**Research paper**

# Numerical investigation on cooling performance of a twisted gas turbine blade with leading edge cooling holes

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**Abstract**

The new and advance technologies for higher performance and lower maintenance are required to operate gas turbines at higher operating temperatures. Higher turbine inlet temperature results in higher blade metal temperatures. These variations in temperatures of the blade material must be limited such that the blades have a sufficient life span. To make blade material temperature within the limits, the coolant air is bled from the compressor to protect the outer surface of the turbine blade from the hot gases. The purpose of this study is to investigate the cooling performance of a blade with leading edge cooling holes. The numerical simulation approach using ANSYS Fluent has been considered. The analysis is performed by taking different hole geometries namely cylindrical (model 1) and tapered (model 2) on the leading edge of the turbine blade for different blowing ratios. The analysis also compares the cooling effectiveness of the blade for two different coolants namely air and nitrogen. The results show that for highest effectiveness hole (E3 hole), Model 1 and Model 2 comparison suggest that Model 1 has 1.2% more cooling effectiveness for air as coolant. For E3 hole, the comparison of Model 1 between two coolants show that film cooling effectiveness of the air gives 0.6% more film cooling effectiveness compared to nitrogen. The presented work helps researchers and blade manufacturers to select the correct hole geometry, coolant type, and determine the best blowing ratio to improve the film cooling efficiency of gas turbine blades with leading edge holes.

## 1. Introduction

High operating temperatures lead to higher thermal stresses and results in the ultimate failure of the blade. Typically to minimize the thermal stress on blade material, blade cooling is provided. The turbine blades are cooled by extracting some of the compressed air through bleed and passing into the hollow gas turbine blades from the blade root. The coolant travels through a serpentine path and emerges out from the hollow blade through the holes provided in the leading edge, the periphery of the blade, and from the radial holes. This emerged coolant air

creates a film of air that acts as a thermal barrier between mainstream hot gases and the metal surface of the turbine blade. The coolant thereafter mixes with the mainstream hot gases, which gives an advantage of a higher mass flow rate and results in greater power output during expansion in subsequent stages. Many numerical and experimental studies have been carried out on gas turbine blades used for metal temperature prediction. Schmidt et al. [1] carried out an experiment on a plate with holes provided with 3 compound corners. The work reports the film cooling effect of circular holes and diffusion holes with a composite angle of 60° under flux variation on the plate. Chowdhury et al. [2] performed some experiments on the film cooling with 3 different leadingedge models and also compared the 2 cooling configurations at 3 blowing ratios. They reported that the film cooling effectiveness with typical radial angle cylindrical holes increases with increasing blowing ratios. Nandakumar et al. [3] investigated the base temperature of a turbine blade when coolant is ejected from the trailing edge at a span-wise angle. They reported that the temperature reduction rate of the turbine blade without the cooling passages is 3.727% and for the blade with the cooling passages is 53.67%. Paregouda and Rao [4] also carried out some experiments on a plate and reported that by increasing the exit angle of the cooling holes, the film effectiveness increases and this in turn reduces the metal surface temperature. Brahmaiah et al. [5] conducted a heat transfer analysis on different types of gas turbine blades including blades with and without holes and blades with different numbers of cooling holes. The work stated that for a blade composed of 13 holes, the rate of the heat transfer was maximum, while the leading edge had the minimum temperature. The thermal and structural analysis of two different materials, namely chrome steel and Inconel718 were studied. It is found that the Inconel718 blade composed of 13 holes had the maximum heat flux, and the induced von misses stress and strain were within the allowable range. The y concluded that Inconel718 is better than chrome steel in blade material. Zeng, et al. [6] examined the cooling effectiveness on 3 simplifications of the blade (straight blade Vs twisted blade, linear cascade Vs annular

