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# An Approach for Solving Signal Cancellation Problem in Spherical Microphone Array

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## Abstract

**Background and Objectives:** One major problem in the minimum power distortionless response (MPDR) beamformer is the signal cancellation problem, i.e., the desired signal is canceled by the reflected signal, even though the distortionless response constraint is satisfied. Solving this problem is the objective of this paper.
**Methods:** It is well known that the signal cancellation problem can be avoided by minimizing the cross-spectrum matrix of noise, i.e., using the minimum variance distortionless response (MVDR) beamformer. But, in the case of disturbance signals which have correlation with the desired signal, estimation of this matrix is a challenging problem. In this paper we propose an approach for estimating the cross-spectrum matrix of noise signal from which we can solve the signal cancellation problem.
**Results:** Simulation examples show that using the proposed method we can bypass the signal cancellation problem completely.
**Conclusion:** A common belief is that in the case of a disturbance that is a reflected version of the desired signal, due to cohesive appearance and disappearance of both the disturbance and the desired signal, the estimation of cross-spectrum matrix of noise signal is typically not possible in practice. So, based on this common belief, we can't use the MVDR beamformer in this case. In this paper we show that this common belief is a fault. We propose a general approach for estimating the cross-spectrum matrix of noise signal that is applicable even in the case of correlated disturbances.



## Introduction

Spherical microphone array is a type of microphone arrays that has a spherical structure in which microphones are placed on a sphere surface. This kind of microphone arrays has been an interesting field of study for the past decade [1], [2], [3], [4], [5], [6]. The microphone array produces an output signal with some desired properties from the microphones input signals. One such desired property is to enhance signal coming from a particular direction and to weaken signals coming from other directions, therefore forming a directional, or spatial filter. This filter forms the beam that looks at a desired direction, so it is called beamformer [1], [2], [3], [7]. Noise minimization bemformer is one important type of beamformer in which the beam pattern is adapted to the actual sound field.

This beamformer discerns the desired signal from the noise and, therefore achieves better performance in real, and noisy sound fields. When the noise field is not perfectly diffuse, the optimality of the beamformers which are optimal in decreasing noise due to diffuse sound field, such as maximum directivity beamformer, is not maintained any more [1]. In this case an optimum beam pattern, adapted to the true measured noise, should be constructed. The minimum power distortionless response (MPDR) is one such a construction in which the beam pattern is restricted to be unity in the look direction, meanwhile the power of the array output is minimized [1], [2]. The MPDR beamformer is beneficial especially when the desired signal propagates with a plane wave coming from the look direction, and all other signals considered as noise that to be minimized.

One of the main problems in the MPDR beamformer is signal cancellation [1], [8], [9], [10], [11], [12], [13]. This problem occurs when the disturbance signals contain some signals that have correlation with the desired signal. For example, whenever the desired signal being reflected from neighboring surfaces such as walls in a room, signal cancellation may happen. So, these disturbance signals are the weakened and phase shifted versions of the desired signal. The signal cancellation means that, in place of retaining the desired signal unchanged and weakening the disturbances, the distortionless response constraint in the look direction is satisfied by the beamformer, but then it uses the correlated disturbances to remove the desired signal via minimizing the cross-spectrum matrix of the overall signal that involves contributions from both [1].

One approach to overcome the signal cancellation problem is by introducing nulls at the directions of the disturbances through additional constraints. The linearly constrained minimum variance (LCMV) beamformer employs this approach [14], [15], [16]. Although the null is obtained irrespective of the characteristic of disturbance signals, we need to determine the arriving direction of the disturbances in the LCMV beamformer. On the other hand, in the MPDR beamformer the nulls in the direction of disturbances are achieved via minimizing the cross-spectrum matrix of the overall signal, and as we have mentioned above, if the disturbance has correlation with the desired signal, MPDR design is remarkably degraded, due to signal cancellation [1].

Another approach to avoid the signal cancellation problem is by minimizing the cross-spectrum matrix of the noise. This approach is called minimum variance distortionless response (MVDR) beamforming [1], [17], [18]. Estimation of this matrix in presence of correlated disturbance is a challenging problem. For example, a common belief is that in a situation where a disturbance is a reflected form of the desired signal, due to cohesive appearance and disappearance of both the disturbance and the desired signal, estimation of cross-spectrum matrix of the noise signal is typically not possible in practice. So, based on this common belief, we cannot use the MVDR beamformer in this case for the purpose of solving the signal cancellation problem.

In this paper we show that this common belief is a fault. We propose a general approach for estimating the cross-spectrum matrix of noise signal that is applicable even on situation of correlated disturbance. For that, at first we determine the amplitude densities and arrival directions of the disturbance signals from which we can estimate the cross-spectrum matrix of overall noise. Then, using this matrix we are able to bypass the signal cancellation problem effectively using MVDR beamformer equipped with this matrix. Note that, it is in contrast to the classical MVDR beamformer in which we don't need to specify the arriving direction of disturbance signals.

