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On multiplicative degree based topological indices of single-walled titania nanotubes

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Abstract. A topological index is a special number which describes the whole structure of a graph. The topological indices are categorized on the basis of their logical roots from topological invariant. A topological index which depends on the degree of a vertex is called degree based topological index. In this paper we will calculate some degree based multiplicative topological indices of single-walled Titania nanotubes.

Keywords: chemical graph theory, Titania nanotubes, multiplicative topological indices, degree based topological indices

Mathematics Subject Classification (2010): 05C92.

1 Introduction

Graph theory is a subject of mathematics in which we study the graphs. Presently graph theory is one of the richest and most cited areas of mathematics, especially chemical graph theory. In chemical graph theory we study the structure of many chemical compounds and calculate some physicals as well as chemical properties of these compounds. Firsly chemical

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graph theory that we studied in book [1] and after that in book [2] gave us more concepts about it. Secondly in book [3] by N. Trinajstić, we found detailed ideas about chemical graph theory, its uses, and applications.

Nowadays many papers has published in chemical graph theory in which authors studied the chemical structures and calculated topological indices of some important chemical compounds like in paper [4]. In papers [5, 6] authors has calculated some degree based topological indices of graphs. Some works has done on multiplicative topological indices of graphs in papers [7–9]. Recently in paper [10] Z. Hussain et al, calculated the degree based multiplicative topological indices of alcohol. In papers [11, 12] authors has calculated the topological indices of single-walled titania nanotubes. The main idea to calculate the degree based multiplicative topological indices of single-walled titania nanotube directly came from the papers [10, 11].

2 Definitions and preliminaries

Let $G\left(V(G),E(G)\right)$ be a simple, finite and connected graph with $V(G)$ the set of vertices and $E(G)$ is the set of edges among the vertices of the graph. A metric space $d_G : V(G) \times V(G) \to \mathbb{R}$, define such as $d_G\left(i,j\right)$ is the number of edges between i and j in shortest path, where $i,j \in V(G)$. In the neighborhood of i in graph G it defines as:

$$N_G(i) = \{j \in V(G) \mid d_G(i,j) = 1\}.$$

The cardinality of neighborhood set of $i \in V(G)$ is called its degree and we will denote it in this paper as $\triangle_i$.
Now we will define some multiplicative topological indices. First, multiplicative Zagreb index defines as:

$$II_1^*(G) = \prod_{rt \in E(G)} \left(\triangle_r + \triangle_t\right). \tag{1}$$

Second, multiplicative Zagreb index defines as:

$$II_2(G) = \prod_{rt \in E(G)} \left(\triangle_r . \triangle_t\right). \tag{2}$$

The multiplicative first and second hyper-Zagreb indices for graphs are define as:

$$HII_1(G) = \prod_{rt \in E(G)} \left(\triangle_r + \triangle_t\right)^2. \tag{3}$$

$$HII_2(G) = \prod_{rt \in E(G)} \left(\triangle_r . \triangle_t\right)^2. \tag{4}$$

First and second multiplicative generalized Zagreb indices are the generalized form of first and second multiplicative Zagreb indices as well as first and second multiplicative hyper-Zagreb indices. First and second multiplicative generalized Zagreb indices are define such as:

$$MZ_1^{\alpha}(G) = \prod_{rt\in E(G)} (\triangle_r + \triangle_t)^{\alpha}. \tag{5}$$

$$MZ_2^{\alpha}(G) = \prod_{rt\in E(G)} (\triangle_r.\triangle_t)^{\alpha}. \tag{6}$$

Multiplicative sum and product connectivity indices are define as:

$$SCII(G) = \prod_{rt\in E(G)} \frac{1}{\sqrt{\triangle_r + \triangle_t}}. \tag{7}$$

$$PCII(G) = \prod_{rt\in E(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}}. \tag{8}$$

The multiplicative atomic bond connectivity index and geometric arithmetic index are defined as:

$$ABCII(G) = \prod_{rt\in E(G)} \sqrt{\frac{\triangle_r + \triangle_t - 2}{\triangle_r.\triangle_t}}. \tag{9}$$

$$G^*AII(G) = \prod_{rt\in E(G)} \left(\frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r + \triangle_t}\right). \tag{10}$$

The general multiplicative geometric arithmetic index is the generalized shape of multiplicative geometric arithmetic index which is defined as:

$$G^*A^{\alpha}II(G) = \prod_{rt\in E(G)} \left(\frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r + \triangle_t}\right)^{\alpha}. \tag{11}$$

These are the topological indices which will be calculated in this paper.

Fact 2.1. *Let $r_1, r_2, \ldots, r_n$ be a sequence. Then*

$$\prod_{i=1}^{n} (r_i)^{\alpha} = \left(\prod_{i=1}^{n} r_i\right)^{\alpha},$$

where α is a constant.

For more details, definitions, and topological indices on graphs, we strongly offer, suggest paper [10].

3 Titania nanotube

Titania is one of the most expansively discussed metal oxide substances. Naturally, titania nanotubes exist in two forms, single-walled nanotubes $(SW\ Tio_2\ NT_s)$ and multi-walled titania nanotubes$(MW\ Tio_2\ NT_s)$.