cascade, and non-rotation Vs rotation). The result of the analysis showed that due to change in the geometry of straight and twisted blade, the stagnation line on the leading edge changed. It also reported that the pressure side had more cooling effectiveness compared to the suction side. Li et al. [7] experimentally studied the effects of injection angle and blowing ratio on the film cooling of the leading edge of a twisted gas turbine blade. The article reported that for an injection angles of 30° and 45°, as the BR (blowing ratio) increased, the average film cooling efficiency at all locations also increased. The main factors to improve efficiency were the angle of the hole to the surface and the distance between the holes. Bacci et al. [8] presented the numerical and experimental investigations on a gas turbine and concluded that metal temperature decreases as cooling flow increases. After a thorough literature review, the scope for the present work is identified. The authors in previous works [9-15] have analyzed different cooled gas turbine configurations on an exergoeconomic basis. All the configurations were analyzed with the air film cooling method of turbine blade cooling. Andrews et al. [16] experimentally compared the effusion and transpiration cooling to determine the better choice between two. The primary factors for improving effectiveness were the hole angle in contradiction of the surface and spacing of the holes. In this regard Andrews et al. [17-19] reported the importance of discrete holes on film cooling effectiveness. Andrews et al. [18] experimentally analyzed the influence of hole size on cooling effectiveness for plate. They also analyzed the coolant density variation and the effect of same on cooling effectiveness [19]. Mendez et al. [20] reported that boundary layer developed by effusion and transpiration cooling differs from each other because in transpiration cooling impulse was much higher. Sinha et al. [21 and 22] have experimentally analyzed the effect of number of rows of cooling holes on cooling effectiveness. Laschet et al. [23] and [24] discussed numerical methods and used conjugate codes to simulate an array of small-sized effusion cooling holes. Sasaki et al. [25] reported the adiabatic cooling effect of a single hole and multiple rows of holes, and compared them with the help of an infrared thermal imaging camera, and compared the results with

the superposition model. Metzger et al. [26] studied an aluminum plate with small cooling holes for in-line and staggered arrangement through experiments, p / D = 4.8. Eckert [27] studied the leading edge area of the blade with 11 rows of cooling holes. Mayle et al. [28] experimented by replacing the cooling holes with heat sinks. The results obtained were not found to be at a satisfactory level. Ekkad et al. [29] analyzed the effect of density for orientation angles of 0°, 45°, & 90° and stated that compound angle injection provides higher cooling effectiveness. Mehendale et al. [30] have reported the turbulence effect on leading edge heat transfer.

The review of literature in the field suggests that researchers have performed the experimental and numerical analysis on a flat plate instead of a real gas turbine blade. In this regard, the present work numerically investigated the twisted gas turbine blade incorporating leading edge cooling holes. The work examined the effect of change in hole geometry (cylindrical holes and tapered holes) for the same angle of inclination. The study compares the two aforesaid models based on cooling effectiveness for varying blowing ratios. The present work also analyses the film cooling effectiveness for two different coolants namely air and nitrogen. The novelty of this work is to perform the analysis of unique models (Model 1 cylindrical holes and Model 2 trapezoidal holes for the same angle of inclination). The work reported the unique results and has significant scientific merits as the presented results can be used as a helpful source of knowledge for power utility developers.

## 2. Geometry creation

A new turbine blade was created using the Eppler airfoil code. In the creation of a twisted turbine blade: first, all points were plotted on 3-dimensional space and by the cross wire, all points were joint to achieve the geometry. Fig. 1 shows the blade with leading edge holes. It also presents the top and bottom view of the blade profile showing the leading amd trailing edge; and pressure and suction side of the blade. A total of 20 leading edge cooling holes with two different geometric hole

profiles/models (cylindrical holes and tapered holes) were considered. The diameter of the cylindrical hole is 0.375mm while the diameters of tapered holes are 0.375mm and 0.250mm. The twisted blade model has a twisted profile in the radial direction with a total angle of twist of 7°. Fig. 2 shows the hole positioning and the difference between model 1 and model 2. The computational domain consists of the mainstream fluid flow passage, the leading edge cooling holes, and the radial cooling holes which act as plenums for coolant. They are presented in Fig. 3.

