.

3 Orbit based-measures

The aim of this section is to verify the orbit-entropy measure and the symmetry index of several classes of graphs.

The main result of this section is as follows.

Theorem 3.1. *For the graph $\mathcal{K}_n^r$, we have $O_{\mathcal{K}_n^r}(x) = x^n(1 + x^{(r-1)})$. In addition, the symmetry index is greater than the orbit-entropy measure.*

Proof. It is clear that $\mathcal{K}_n^r$ has a clique of order n in which each vertex u_i is adjacent to exactly r vertices $v_1^i, \ldots, v_r^i$. The induced subgraph $\langle v_1^i, \ldots, v_r^i \rangle$ is similarly a clique in $\mathcal{K}_n^r$. This means that the structure of $\mathcal{K}_n^r$ is $K_n \circ K_{r-1}$ gives $\mathrm{Aut}(\mathcal{K}_\mathrm{n}^\mathrm{r}) \cong \mathcal{S}_\mathrm{n} \wr \mathcal{S}_{\mathrm{r}-1}$. This means that $\mathcal{K}_n^r$ has two orbits. The vertices of degree $n + r - 2$ give rise to an orbit of size n, and the other vertices give rise to an orbit of order $n(r-1)$. Hence, we have that

$$S(\mathcal{K}_n^r) = \log(n) + \frac{r-1}{r}\log(r-1) + \log(n!) + n\log((r-1)!),$$
$$I_a(\mathcal{K}_n^r) = \log(r) - \frac{r-1}{r}\log(r-1).$$

This leads to the proof. □

Now consider the graph $\mathcal{L}_n^r$ as depicted in Fig. 2.. This graph has $n + r$ vertices, where r is the number of outer triangles. Then we have the following.

Theorem 3.2. *The automorphism group of $\mathcal{L}_n^r$ has the following structure:*

$$Aut(\mathcal{L}_n^r) \cong \begin{cases} \mathcal{Z}_2 \times \mathcal{Z}_2 & r = 1, n = 4,5,7 \\ D_{12} & r = 1, n = 6 \\ \mathcal{Z}_2 \times \mathcal{Z}_2 & r = 2, n = 7 \\ D_{2n} & n = r, n \geq 8 \\ \mathcal{Z}_2 & otherwise \end{cases}.$$

Proof. If $n = r$, then we have that $Aut(\mathcal{L}_n^r) \cong D_{2n}$. Suppose that $n \neq 2r$. It is obvious that the blue lines in Fig. 3. are symmetry lines, showing that $Aut(\mathcal{L}_n^r) \leq \mathcal{Z}_2$. Meanwhile, because an

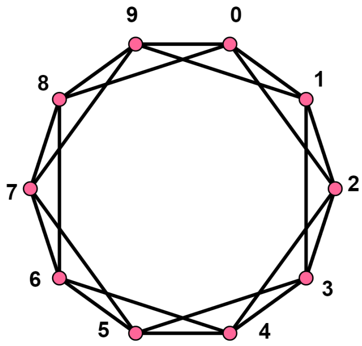

Figure 2. The graph $\mathcal{L}_n^r$, where $n = 10$.

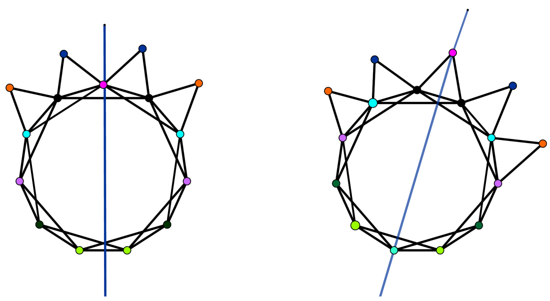

Figure 3. The graph $\mathcal{L}_n^r$, where $n = 11$.

automorphism preserves the degree of the vertices, there are no other symmetry elements, thereby giving $Aut(\mathcal{L}_n^r) \cong \mathcal{Z}_2$. If $r = 1$ and $n = 5,6,7$ or $r = 2$ and $n = 7$, then it is clear that $Aut(\mathcal{L}_n^r) \cong \mathcal{Z}_2 \times \mathcal{Z}_2$. $\square$

Corollary 3.3. *For the graph $\mathcal{L}_n^r$, if $n \geq 16$ and $1 \leq r \leq n$, then $I_a(\mathcal{L}_n^r) > S(\mathcal{L}_n^r)$.*