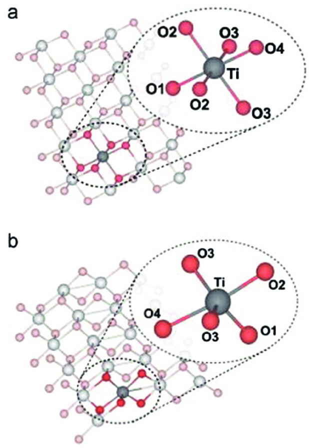

Figure 1. Titania Nanotubes.

We can see the differences between these two types of nanotubes in Figure 1. In this paper we will only study the single-walled Titania nanotubes only. Single-walled Titania nanotubes have further two types which are single-walled three layered Titania nanotube and single-walled six layered Titania nanotube. For more details of concepts on single-layered titania nanotube readers shoud review [11].

3.1 Three layered titania nanotubes

A three layered single-walled titania nanotube is written as $TNT_3[m,n]$, where m and n are the number of atoms in each column and row respectively. It is easy for the readers to check that the total number of edges in $TNT_3[m,n]$ are $12mn - 4m - 3n + 1$.

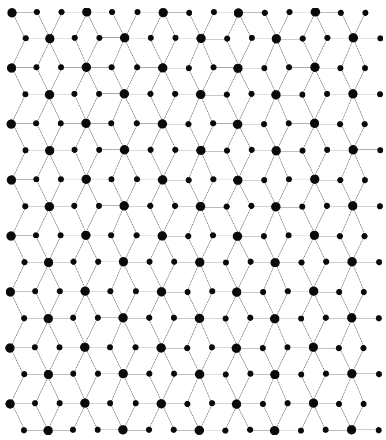

Figure 2. Single walled three layered Titania naotube

Let G be the $TNT_3[m,n]$ nanotube of single-walled, there are total nine types of edges based on the end degree for each vertex so we can decompose the set of edges such as

$$E(G) = E_1(G)\bigcup E_2(G)\bigcup E_3(G)\bigcup E_4(G)\bigcup E_5(G)\bigcup E_6(G)\bigcup E_7(G)\bigcup E_8(G)\bigcup E_9(G),$$

where

$$\begin{aligned}
E_1(G) &= \{e = rt \in E(G) \mid \triangle_r = 1, \triangle_t = 4\},\\
E_2(G) &= \{e = rt \in E(G) \mid \triangle_r = 1, \triangle_t = 6\},\\
E_3(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 2\},\\
E_4(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 3\},\\
E_5(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 4\},\\
E_6(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 6\},\\
E_7(G) &= \{e = rt \in E(G) \mid \triangle_r = 3, \triangle_t = 3\},\\
E_8(G) &= \{e = rt \in E(G) \mid \triangle_r = 3, \triangle_t = 4\},\\
E_9(G) &= \{e = rt \in E(G) \mid \triangle_r = 3, \triangle_t = 6\}.
\end{aligned}$$

It is easy to check that
$|E_1(G)| = 1$, $|E_2(G)| = n$, $|E_3(G)| = 1$, $|E_4(G)| = 2$, $|E_5(G)| = 4m-2$, $|E_6(G)| = 4m+2n-6$, $|E_7(G)| = 3n-4$, $|E_8(G)| = 4m-3$ and $|E_9(G)| = 12mn-16m-9n+12$.

Theorem 3.1. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of three layered $TNT_3[m,n]$. The first multiplicative Zagreb index for G is $2^{16m+9n-22} \times 3^{24mn-28m-15n+18} \times 5^3 \times 7^{4m+n-3}$.*

Proof. From Eq. (1)

$$\begin{aligned}
II_1^*(G) &= \prod_{rt\in E(G)} (\triangle_r + \triangle_t),\\
&= \prod_{rt\in E_1(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_2(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_3(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_4(G)} (\triangle_r + \triangle_t)\\
&\times \prod_{rt\in E_5(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_6(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_7(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_8(G)} (\triangle_r + \triangle_t)\\
&\times \prod_{rt\in E_9(G)} (\triangle_r + \triangle_t),\\
&= (1+4)^{|E_1(G)|} \times (1+6)^{|E_2(G)|} \times (2+2)^{|E_3(G)|} \times (2+3)^{|E_4(G)|} \times (2+4)^{|E_5(G)|}\\
&\times (2+6)^{|E_6(G)|} \times (3+3)^{|E_7(G)|} \times (3+4)^{|E_8(G)|} \times (3+6)^{|E_9(G)|},\\
&= 5 \times 7^n \times 2^2 \times 5^2 \times 2^{4m-2}.3^{4m-2} \times 2^{3(4m+2n-6)} \times 2^{3n-4}.3^{3n-4} \times 7^{4m-3}\\
&\times 3^{2(12mn-16m-9n+12)}.
\end{aligned}$$

After some simple calculations we get

$$II_1^*(G) = 2^{16m+9n-22} \times 3^{24mn-28m-15n+18} \times 5^3 \times 7^{4m+n-3}.$$