## 3. Modelling and boundary conditions
### 3.1. Model selection

The Standard k–ε model has been implemented for the investigation. This model is widely used for turbulent flow conditions in CFD simulation to simulate the average flow characteristics. This is a two-equation model that provides a general description of turbulence through two transport equations. The original motivation of the k-ε model is to improve the mixing length model and find an algebraic method to algebraically specify the turbulence length scale in the medium to high complexity flow. The adopted blade material is Inconel 718. The operational parameters are tabulated in Table 1. The blowing ratio is the mass flux ratio of coolant stream and mainstream hot gases. Fig. 4 is a schematic representation of the blowing ratio. Mathematically, it is represented in Eq. (1) as below:

$$\text{Blowing Ratio (BR)} = \frac{(\rho_c \times U_c)}{(\rho_\infty \times U_\infty)} \tag{1}$$

where,
$U_\infty$ = 262 m/s
$U_c$ will be calculated based on assumed blowing ratio
$\rho_\infty$ = 0.275 kg/m³
$\rho_c$ = 0.456 kg/m³

### 3.2. Governing equations

The three basic governing equations (continuity, momentum and energy) are considered to simulat the problem. Along with these equations the turbulent equations are used to simulate turbulent flow.
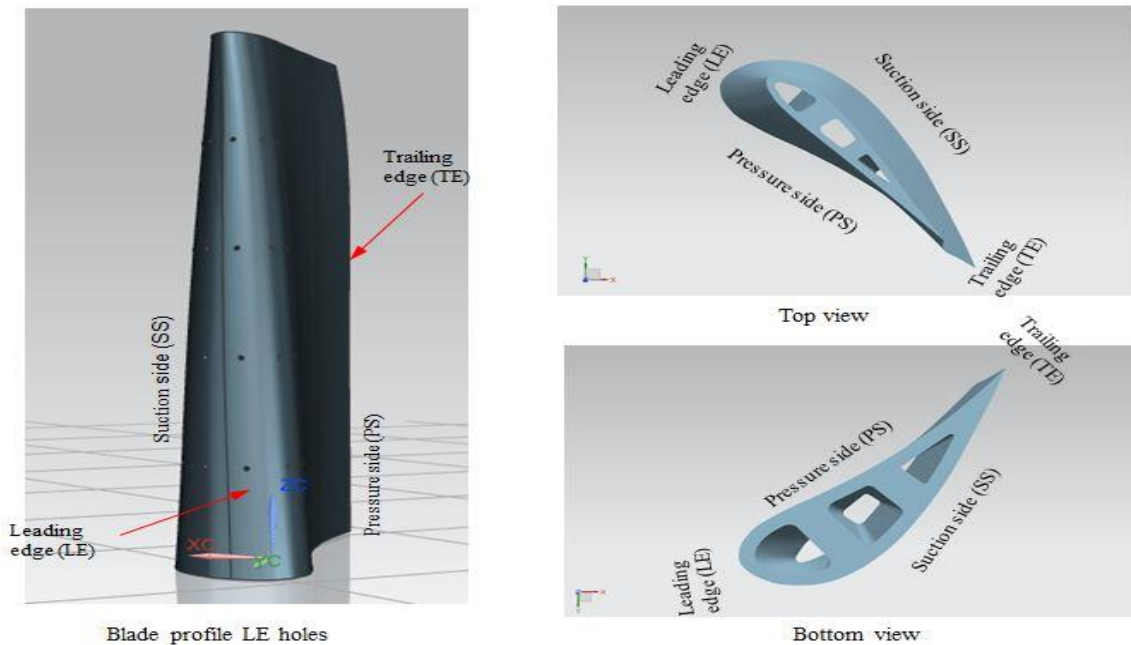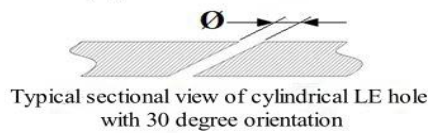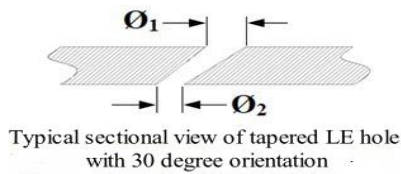
**Fig. 1.** The turbine blade showing leading edge cooling holes and different views of blades.
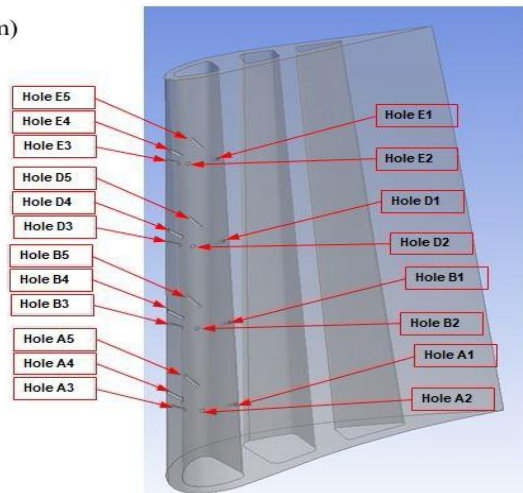