Proof. The special cases are as follows. If $r = 1$ and $n = 4$, then $S(\mathcal{L}_4^1) = 3$ and $I_a(\mathcal{L}_4^1) = 1$. If $r = 1$ and $n = 5$, then $S(\mathcal{L}_5^1) = \frac{14}{5}$ and $I_a(\mathcal{L}_5^1) = \log 5 - \frac{4}{5}$. Additionally, we have that $S(\mathcal{L}_6^1) = \frac{3}{2}\log 3 + \frac{7}{3}$ and $I_a(\mathcal{L}_6^1) = \frac{1}{2}\log 3 + \frac{2}{3}$. If $r = 1$ and $n = 7$, then the symmetry index and orbit-entropy measure are $\frac{20}{7}$ and $\log 7 - \frac{6}{7}$, respectively. Also, we have that $S(\mathcal{L}_7^2) = \frac{20}{7}$ and $I_a(\mathcal{L}_7^2) = \log 7 - \frac{6}{7}$. If $n = r$, then $S(\mathcal{L}_n^n) = 1 + 2\log(n)$ and $\mathrm{I_a}(\mathcal{L}_\mathrm{n}^\mathrm{n}) = 0$.

Consider the following cases.

- If n is even, then the orbits with two elements are $\{l, n-l+2\}$, $\{s, r-s+2\}$, and $\{n-r+i, n-i+1\}$, where $r+2 \leq l \leq \frac{n}{2}$, $1 \leq s \leq \frac{r}{2}$, and $1 \leq i \leq \frac{r}{2}$. Additionally, if r is even, then $\frac{r+2}{2}$ and $\frac{n+2}{2}$ are singleton orbits for $\mathcal{L}_n^r$, and if r is odd, then $\frac{n+2}{2}$ and $\frac{2n-r+1}{2}$ are singleton orbits for $\mathcal{L}_n^r$. Therefore, there are $r+1$ orbits of size two if $n < 2k+4$, r orbits of size two if $n = 2k+4$, and $\frac{n-2}{2}$ orbits of size two if $n > 2k+4$. Again, consider the following two cases.

(a) If r is even, then

$$S(\mathcal{L}_n^r) = \begin{cases} \frac{2r+n}{n} & n < 2r+4 \\ \frac{2n+6}{n} & n = 2r+4 \\ \frac{2(n-1)}{n} & n > 2r+4 \end{cases},$$

and

$$I_a(\mathcal{L}_n^r) = \begin{cases} \frac{2r+2}{n}\log(n) - \frac{2r}{n} & n < 2r+4 \\ \frac{2r+4}{n}\log(n) - \frac{2(r+1)}{n} & n = 2r+4 \\ \log n + \frac{n-2}{n} & n > 2r+4 \end{cases}.$$

(b) If r is odd, then

$$S(\mathcal{L}_n^r) = \begin{cases} \frac{2r+n+2}{n} & n < 2r+4 \\ \frac{2r+n}{n} & n = 2r+4 \\ \frac{n-2}{n} & n > 2r+4 \end{cases},$$

and

$$I_a(\mathcal{L}_n^r) = \begin{cases} \frac{2r+4}{n}\log(n) + \frac{2r+2}{n} & n < 2r+4 \\ \frac{2+2nr}{n}\log(n-2k) & n = 2r+4 \\ n + p\log(n-2) & n > 2r+4 \end{cases},$$

where $p = (n^2 - 2n + 2)$.

- If n is odd, then $\{l, n-l+2\}$, $\{s, r-s+2\}$ and $\{n-r+i, n-i+1\}$ are orbits of size two, where $r+2 \leq l \leq \frac{n}{2}$, $1 \leq s \leq \frac{r}{2}$, and $1 \leq i \leq \frac{2n-r+1}{2}$. Also if r is even, then $\mathcal{L}_n^r$ has $\frac{r+2}{2}$ singleton orbits, and if r is odd, then it has $\frac{r-1}{2}$ singleton orbits. Thus, there are r orbits of size two if $n < 2k+3$, $r+1$ orbits of size two if $n = 2k+3$, and $\frac{n-5}{2}$ orbits of size two if $n > 2k+3$. Accordingly, the following subcases can be concluded.

(a) If r is even, then

$$S(\mathcal{L}_n^r) = \begin{cases} \frac{2r+n}{n} & n < 2r+3 \\ \frac{2r+n+2}{n} & n = 2r+3 \\ \frac{2n-5}{n} & n > 2r+3 \end{cases},$$

and

$$I_a(\mathcal{L}_n^r) = \begin{cases} \frac{2r+1}{n}\log(n) + \frac{2r}{n} & n < 2r+3 \\ \frac{2r+3}{n}\log(n) - \frac{2r+2}{n} & n = 2r+3 \\ \frac{n-4}{n}\log(n) - \frac{n-5}{n} & n > 2r+3 \end{cases}.$$

(b) If r is odd, then

$$\mathrm{S}(\mathcal{L}_{\mathrm{n}}^{\mathrm{r}}) = \begin{cases} \frac{2r+n+2}{n} & n < 2r+3 \\ \frac{2r+n+4}{n} & n = 2r+3 \\ \frac{2n+1}{n} & n > 2r+3 \end{cases},$$

and

$$I_a(\mathcal{L}_n^r) = \begin{cases} \frac{2r+3}{n}\log(n) + \frac{2r+2}{n} & n < 2r+3 \\ \frac{2r+5}{n}\log(n) - \frac{2r+4}{n} & n = 2r+3 \\ \frac{n+2}{n}\log(n) - \frac{n+1}{n} & n > 2r+3 \end{cases}.$$