□

Theorem 3.2. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of single−walled Titania nanotubes of three layered $TNT_3[m,n]$. The second multiplicative Zagreb index for G is $2^{12mn+12m-4n-6}\times 3^{24mn-24m-9n+9}$.*

Proof. From Eq. (2)

$$\begin{aligned}
II_2(G) &= \prod_{rt\in E(G)}\left(\triangle_r.\triangle_t\right),\\
&= \prod_{rt\in E_1(G)}\left(\triangle_r.\triangle_t\right)\times\prod_{rt\in E_2(G)}\left(\triangle_r.\triangle_t\right)\times\prod_{rt\in E_3(G)}\left(\triangle_r.\triangle_t\right)\times\prod_{rt\in E_4(G)}\left(\triangle_r.\triangle_t\right)\\
&\times\prod_{rt\in E_5(G)}\left(\triangle_r.\triangle_t\right)\times\prod_{rt\in E_6(G)}\left(\triangle_r.\triangle_t\right)\times\prod_{rt\in E_7(G)}\left(\triangle_r.\triangle_t\right)\times\prod_{rt\in E_8(G)}\left(\triangle_r.\triangle_t\right)\\
&\times\prod_{rt\in E_9(G)}\left(\triangle_r.\triangle_t\right),\\
&= (1.4)^{|E_1(G)|}\times(1.6)^{|E_2(G)|}\times(2.2)^{|E_3(G)|}\times(2.3)^{|E_4(G)|}\times(2.4)^{|E_5(G)|}\\
&\times(2.6)^{|E_6(G)|}\times(3.3)^{|E_7(G)|}\times(3.4)^{|E_8(G)|}\times(3.6)^{|E_9(G)|},\\
&= 2^2\times 2^n.3^n\times 2^2\times 2^2.3^2\times 2^{12m-6}\times 2^{8m+4n-12}.3^{4m+2n-6}\times 3^{6n-8}\\
&\times 2^{8m-6}.3^{4m-3}\times 2^{12mn-16m-9n+12}.3^{24mn-32m-18n+24},\\
II_2(G) &= 2^{12mn+12m-4n-6}\times 3^{24mn-24m-9n+9}.
\end{aligned}$$

□

Theorem 3.3. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of single−walled Titania nanotubes of three layered $TNT_3[m,n]$. Then multiplicative sum connectivity index for G is $\sqrt{2^{22-16m-9n}}\times\sqrt{3^{28m+15n-24mn-18}}\times\sqrt{5^{-3}}\times\sqrt{7^{3-4m-n}}$.*

Proof. From Eq. (7)

$$
\begin{aligned}
SCII(G) &= \prod_{rt\in E(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}},\\
&= \prod_{rt\in E_1(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_2(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_3(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_4(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}}\\
&\times \prod_{rt\in E_5(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_6(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_7(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_8(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}}\\
&\times \prod_{rt\in E_9(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}},\\
&= \left(\frac{1}{\sqrt{1+4}}\right)^{|E_1(G)|} \times \left(\frac{1}{\sqrt{1+6}}\right)^{|E_2(G)|} \times \left(\frac{1}{\sqrt{2+2}}\right)^{|E_3(G)|} \times \left(\frac{1}{\sqrt{2+3}}\right)^{|E_4(G)|}\\
&\times \left(\frac{1}{\sqrt{2+4}}\right)^{|E_5(G)|} \times \left(\frac{1}{\sqrt{2+6}}\right)^{|E_6(G)|} \times \left(\frac{1}{\sqrt{3+3}}\right)^{|E_7(G)|} \times \left(\frac{1}{\sqrt{3+4}}\right)^{|E_8(G)|}\\
&\times \left(\frac{1}{\sqrt{3+6}}\right)^{|E_9(G)|},\\
&= \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{7^n}} \times \frac{1}{2} \times \frac{1}{5} \times \frac{1}{2^{2m-1}.3^{2m-1}} \times \frac{1}{\sqrt{2^{12m+6n-18}}}\\
&\times \frac{1}{\sqrt{2^{3n-4}.3^{3n-4}}} \times \frac{1}{\sqrt{7^{4m-3}}} \times \frac{1}{3^{12mn-16m-9n+12}},\\
SCII(G) &= \sqrt{2^{22-16m-9n}} \times \sqrt{3^{28m+15n-24mn-18}} \times \sqrt{5^{-3}} \times \sqrt{7^{3-4m-n}}.
\end{aligned}
$$

□

Theorem 3.4. *Let $G\left(V\left(G\right), E\left(G\right)\right)$ be the graph of single−walled Titania nanotubes of three layered $TNT_3[m,n]$. Then multiplicative product connectivity index for G is $2^{3+2n-6mn-6m} \times \sqrt{3^{24m+9n-24mn-9}}$.*