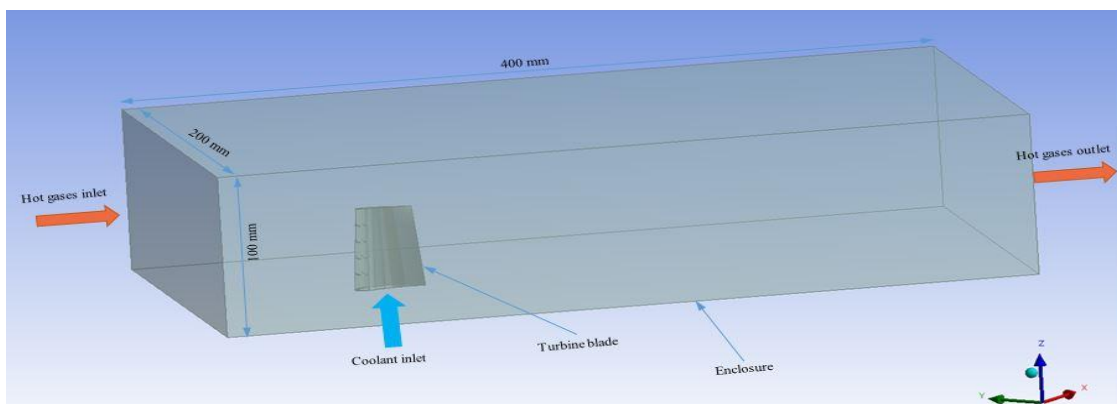**Fig. 2.** Details of model 1 and model 2 with hole positions.



**Fig. 3.** Isometric view of a turbine blade with enclosure dimensions.

**Table 1.** Operational parameters for film-cooling performance analysis.

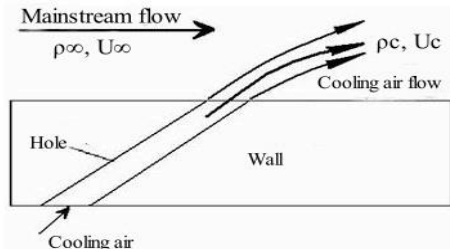| Parameter | Value | Unit |
|---|---|---|
| Ratio of mainstream to coolant temperature | 2:1 | |
| Mainstream inlet temperature | 1273 | K |
| Coolant inlet temperature | 773 | K |
| Blowing ratios | 0.9, 1, 1.1, 1.2 | |
| Mainstream inlet Reynolds number | 200000 | |



**Fig. 4.** Blowing ratio model.

The equations are formulated by taking the assumptions of inviscid dissipation, incompressible and steady flow and in the end by taking the buoyancy effects into consideration. The governing Eqs. (2-4) are as follows

Continuity equation:

$$\nabla \cdot (\rho u) = 0 \tag{2}$$

Momentum equation:

$$(u \cdot \rho \nabla) u + \frac{1}{\rho} \nabla p - \nu \nabla^2 u = 0 \tag{3}$$

Energy equation:

$$\rho c_P (u \cdot \rho \nabla T) - k \nabla^2 T = 0 \tag{4}$$

*3.3. Mesh quality*

Mesh is done in ANSYS meshing. The mesh on the turbine blade is tetrahedral elements. The mainstream hot gases that have tetrahedral elements with inflation (sub-layers) are used near the turbine blades and mainstream hot gases interface. Five inflation layers are used to get fine results. The cold gas fluid interface is the combination of tetrahedral grids and hexahedron elements to get the highest quality of results at boundary layers. The total numbers of elements are 212969 and a grid independence test is performed.