This paper is organized as follows. The second section reviews the spherical array processing fundamentals. The third section presents the proposed method for solving the signal cancellation problem. Simulation examples are presented in the fourth section, and the end section concludes the paper.

## Spherical Array Processing

This section shortly explains the theory of spherical microphone array processing [1], [4], [19], [20]. The formulation provided in this section will be utilized in the following sections to develop the proposed method.

*A. Array processing in Spherical Harmonic Domain*

Consider a sound field composed of multiple "single frequency plane wave" each with amplitude density denoted by $a(k,\theta_k,\phi_k)$ arriving from direction $(\theta_k,\phi_k)$ with a wave vector $\tilde{\mathbf{k}} = -\mathbf{k} = (k,\theta_k,\phi_k)$ and wave number $k$. The sound pressure at $\mathbf{r} = (r,\theta,\phi)$ due to this sound field can be written as follows [1]

$$p(k,r,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} p_{nm}(k,r)Y_n^m(\theta,\phi), \tag{1}$$

where $p_{nm}(k,r)$ are the spherical harmonic coefficients of the sound pressure, and $Y_n^m(\theta,\phi)$ are the spherical harmonics.

The relation between the amplitude of the plane waves producing the sound field and the pressure on the sphere in the spherical harmonic domain is

$$p_{nm}(k,r) = b_n(kr)a_{nm}(k), \tag{2}$$

where $a_{nm}(k)$ is the spherical Fourier transform of $a(k,\theta_k,\phi_k)$, i.e.,

$$a_{nm}(k) = \int_0^{2\pi}\int_0^{\pi} a(k,\theta_k,\phi_k)[Y_n^m(\theta_k,\phi_k)]^* \sin\theta_k d\theta_k d\phi_k, \tag{3}$$

and $b_n(kr)$ defines the projection of the sound field onto the surface of sphere. The expression for $b_n(kr)$ depends on the array configuration. For example, in the case of a single open sphere, we have

$$b_n(kr) = 4\pi i^n j_n(kr), \tag{4}$$

where $j_n(x)$ is the spherical Bessel function of the first kind [1].

In the case of order-limited pressure function, we have $p_{nm}(k,r) = 0 \ \forall n > N$, so we can represent the function by a limited number of spherical harmonics and we have

$$p(k,r,\theta,\phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} p_{nm}(k,r) Y_n^m(\theta,\phi). \tag{5}$$

Equation (1) is, in fact, the inverse spherical Fourier transform of the pressure function [5]. So, we have

$$p_{nm} = \int_0^{2\pi} \int_0^{\pi} p(\theta,\phi)[Y_n^m(\theta,\phi)]^* \sin\theta d\theta d\phi, \tag{6}$$

which is the spherical Fourier transform of $p(\theta,\phi)$. For simplicity, we omitted the parameters $k, r$.

According to the Cubature method, it can be possible to compute the multiple integrations of a specified function using a summation over samples of that function [1]. So,

$$p_{nm} \approx \sum_{q=1}^{Q} \alpha_q \, p(\theta,\phi)[Y_n^m(\theta,\phi)]^* = \hat{p}_{nm}, \tag{7}$$

where $Q$ is the total number of samples and $\alpha_q$ is the sampling weight whose value depends on the sampling method. The approximation becomes equality for order-limited function provided a sufficiently large $Q$. In this case, using the inverse spherical Fourier transform, $p(\theta,\phi)$ can be reconstructed perfectly on the sphere. But, in the case of $p_{nm}$ of infinite order, due to aliasing, perfect reconstruction is not possible [4], [21], [22], [23], [24].

Several sampling methods, such as Gaussian, equal-angle, and uniform sampling, have been previously presented [4], [25], for which the sampling points $(\theta_q,\phi_q)$ and sampling weight $\alpha_q$ have been derived such that for order-limited functions (7) is maintained with equality. Due to some constraints, we may want to use any arbitrary given sampling set. So, Assume that the samples of the function, $p(\theta_q,\phi_q)$, are given, together with the positions of the samples, $(\theta_q,\phi_q)$, for $q = 1,\dots,Q$. Using (5) we have

$$p(\theta_q,\phi_q) = \sum_{n=0}^{N} \sum_{m=-n}^{n} p_{nm} Y_n^m(\theta_q,\phi_q), 1 \le q \le Q \tag{8}$$