Hence, we conclude that for $n \geq 16$, $S(\mathcal{L}_n^r) < I_a(\mathcal{L}_n^r)$. □

Suppose $I(G)$ and $I(H)$ are two graph invariants of graphs G and H, respectively. Then the graph distance measure between $I(G)$ and $I(H)$ is defined as $d_I(G,H) = 1 - e^{-(\frac{I(G)-I(H)}{\sigma})^2}$, see [7]. In Table 1., the size of automorphism group, the values of orbit-entropy I_a, symmetric index $S(G)$, the unique positive root δ, distance measure between δ and I_a, and distance measure between δ and $S(G)$ of biological networks and technological networks are reported.

Table 1.

	n	$\lvert\mathbb{A}\rvert$	δ	I_a	S	$d(\delta, I_a)$	$d(\delta, S)$
Human B Cell Genetic	5930	5.94×10^{13}	1710×10^{-7}	3.77	13.8	14.20	189.83
Caenorhabditis elegans Genetic	2060	6.99×10^{161}	6258×10^{-7}	3.20	161.96	10.22	26.23×10^{3}
BioGRID Human	7013	1.26×10^{485}	1764×10^{-7}	3.73	485.2	13.93	23.54×10^{4}
BioGRID Saccharomy-ccs cerevisiae	5295	6.86×10^{64}	1958×10^{-7}	3.70	64.86	13.71	4206.4
BioGRID Drosophila	7371	3.07×10^{493}	1690×10^{-7}	3.76	493.6	14.10	24.36×10^{4}
BioGRID Mus musculus	209	5.35×10^{125}	221440×10^{-7}	1.46	126.59	2.06	16.02×10^{3}
Yeast Protein Interactions	1458	1.07×10^{254}	11599×10^{-7}	2.88	254.30	8.31	64.67×10^{3}
c. elegans metabolic	453	1.93×10^{10}	25702×10^{-7}	2.60	10.33	6.78	106.79
Internet	22332	$1.28 \times 10^{11,298}$	1035×10^{-7}	3.67	11.3×10^{3}	13.44	12.77×10^{7}
US Power Grid	4941	5.18×10^{152}	2380×10^{-7}	3.63	152.77	13.20	23.34×10^{3}
US Airports	332	2.59×10^{24}	40472×10^{-7}	2.39	24.54	5.71	602.07
www California search subnet	5925	$1.24 \times 10^{1,298}$	2820×10^{-7}	3.45	1298.4	11.93	16.86×10^{5}
www EPA.gov subnet	4253	$1.28 \times 10^{2,321}$	4992×10^{-7}	2.91	2321.8	8.47	53.91×10^{5}
www Political Blogs	1222	2.40×10^{35}	8741×10^{-7}	3.04	35.43	9.25	1254.9
Email	1133	1.53×10^{9}	9216×10^{-7}	3.04	9.20	9.23	84.62
Media ownership	4475	$3.36 \times 10^{4,818}$	13278×10^{-7}	2.16	4820.01	4.68	23.23×10^{6}
Geometry Co-authorship	3621	1.90×10^{320}	4419×10^{-7}	3.38	320.45	11.45	10.27×10^{4}
Erdös Collaboration	6927	$3.46 \times 10^{4,222}$	5491×10^{-7}	2.956	4223.4	8.73	17.84×10^{6}
PhD network	1025	2.98×10^{292}	25245×10^{-7}	2.55	292.93	6.49	85.81×10^{3}

The comparing these results shows that the correlation values of δ and I_a in biological networks and technological networks are very close. Therefore, it seems that there are differences in the effective variables in the structure of social networks with other networks which is suggested to be examined in future research.

4 Summary and Conclusion

In this paper, we studied the orbit-entropy measure $I_a(G)$ based on the size of orbits of a graph under the action of an automorphism group on the set of vertices. We compared this measure with the symmetry index $S(G)$ of several types of graphs and obtained certain inequalities such as $I_a(G) < S(G)$ and $S(G) < I_a(G)$. However, it is clear that it would be quite challenging to prove general inequalities.

If G is a random graph with sufficiently many vertices, then the probability that $Aut(G) = id$ is approximately one. This leads to the well-known fact that among all classes of graphs, most of them are asymmetric. By using the results of this paper, if $Aut(G) = id$ then $S(G) < I_a(G)$. This leads to the conclusion that for random graphs, we obtain $S(G) < I_a(G)$ as the number of vertices tends to infinity. However, for most of the examples shown in this paper, it was proved that $S(G) > I_a(G)$.

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