*3.4. Film cooling effectiveness ( η)*

It is a non-dimensional parameter used to determine the film cooling performance. It is given by Eq. (5) as below:

$$\eta = \frac{(T_m - T_{aw})}{(T_m - T_C)} \tag{5}$$

*3.5. Grid independence*

In the present work, one of the models has been tested for grid independence to find the optimal number of elements which can provide optimal solution of the problem. Having optimal number of elements also results in reduction in simulation time. The grid convergence test was performed for model 1 with a blowing ratio of 1. The test was carried out by changing the mesh size by body size operation of the components, and by comparing the results of the adjacent wall temperature of the turbine blade.

*3.6. Solution convergence*

Solution convergence is one of the most important tests to check the feasibility and accuracy of the results of the analysis. In this particular work, the convergence is achieved for model 1 and model 2 respectively in a limited number of iterations.

**4. Results and discussion**

In this work, the spanwise film cooling effectiveness was numerically analyzed to study the effects of blowing ratios and variation of coolant densities at the leading edge of the twisted gas turbine blade. The horizontal ordinate represents the geometric distance along the X-Plane measured from the centerline.
The negative value represents the holes at the pressure side and a positive value represents the holes at the suction side. The vertical abscissa η represents the Film cooling effectiveness at the leading edge of the blade. Fig. 5 depicts the dynamics of the flow and the streamlines for gas turbine blade cooling. Fig. 6 shows the temperature distribution for both models, i.e. for cylindrical holes and tapered holes for a blowing ratio of 1.
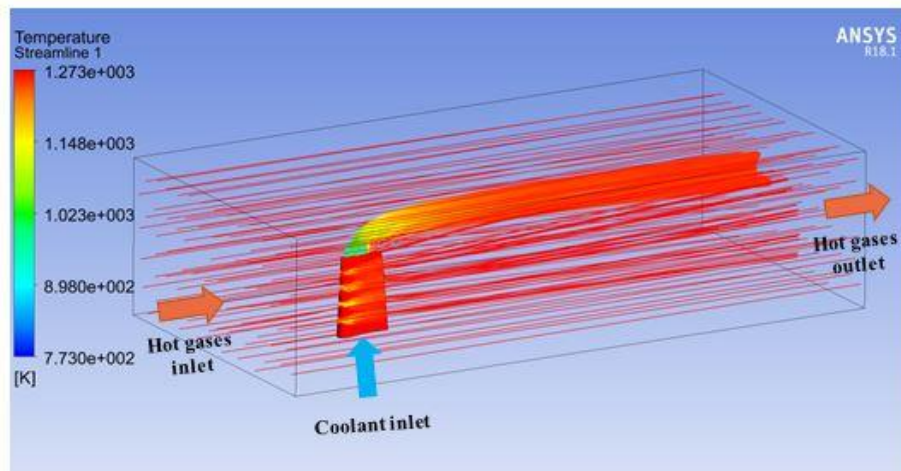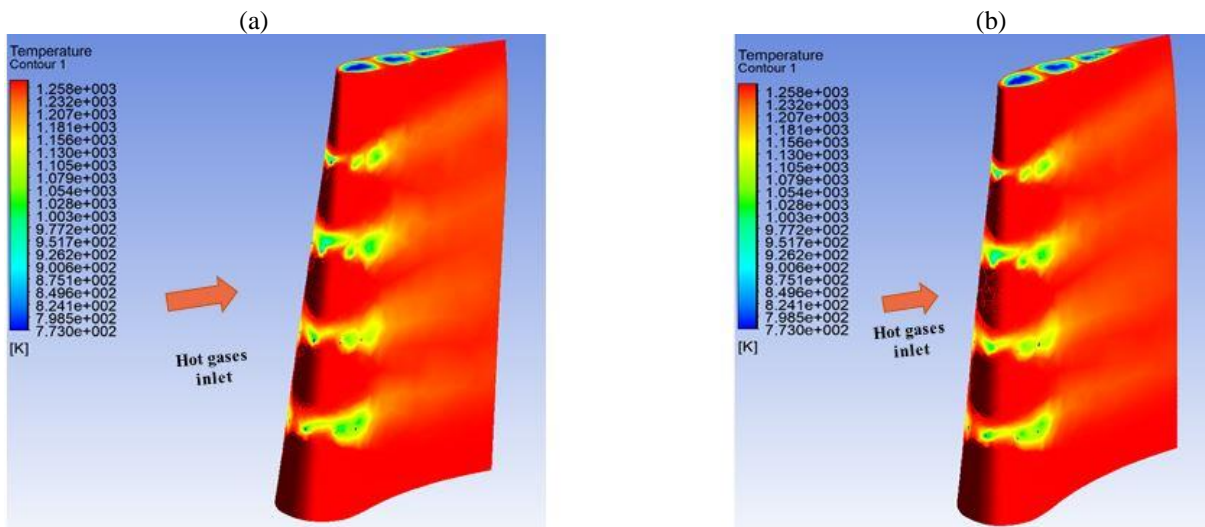
**Fig. 5.** Streamlines for gas turbine blade cooling.