Equation (8) may be written in matrix form as

$$\mathbf{p} = \mathbf{Y}\mathbf{p_{nm}}, \tag{9}$$

where vectors $\mathbf{p}$ of length $Q$ and $\mathbf{p_{nm}}$ of length $(N+1)^2$ are defined as

$$\mathbf{p} = \left[p(\theta_1,\phi_1), p(\theta_2,\phi_2), \dots, p(\theta_Q,\phi_Q)\right]^T \tag{10}$$

and

$$\mathbf{p_{nm}} = \left[p_{00}, p_{1(-1)}, p_{10}, p_{11}, \dots, p_{NN}\right]^T, \tag{11}$$

and the matrix $\mathbf{Y}$ of dimensions $Q \times (N+1)^2$ is given by

$$\mathbf{Y} = \begin{bmatrix} Y_0^0(\theta_1,\phi_1) & Y_1^{-1}(\theta_1,\phi_1) & Y_1^0(\theta_1,\phi_1) & Y_1^1(\theta_1,\phi_1) & \dots & Y_N^N(\theta_1,\phi_1) \\ Y_0^0(\theta_2,\phi_2) & Y_1^{-1}(\theta_2,\phi_2) & Y_1^0(\theta_2,\phi_2) & Y_1^1(\theta_2,\phi_2) & \dots & Y_N^N(\theta_2,\phi_2) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_0^0(\theta_Q,\phi_Q) & Y_1^{-1}(\theta_Q,\phi_Q) & Y_1^0(\theta_Q,\phi_Q) & Y_1^1(\theta_Q,\phi_Q) & \dots & Y_N^N(\theta_Q,\phi_Q) \end{bmatrix} \tag{12}$$

Equation (9) is called inverse discrete spherical Fourier transform [26]. Also,

$$\mathbf{p_{nm}} = \mathbf{Y}^\dagger \mathbf{p}, \tag{13}$$

is called discrete spherical Fourier transform, where $\mathbf{Y}^\dagger = (\mathbf{Y}^H\mathbf{Y})^{-1}\mathbf{Y}^H$ is the pseudo-inverse of $\mathbf{Y}$. Generally, the discrete spherical Fourier transform may be written as

$$\mathbf{p_{nm}} = \mathbf{S}\mathbf{p} \tag{14}$$

In the situation of a general sampling, matrix $\mathbf{S}$ is given by $\mathbf{S} = \mathbf{Y}^\dagger$, in the situation of Gaussian and equal-angle sampling methods is given by $\mathbf{S} = \mathbf{Y}^H \operatorname{diag}(\boldsymbol{\alpha})$, where $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \dots, \alpha_Q]^T$ holds the sampling weights, and for nearly uniform and uniform sampling methods is given by $\mathbf{S} = \frac{4\pi}{Q}\mathbf{Y}^H$ .

*B. Spherical Array Beamforming*

Array equations or beamforming equations is as follows [1], [4], [25], [27], [28], [29],

$$y = \int_0^{2\pi} \int_0^{\pi} w^*(k,\theta,\phi) p(k,r,\theta,\phi) \sin\theta d\theta d\phi \\ = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} w_{nm}^*(k) p_{nm}(k,r). \tag{15}$$

where $w^*(k,\theta,\phi)$ are the beamforming coefficient. The standard discrete form of beamforming in the space domain is

$$y = \mathbf{w}^H \mathbf{p}, \tag{16}$$

where $\mathbf{p}$ is as (10) with a little modification in notation

$$\mathbf{p} = [p_1(k), p_2(k), \dots, p_Q(k)]^T \tag{17}$$

with $p_q(k) = p(k, r, \theta_q, \phi_q)$, $q = 1, \dots, Q$, and $\mathbf{w}$ is the $Q \times 1$ weight vector as follows

$$\mathbf{w} = [w_1(k), w_2(k), \dots, w_Q(k)]^T \tag{18}$$

Assuming $w_{nm} = 0\ \forall n > N$, the discrete form of beamforming in spherical harmonic domain is as

$$y = \mathbf{w}_{\mathbf{nm}}^H \mathbf{p}_{\mathbf{nm}}, \tag{19}$$

where the $(N+1)^2 \times 1$ vector $\mathbf{w}_{\mathbf{nm}}$ is given by

$$\mathbf{w}_{\mathbf{nm}} = [w_{00}(k), w_{1(-1)}(k), w_{10}(k), w_{11}(k)\ \dots, w_{NN}(k)]^T \tag{20}$$

and the $(N+1)^2 \times 1$ vector $\mathbf{p}_{\mathbf{nm}}$ is given by

$$\mathbf{p}_{\mathbf{nm}} = [p_{00}(k,r), p_{1(-1)}(k,r), p_{10}(k,r), \dots, p_{NN}(k,r)]^T \tag{21}$$

In these equations $p_{nm}(k)$ and $w_{nm}(k)$ are the spherical Fourier transform of $p(k, r, \theta, \phi)$ and $w(k, \theta, \phi)$ respectively and $N$ is called the effective order of the array.