Proof. From Eq. (8)

$$
\begin{aligned}
PCII(G) &= \prod_{rt\in E(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}},\\
&= \prod_{rt\in E_1(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_2(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_3(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_4(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_5(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_6(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_7(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_8(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_9(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}},\\
&= \left(\frac{1}{\sqrt{1.4}}\right)^{|E_1(G)|} \times \left(\frac{1}{\sqrt{1.6}}\right)^{|E_2(G)|} \times \left(\frac{1}{\sqrt{2.2}}\right)^{|E_3(G)|} \times \left(\frac{1}{\sqrt{2.3}}\right)^{|E_4(G)|}\\
&\times \left(\frac{1}{\sqrt{2.4}}\right)^{|E_5(G)|} \times \left(\frac{1}{\sqrt{2.6}}\right)^{|E_6(G)|} \times \left(\frac{1}{\sqrt{3.3}}\right)^{|E_7(G)|} \times \left(\frac{1}{\sqrt{3.4}}\right)^{|E_8(G)|}\\
&\times \left(\frac{1}{\sqrt{3.6}}\right)^{|E_9(G)|},\\
&= \frac{1}{2} \times \frac{1}{\sqrt{2^n.3^n}} \times \frac{1}{2} \times \frac{1}{2.3} \times \frac{1}{2^{6m-3}} \times \frac{1}{2^{4m+2n-6}} \times \frac{1}{3^{2m+n-3}}\\
&\times \frac{1}{3^{3n-4}} \times \frac{1}{2^{4m-3}.\sqrt{3^{4m-3}}} \times \frac{1}{\sqrt{12^{12mn-16m-9n+12}}} \times \frac{1}{3^{12mn-16m-9n+12}},\\
PCII(G) &= 2^{3+2n-6mn-6m} \times \sqrt{3^{24m+9n-24mn-9}}.
\end{aligned}
$$

□

Theorem 3.5. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of three layered $TNT_3[m,n]$. Then multiplicative atomic bond connectivity index for G is* $\sqrt{2^{12n-12mn-11}} \times \sqrt{3^{28m+11n-24mn-12}} \times \sqrt{5^{4m+n-3}} \times \sqrt{7^{12mn-16m-9n+12}}$.

Proof. From Eq. (9)

$$\begin{aligned}
ABCII(G) &= \prod_{rt\in E(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}},\\
&= \prod_{rt\in E_1(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_2(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_3(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_4(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_5(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_6(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_7(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_8(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_9(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}},\\
&= \left(\sqrt{\frac{1+4-2}{4}}\right)^{|E_1(G)|} \times \left(\sqrt{\frac{1+6-2}{6}}\right)^{|E_2(G)|} \times \left(\sqrt{\frac{2+2-2}{2.2}}\right)^{|E_3(G)|}\\
&\times \left(\sqrt{\frac{2+3-2}{2.3}}\right)^{|E_4(G)|} \times \left(\sqrt{\frac{2+4-2}{2.4}}\right)^{|E_5(G)|} \times \left(\sqrt{\frac{2+6-2}{2.6}}\right)^{|E_6(G)|}\\
&\times \left(\sqrt{\frac{3+3-2}{3.3}}\right)^{|E_7(G)|} \times \left(\sqrt{\frac{3+4-2}{3.4}}\right)^{|E_8(G)|} \times \left(\sqrt{\frac{3+6-2}{3.6}}\right)^{|E_9(G)|},\\
&= \frac{\sqrt{3}}{2} \times \frac{\sqrt{5^n}}{\sqrt{2^n}.\sqrt{3^n}} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} \times \frac{2^{4m-2}}{2^{12m-6}} \times \frac{1}{2^{2m+n-3}}\\
&\times \frac{2^{3n-4}}{3^{3n-4}} \times \frac{\sqrt{5^{4m-3}}}{2^{4m-3}.\sqrt{3^{4m-3}}} \times \frac{\sqrt{7^{12mn-16m-9n+12}}}{3^{12mn-16m-9n+12}.\sqrt{2^{12mn-16m-9n+12}}},\\
ABCII(G) &= \sqrt{2^{12n-12mn-11}} \times \sqrt{3^{28m+11n-24mn-12}} \times \sqrt{5^{4m+n-3}} \times \sqrt{7^{12mn-16m-9n+12}}.
\end{aligned}$$

□

Theorem 3.6. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of three layered $TNT_3[m,n]$. Then multiplicative geometric arithmetic index of G is*

$$2^{18mn-14m-14n+20} \times \sqrt{3^{32m+21n-24mn-27}} \times 5^{-3} \times 7^{3-4m-n}.$$

Proof. From Eq. (10)