**Fig. 6.** Temperature distribution: (a) model 1 and (b) model 2 for blowing ratio of 1.

### 4.1. The positioning of leading edge holes for model 1 and model 2

Table 2 shows the positioning of leading edge holes for model 1 and model 2.

### 4.2. Effect of blowing ratios

Fig. 7 depicts the variation of film cooling effectiveness for different blowing ratios considered (0.9, 1.0, 1.1, and 1.2 for Model 1). The arrows shown in Fig. 7 show the location of the hole on the leading edge of the blade. Fig. 7 shows that when the blowing ratio increases from 0.9 to 1.1, the efficiency increases, and then when the blowing ratio is 1.2, a slight decrease in efficiency is observed. As the blowing ratio increases, the momentum of the coolant jet increases and gradually it becomes equal to the momentum of the mainstream. Therefore, the traces of the coolant jet can better withstand the pressure of the main flow, so that the coolant jet can be ejected, thereby playing a better protective effect. The membrane effectiveness decreases on the pressure side, tends to increase when stagnant, and then decreases again when it reaches the suction side of the blade. Compared with the suction side, the film cooling effect has a greater influence on the pressure side.

Fig. 8 shows the effect of varying blowing ratios (0.9 to 1.2 for Model 1) on film cooling effectiveness. The holes of model 1 are arranged as per their geometric distance from negative to positive (See Table 2). The distance zero represents the stagnation line for models. Negative geometric distances represent the pressure side while positive distances denote the suction side of the blade. From Fig. 8, it can be seen that with increasing blowing ratio the film cooling effectiveness also increases. This increase is limited for a blowing ratio of 0.9 to 1.1 and it starts decreasing for the presented model. The highest peak was observed at the stagnation line of E3 hole for cooling effectiveness. The film cooling efficiency decreases on the pressure side, tends to increase when stagnant, and then decreases again when it reaches the suction side of the blade.

In Fig. 9, the variation of cooling effectiveness for tapered holes is presented for varying blowing ratios (0.9 to 1.2). Fig. 9 shows the cooling effectiveness of each hole positioned at a different geometric distance. The effectiveness curve resembles the same nature as in model 1 with lower magnitudes.

In Fig. 9, the variation of cooling effectiveness for tapered holes is presented for varying blowing ratios (0.9 to 1.2). Fig. 9 shows the cooling effectiveness of each hole positioned at

a different geometric distance. The effectiveness curve resembles the same nature as in model 1 with lower magnitudes. From Fig. 9, it can be seen that with increasing the blowing ratio, the film cooling effectiveness increases up to BR value of 1.1 and afterward it starts decreasing. The highest peak was observed at the stagnation line of E3 hole for cooling effectiveness. The film cooling efficiency decreases on the pressure side, tends to increase when stagnant, and then decreases again when it reaches the suction side of the blade.

Fig. 10 depicts the variation of film cooling effectiveness for tapered holes with varying BR of 0.9 to 1.2. The pressure and suction sides are defined based on negative and positive geometric distances. The curve shows, the same trend as shown for model 1 with slightly lower values of film cooling effectiveness. Fig. 10 shows that when the blowing ratio increases from 0.9 to 1.1, the effectiveness increases, and then, when the BR is 1.2, a slight decrease in effectiveness is observed. Hole E3 shows the highest peak of film cooling effectiveness at the stagnation line. The film cooling efficiency on the pressure side decreases and tends to increase when stagnant, and then decreases again when it reaches the suction side of the blade.