$$\begin{aligned}
G^*AII(G) &= \prod_{rt\in E(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t},\\
&= \prod_{rt\in E_1(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_2(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_3(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t}\\
&\times \prod_{rt\in E_4(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \prod_{rt\in E_5(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_6(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t}\\
&\times \prod_{rt\in E_7(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_8(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_9(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t},\\
&= \left(\frac{2\sqrt{4}}{1+4}\right)^{|E_1(G)|} \times \left(\frac{2\sqrt{6}}{1+6}\right)^{|E_2(G)|} \times \left(\frac{2\sqrt{2.2}}{2+2}\right)^{|E_3(G)|} \times \left(\frac{2\sqrt{2.3}}{2+3}\right)^{|E_4(G)|}\\
&\times \left(\frac{2\sqrt{2.4}}{2+4}\right)^{|E_5(G)|} \times \left(\frac{2\sqrt{2.6}}{2+6}\right)^{|E_6(G)|} \times \left(\frac{2\sqrt{3.3}}{3+3}\right)^{|E_7(G)|} \times \left(\frac{2\sqrt{3.4}}{3+4}\right)^{|E_8(G)|}\\
&\times \left(\frac{2\sqrt{3.6}}{3+6}\right)^{|E_9(G)|},\\
&= \frac{2^2}{5} \times \frac{\sqrt{2^{3n}}.\sqrt{3^n}}{7^n} \times \frac{2^3.3}{5^2} \times \frac{2^{3(2m-1)}}{3^{4m-2}} \times \frac{3^{2m+n-3}}{2^{4m+2n-6}}\\
&\times \frac{2^{2(4m-3)}.\sqrt{3^{4m-3}}}{7^{4m-3}} \times \frac{\sqrt{2^{3(12mn-16m-9n+12)}}}{3^{12mn-16m-9n+12}},\\
G^*AII(G) &= 2^{18mn-14m-14n+20} \times \sqrt{3^{32m+21n-24mn-27}} \times 5^{-3} \times 7^{3-4m-n}.
\end{aligned}$$

□

3.2 Six layered titania nanotubes

A single-walled six layered titania nanotube written as $TNT_6[m,n]$, where m is defined periodically as shown in Figure (3) and n is the number of titania atoms in each row. The number of edges in $TNT_6[m,n]$ are $20mn-4m-2n$.

In this section we consider G is $TNT_6[m,n]$ nanotube of single-walled. There are nine types of edges based on the end degrees for each vertex, so we can rift the set of edges such as

$$E(G) = E_1(G)\bigcup E_2(G)\bigcup E_3(G)\bigcup E_4(G)\bigcup E_5(G)\bigcup E_6(G)\bigcup E_7(G)\bigcup E_8(G)\bigcup E_9(G),$$

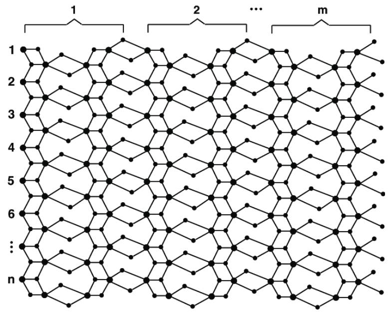

Figure 3. Single walled six layered Titania naotube

where

$$
\begin{aligned}
E_1(G) &= \{e = rt \in E(G) \mid \triangle_r = 1, \triangle_t = 4\},\\
E_2(G) &= \{e = rt \in E(G) \mid \triangle_r = 1, \triangle_t = 5\},\\
E_3(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 2\},\\
E_4(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 3\},\\
E_5(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 4\},\\
E_6(G) &= \{e = rt \in E(G) \mid \triangle_r = 2, \triangle_t = 5\},\\
E_7(G) &= \{e = rt \in E(G) \mid \triangle_r = 3, \triangle_t = 3\},\\
E_8(G) &= \{e = rt \in E(G) \mid \triangle_r = 3, \triangle_t = 4\},\\
E_9(G) &= \{e = rt \in E(G) \mid \triangle_r = 3, \triangle_t = 5\}.
\end{aligned}
$$

It is easy for the reader to check that
$|E_1(G)| = 2$, $|E_2(G)| = 2n - 2$, $|E_3(G)| = 1$, $|E_4(G)| = 2$, $|E_5(G)| = 12m - 5$, $|E_6(G)| = 8mn - 4m - 4n + 3$, $|E_7(G)| = 3n - 4$, $|E_8(G)| = 4m - 1$ and $|E_9(G)| = 12mn - 16m - 3n + 4$.

Theorem 3.7. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of six layered $TNT_6[m,n]$. Then first multiplicative Zagreb index for G is $2^{36mn-36m-4n+3} \times 3^{12m+5n-11} \times 5^4 \times 7^{8mn-4n+2}$.*

Proof. From Eq. (1)

$$
\begin{aligned}
II_1^*(G) &= \prod_{rt\in E(G)} (\triangle_r + \triangle_t),\\
&= \prod_{rt\in E_1(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_2(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_3(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_4(G)} (\triangle_r + \triangle_t)\\
&\times \prod_{rt\in E_5(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_6(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_7(G)} (\triangle_r + \triangle_t) \times \prod_{rt\in E_8(G)} (\triangle_r + \triangle_t)\\
&\times \prod_{rt\in E_9(G)} (\triangle_r + \triangle_t),\\
&= (1+4)^{|E_1(G)|} \times (1+5)^{|E_2(G)|} \times (2+2)^{|E_3(G)|} \times (2+3)^{|E_4(G)|} \times (2+4)^{|E_5(G)|}\\
&\times (2+5)^{|E_6(G)|} \times (3+3)^{|E_7(G)|} \times (3+4)^{|E_8(G)|} \times (3+5)^{|E_9(G)|},\\
&= 5^2 \times 2^{2n-2}.3^{2n-2} \times 2^2 \times 5^2 \times 2^{12m-5}.3^{12m-5} \times 7^{8mn-4m-4n+3}\\
&\times 2^{3n-4}.3^{3n-4} \times 7^{4m-1} \times 2^{36mn-48m-9n+12},\\
II_1^*(G) &= 2^{36mn-36m-4n+3} \times 3^{12m+5n-11} \times 5^4 \times 7^{8mn-4n+2}.
\end{aligned}
$$

□

Theorem 3.8. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of six layered $TNT_6[m,n]$. The second multiplicative Zagreb index for G is $2^{8mn+40m-4n-6} \times 3^{12mn-12m+3n-3} \times 5^{20mn-20m-5n+5}$.*

Proof. From Eq. (2)

$$
\begin{aligned}
II_2(G) &= \prod_{rt\in E(G)} (\triangle_r.\triangle_t),\\
&= \prod_{rt\in E_1(G)} (\triangle_r.\triangle_t) \times \prod_{rt\in E_2(G)} (\triangle_r.\triangle_t) \times \prod_{rt\in E_3(G)} (\triangle_r.\triangle_t) \times \prod_{rt\in E_4(G)} (\triangle_r.\triangle_t)\\
&\times \prod_{rt\in E_5(G)} (\triangle_r.\triangle_t) \times \prod_{rt\in E_6(G)} (\triangle_r.\triangle_t) \times \prod_{rt\in E_7(G)} (\triangle_r.\triangle_t) \times \prod_{rt\in E_8(G)} (\triangle_r.\triangle_t)\\
&\times \prod_{rt\in E_9(G)} (\triangle_r.\triangle_t),\\
&= (1.4)^{|E_1(G)|} \times (1.5)^{|E_2(G)|} \times (2.2)^{|E_3(G)|} \times (2.3)^{|E_4(G)|} \times (2.4)^{|E_5(G)|}\\
&\times (2.5)^{|E_6(G)|} \times (3.3)^{|E_7(G)|} \times (3.4)^{|E_8(G)|} \times (3.5)^{|E_9(G)|},\\
&= 2^4 \times 5^{2n-2} \times 2^2 \times 2^2.3^2 \times 2^{36m-15} \times 2^{8mn-4m-4n+3}.5^{8mn-4m-4n+3} \times 3^{6n-8}\\
&\times 2^{8m-2}.3^{4m-1} \times 3^{12mn-16m-3n+4}.5^{12mn-16m-3n+4},\\
II_2(G) &= 2^{8mn+40m-4n-6} \times 3^{12mn-12m+3n-3} \times 5^{20mn-20m-5n+5}.
\end{aligned}
$$

□

Theorem 3.9. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of six layered $TNT_6[m,n]$. Then multiplicative sum connectivity index for G is $\sqrt{2^{36m+4n-36mn-3}} \times \sqrt{3^{11-12m-5n}} \times \sqrt{5^{-4}} \times \sqrt{7^{4n-8mn-2}}$.*

Proof. From Eq. (7)

$$
\begin{aligned}
SCII(G) &= \prod_{rt\in E(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}},\\
&= \prod_{rt\in E_1(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_2(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_3(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_4(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}}\\
&\times \prod_{rt\in E_5(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_6(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_7(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}} \times \prod_{rt\in E_8(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}}\\
&\times \prod_{rt\in E_9(G)} \frac{1}{\sqrt{\triangle_r+\triangle_t}},\\
&= \left(\frac{1}{\sqrt{1+4}}\right)^{|E_1(G)|} \times \left(\frac{1}{\sqrt{1+5}}\right)^{|E_2(G)|} \times \left(\frac{1}{\sqrt{2+2}}\right)^{|E_3(G)|} \times \left(\frac{1}{\sqrt{2+3}}\right)^{|E_4(G)|}\\
&\times \left(\frac{1}{\sqrt{2+4}}\right)^{|E_5(G)|} \times \left(\frac{1}{\sqrt{2+5}}\right)^{|E_6(G)|} \times \left(\frac{1}{\sqrt{3+3}}\right)^{|E_7(G)|} \times \left(\frac{1}{\sqrt{3+4}}\right)^{|E_8(G)|}\\
&\times \left(\frac{1}{\sqrt{3+5}}\right)^{|E_9(G)|},\\
&= \frac{1}{5} \times \frac{1}{2^{n-1}.3^{n-1}} \times \frac{1}{2} \times \frac{1}{5} \times \frac{1}{\sqrt{2^{12m-5}.3^{12m-5}}} \times \frac{1}{\sqrt{7^{8mn-4m+4n+3}}}\\
&\times \frac{1}{\sqrt{2^{3n-4}.3^{3n-4}}} \times \frac{1}{\sqrt{7^{4m-1}}} \times \frac{1}{\sqrt{2^{36mn-48m-9n+12}}},\\
SCII(G) &= \sqrt{2^{36m+4n-36mn-3}} \times \sqrt{3^{11-12m-5n}} \times \sqrt{5^{-4}} \times \sqrt{7^{4n-8mn-2}}.
\end{aligned}
$$

□

Theorem 3.10. *Let $G(V(G),E(G))$ be the graph of single−walled Titania nanotubes of six layered $TNT_6[m,n]$. Then multiplicative product connectivity index for G is*

$$2^{2n+3-4mn-20m} \times \sqrt{3^{12m+3-12mn-3n}} \times \sqrt{5^{20m+5n-20mn-5}}.$$