**Table 2.** Leading edge hole positions for model 1 and model 2.

| S. no. | Hole name | Model 1 (cylindrical LE holes) geometric distance X  (mm) | Model 2 (tapered LE holes) geometric distance X  (mm) |
|--------|-----------|------------------------------------------------------------|--------------------------------------------------------|
| 1 | A1 | -3.9352 | -3.9354 |
| 2 | A2 | -3.6798 | -3.6767 |
| 3 | B1 | -3.3351 | -3.3351 |
| 4 | B2 | -2.9767 | -2.9766 |
| 5 | D1 | -2.7050 | -2.7050 |
| 6 | D2 | -2.3009 | -2.3007 |
| 7 | E1 | -1.9336 | -1.9336 |
| 8 | E2 | -1.5039 | -1.5037 |
| 9 | A3 | -0.8870 | -0.8870 |
| 10 | B3 | -0.3423 | -0.3426 |
| 11 | D3 | 0.0903 | 0.0895 |
| 12 | E3 | 0.5386 | 0.5373 |
| 13 | A4 | 2.3059 | 2.3059 |
| 14 | E4 | 2.4212 | 2.4204 |
| 15 | B4 | 2.5242 | 2.5238 |
| 16 | D4 | 2.9242 | 2.9238 |
| 17 | E5 | 3.6830 | 3.6828 |
| 18 | D5 | 4.2268 | 4.2266 |
| 19 | B5 | 4.5557 | 4.5556 |
| 20 | A5 | 4.7178 | 4.7176 |
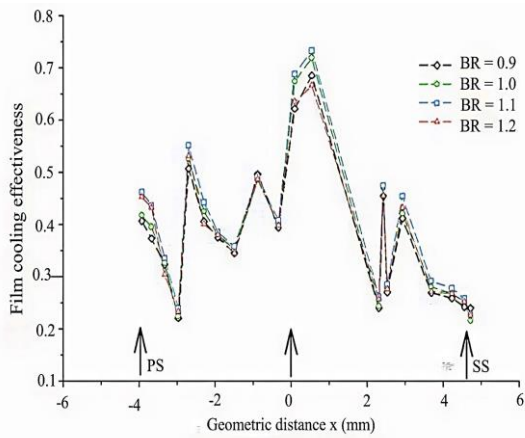
**Fig. 7.** Film cooling effectiveness vs blade geometric distance with varying blowing ratio for cylindrical LE holes with air as coolant.
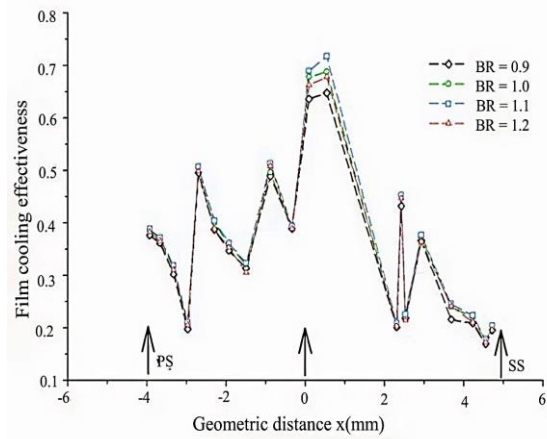


**Fig. 10.** Film cooling effectiveness vs blade geometric distance with varying blowing ratio for tapered LE holes with nitrogen as a coolant.
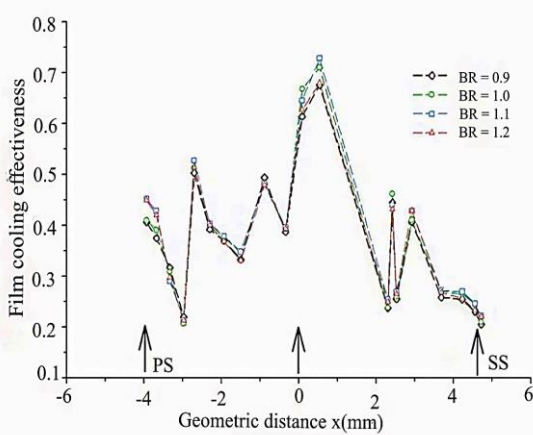