Proof. From Eq. (8)

$$\begin{aligned}
PCII(G) &= \prod_{rt\in E(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}},\\
&= \prod_{rt\in E_1(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_2(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_3(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_4(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_5(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_6(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_7(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}} \times \prod_{rt\in E_8(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_9(G)} \frac{1}{\sqrt{\triangle_r.\triangle_t}},\\
&= \left(\frac{1}{\sqrt{1.4}}\right)^{|E_1(G)|} \times \left(\frac{1}{\sqrt{1.5}}\right)^{|E_2(G)|} \times \left(\frac{1}{\sqrt{2.2}}\right)^{|E_3(G)|} \times \left(\frac{1}{\sqrt{2.3}}\right)^{|E_4(G)|}\\
&\times \left(\frac{1}{\sqrt{2.4}}\right)^{|E_5(G)|} \times \left(\frac{1}{\sqrt{2.5}}\right)^{|E_6(G)|} \times \left(\frac{1}{\sqrt{3.3}}\right)^{|E_7(G)|} \times \left(\frac{1}{\sqrt{3.4}}\right)^{|E_8(G)|}\\
&\times \left(\frac{1}{\sqrt{3.5}}\right)^{|E_9(G)|},\\
&= \frac{1}{2} \times \frac{1}{5^{n-1}} \times \frac{1}{2} \times \frac{1}{2.3} \times \frac{1}{2^{36m-15}} \times \frac{1}{\sqrt{2^{8mn-4m-4n+3}.5^{8mn-4m-4n+3}}} \times \frac{1}{3^{3n-4}}\\
&\times \frac{1}{2^{4m-1}.\sqrt{3^{4m-1}}} \times \frac{1}{\sqrt{3^{12mn-16m-3n+4}.5^{12mn-16m-3n+4}}},\\
PCII(G) &= 2^{2n+3-4mn-20m} \times \sqrt{3^{12m+3-12mn-3n}} \times \sqrt{5^{20m+5n-20mn-5}}.
\end{aligned}$$

□

Theorem 3.11. *Let $G(V(G), E(G))$ be the graph of single−walled Titania nanotubes of six layered $TNT_6[m,n]$. Then multiplicative atomic bond connectivity index for G is*

$$\sqrt{2^{4mn+11n-32m-11}} \times \sqrt{3^{11-4m-6n}} \times \sqrt{5^{20m+n-12mn-3}}.$$

Proof. From Eq. (9)

$$
\begin{aligned}
ABCII(G) &= \prod_{rt\in E(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}},\\
&= \prod_{rt\in E_1(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_2(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_3(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_4(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_5(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_6(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}}\\
&\times \prod_{rt\in E_7(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_8(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}} \times \prod_{rt\in E_9(G)} \sqrt{\frac{\triangle_r+\triangle_t-2}{\triangle_r.\triangle_t}},\\
&= \left(\sqrt{\frac{1+4-2}{4}}\right)^{|E_1(G)|} \times \left(\sqrt{\frac{1+5-2}{5}}\right)^{|E_2(G)|} \times \left(\sqrt{\frac{2+2-2}{2.2}}\right)^{|E_3(G)|}\\
&\times \left(\sqrt{\frac{2+3-2}{2.3}}\right)^{|E_4(G)|} \times \left(\sqrt{\frac{2+4-2}{2.4}}\right)^{|E_5(G)|} \times \left(\sqrt{\frac{2+5-2}{2.5}}\right)^{|E_6(G)|}\\
&\times \left(\sqrt{\frac{3+3-2}{3.3}}\right)^{|E_7(G)|} \times \left(\sqrt{\frac{3+4-2}{3.4}}\right)^{|E_8(G)|} \times \left(\sqrt{\frac{3+5-2}{3.5}}\right)^{|E_9(G)|},\\
&= \frac{3}{2^2} \times \frac{2^{2n-2}}{5^{n-1}} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} \times \frac{2^{12m-5}}{\sqrt{2^{36m-15}}} \times \frac{1}{\sqrt{2^{8mn-4m-4n+3}}}\\
&\times \frac{2^{3n-4}}{3^{3n-4}} \times \frac{\sqrt{5^{4m-1}}}{2^{4m-1}.\sqrt{3^{4m-1}}} \times \frac{\sqrt{2^{12mn-16m-3n+4}}}{\sqrt{5^{12mn-16m-3n+4}}},\\
ABCII(G) &= \sqrt{2^{4mn+11n-32m-11}} \times \sqrt{3^{11-4m-6n}} \times \sqrt{5^{20m+n-12mn-3}}.
\end{aligned}
$$

□

Theorem 3.12. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of single−walled Titania nanotubes of six layered $TNT_6[m,n]$. Then multiplicative geometric arithmetic index of G is*

$$2^{52m-12mn-6} \times \sqrt{3^{12mn-36m-7n+19}} \times \sqrt{5^{20mn-20m-5n-3}} \times 7^{4n-8mn-2}.$$

Proof. From Eq. (10)

$$
\begin{aligned}
G^*AII(G) =& \prod_{rt\in E(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t},\\
=& \prod_{rt\in E_1(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_2(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_3(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t}\\
&\times \prod_{rt\in E_4(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \prod_{rt\in E_5(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_6(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t}\\
&\times \prod_{rt\in E_7(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_8(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t} \times \prod_{rt\in E_9(G)} \frac{2\sqrt{\triangle_r.\triangle_t}}{\triangle_r+\triangle_t},\\
=& \left(\frac{2\sqrt{4}}{1+4}\right)^{|E_1(G)|} \times \left(\frac{2\sqrt{5}}{1+5}\right)^{|E_2(G)|} \times \left(\frac{2\sqrt{2.2}}{2+2}\right)^{|E_3(G)|} \times \left(\frac{2\sqrt{2.3}}{2+3}\right)^{|E_4(G)|}\\
&\times \left(\frac{2\sqrt{2.4}}{2+4}\right)^{|E_5(G)|} \times \left(\frac{2\sqrt{2.5}}{2+5}\right)^{|E_6(G)|} \times \left(\frac{2\sqrt{3.3}}{3+3}\right)^{|E_7(G)|} \times \left(\frac{2\sqrt{3.4}}{3+4}\right)^{|E_8(G)|}\\
&\times \left(\frac{2\sqrt{3.5}}{3+5}\right)^{|E_9(G)|},\\
=& \frac{2^4}{5} \times \frac{5^{n-1}}{3^{2n-2}} \times \frac{2^3.3}{5^2} \times \frac{2^{36m-15}}{3^{12m-5}} \times \frac{\sqrt{2^{24mn-12m-12n+9}.5^{8mn-4m-4n+3}}}{7^{8mn-4m-4n+3}}\\
&\times \frac{2^{8m-2}.\sqrt{3^{4m-1}}}{7^{4m-1}} \times \frac{\sqrt{3^{12mn-16m-3n+4}.5^{12mn-16m-3n+4}}}{2^{24mn-32m-6n+8}},\\
G^*AII(G) =& 2^{52m-12mn-6} \times \sqrt{3^{12mn-36m-7n+19}} \times \sqrt{5^{20mn-20m-5n-3}} \times 7^{4n-8mn-2}.
\end{aligned}
$$

□

Remark 3.1. *By using fact 2.1 we can compute the following very easily.*

i $HII_1(TNT_3[m,n]) = 2^{32m+18n-44} \times 3^{48mn-56m-30n-36} \times 5^6 \times 7^{8m+2n-6}$.

ii $HII_2(TNT_3[m,n]) = 2^{24mn+24m-8n-12} \times 3^{48mn-48m-18n+18}$.

iii $MZ_1^\alpha(TNT_3[m,n]) = 2^{16\alpha m+9\alpha n-22\alpha} \times 3^{24\alpha mn-28\alpha m-15\alpha n-18\alpha} \times 5^{3\alpha} \times 7^{4\alpha m+\alpha n-3\alpha}$.

iv $MZ_2^\alpha(TNT_3[m,n]) = 2^{12\alpha mn+12\alpha m-4\alpha n-6\alpha} \times 3^{24\alpha mn-24\alpha m-9\alpha n+9\alpha}$.

v $G^*A^\alpha(TNT_3[m,n]) = 2^{18\alpha mn-14\alpha m-14\alpha n+20\alpha} \times \sqrt{3^{32\alpha m+21\alpha n-24\alpha mn-27\alpha}} \times 5^{-3\alpha} \times 7^{3\alpha-4\alpha m-\alpha n}$.

vi $HII_1(TNT_6[m,n]) = 2^{72mn-72m-8n+6} \times 3^{24m+10n-22} \times 5^8 \times 7^{16mn-8n+4}$.

vii $HII_2(TNT_6[m,n]) = 2^{16mn+80m-8n-12} \times 3^{24mn-24m+6n-6} \times 5^{40mn-40m-10n+10}$.

viii $MZ_1^\alpha(TNT_6[m,n]) = 2^{36\alpha mn-36\alpha m-4\alpha n+3\alpha} \times 3^{12\alpha m+5\alpha n-11\alpha} \times 5^{4\alpha} \times 7^{8\alpha mn-4\alpha n+2\alpha}$.

ix $MZ_2^{\alpha}(TNT_6[m,n]) = 2^{8\alpha mn+40\alpha m-4\alpha n-6\alpha} \times 3^{\alpha mn-12\alpha m+3\alpha n-3\alpha} \times 5^{20\alpha mn-20\alpha m-5\alpha n+5\alpha}$.

x $G^*A^{\alpha}(TNT_6[m,n]) = 2^{52\alpha m-12\alpha mn-6\alpha} \times \sqrt{3^{12\alpha mn-36\alpha m-7\alpha n+19\alpha}} \times \sqrt{5^{20\alpha mn-20\alpha m-5\alpha n-3\alpha}} \times 7^{4\alpha n-8\alpha mn-2\alpha}$.

Conclusion

In this paper we have calculated some degree based multiplicative topological indices of single−walled three layered and six layered Titania nanotubes.

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