**Fig. 8.** Film cooling effectiveness vs blade geometric distance with varying blowing ratio for cylindrical LE holes with nitrogen as a coolant.
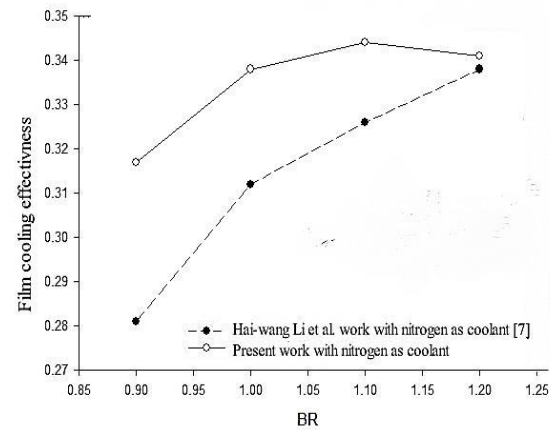


**Fig. 11.** Film cooling effectiveness vs blowing ratio for cylindrical LE holes with nitrogen as a coolant (validation graph).
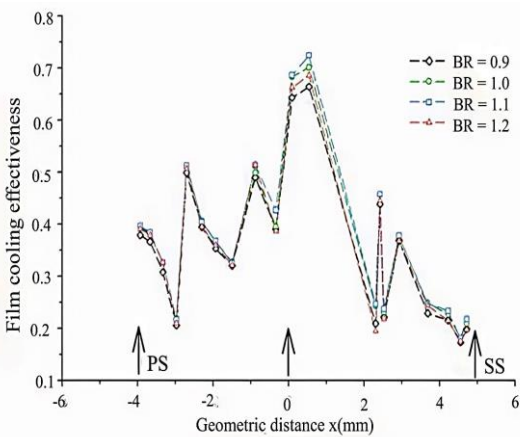
### 4.3. Effect of coolant densities

- It has been observed that the film cooling effectiveness is more for higher density coolants compared to lower density coolant.
- In present work, air as a coolant is showing greater effectiveness as compared to the nitrogen as a coolant.

### 4.4. Validation of results

As shown in Fig. 11, the present work is validated with Hai-wang Li et al's [7] work on a twisted gas turbine blade with nitrogen as a coolant. The results are compared for cooling effectiveness with varying blowing ratio and



**Fig. 9.** Film cooling effectiveness vs blade geometric distance with varying blowing ratio for tapered LE holes with air as coolant.

were found to be within an acceptable limit, with the highest percentage of error 12.81%.

## 5. Conclusions

The numerical simulation has been performed and results have been discussed. To provide insight information, the major conclusions are drawn and presented as follows:

- Different blowing ratios studied (0.9 to 1.2) were taken for analysis with two different coolants namely air and nitrogen.
- The film cooling efficiency of the blade increases upto a specific level of blowing ratio (here, 1.1), and then begins to decrease.
- As the density of the coolant increases, the efficiency of film cooling also increases.
- Here, it is found that a cylindrical hole provides better spanwise film cooling effect than a tapered hole. Ccoolant as air has better spanwise cooling when compared with nitrogen for both the models.
- Considering E3 hole (the most effective hole), the cooling efficiency of the cylindrical LE hole using air as the coolant is 1.2% higher than that of the tapered LE hole using air as the coolant.
- Comparing hole E3 of Model 1 (cylindrical LE hole) between the two coolants shows that the film cooling efficiency of air as a coolant is 0.6% higher than that of nitrogen as a coolant.

The articles discussed here will provide a useful source of knowledge for researchers and gas turbine blade designers in the field. The methodology given will help them to simulate the conditions and to get information regarding the coolant type and best blowing ratio; and to determine the correct hole geometry for improving the cooling effectiveness of GT blades with LE holes.

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