The output of the array due to a unit-amplitude plane-wave, i.e, array beam pattern, is defined as

$$y = \mathbf{w}_{\mathbf{nm}}^H \mathbf{v}_{\mathbf{nm}}, \tag{22}$$

where $\mathbf{v}_{\mathbf{nm}}$ is a $(N+1)^2 \times 1$ column vector as

$$\mathbf{v}_{\mathbf{nm}} = [\mathrm{v}_{00}(\mathrm{k},\mathrm{r}), \mathrm{v}_{1(-1)}(\mathrm{k},\mathrm{r}), \mathrm{v}_{10}(\mathrm{k},\mathrm{r}), \dots, \mathrm{v}_{\mathrm{NN}}(\mathrm{k},\mathrm{r})]^{\mathrm{T}} \tag{23}$$

with $v_{nm}$ represents the array input owing to the sound field created by plane wave. Since for unit amplitude plane wave we have [1],

$$a_{nm}(k) = [Y_n^m(\theta_k, \phi_k)]^*, \tag{24}$$

According to (2) we have

$$v_{nm}(k,r) = b_n(kr)[Y_n^m(\theta_k, \phi_k)]^* \tag{25}$$

where $(\theta_k, \phi_k)$ is the plane wave arrival direction.

Using a different set of beamforming coefficients, different beam patterns can be designed. For instance, axis-symmetric beamformers with $w_{nm}^*(k) = \frac{d_n(k)}{b_n(kr)} Y_n^m(\theta_l, \phi_l)$ of which two famous beamformers are the maximum directivity (MD) beamformer and the maximum white noise gain (WNG) beamformer [20]. Note that the beamformer coefficients are function of look direction which denoted by $(\theta_l, \phi_l)$ in the above relation.

### *C. The MPDR and MVDR Beamformers*

Consider a desired signal $s(k)$, arriving from direction $(\theta_k, \phi_k)$. The corresponding distant source creates a plane wave at the location of array. Array input may be written as follows

$$\mathbf{x} = \mathbf{p} + \mathbf{n} = \mathbf{v}s + \mathbf{n} \tag{26}$$

where $\mathbf{v}$ is the transfer function from the source to the microphone array input, also called the steering vector,

$$\mathbf{p} = [p_1(k), p_2(k), \dots, p_Q(k)]^T \tag{27}$$

is the sound pressure at the $Q$ microphones due to the desired source and,

$$\mathbf{n} = [n_1(k), n_2(k), \dots, n_Q(k)]^T \tag{28}$$

is the noise at the microphones [1], [14]. The array output is as follows

$$\mathrm{y} = \mathbf{w}^{\mathrm{H}}\mathbf{x} \tag{29}$$

The array output variance is

$$E[|y|^2] = E[\mathbf{w}^H \mathbf{x}\mathbf{x}^H \mathbf{w}] = \mathbf{w}^H \mathbf{S}_{\mathbf{xx}} \mathbf{w} \tag{30}$$

where

$$\mathbf{S}_{\mathbf{xx}} = E[\mathbf{x}\mathbf{x}^H] \tag{31}$$

is the spatial spectral matrix of the array input. Each element in this matrix represents the cross-spectral density between the signals at two microphones at wave number $k$. From (26) and (31) the spatial cross-spectral density matrix of the array input may be written as

$$\mathbf{S}_{\mathbf{xx}} = \mathbf{S}_{\mathbf{pp}} + \mathbf{S}_{\mathbf{nn}} + \mathbf{S}_{\mathbf{pn}} + \mathbf{S}_{\mathbf{np}} \tag{32}$$

with

$$\mathbf{S}_{\mathbf{pp}} = E[\mathbf{p}\mathbf{p}^H] \tag{33}$$

and

$$\mathbf{S}_{\mathbf{nn}} = E[\mathbf{n}\mathbf{n}^H] \tag{34}$$

With the assumption of independent pressure signal and the noise signal, (32) is rewritten as

$$\mathbf{S_{xx}} = \mathbf{S_{pp}} + \mathbf{S_{nn}} \tag{35}$$

So, we can rewrite (30) as follows

$$E[|y|^2] = \mathbf{w}^H\mathbf{S_{xx}}\mathbf{w} = \mathbf{w}^H\mathbf{S_{pp}}\mathbf{w} + \mathbf{w}^H\mathbf{S_{nn}}\mathbf{w} \\ = |\mathbf{w}^H\mathbf{v}|^2E[|s|^2] + \mathbf{w}^H\mathbf{S_{nn}}\mathbf{w} \tag{36}$$

We have the following design objective

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H\mathbf{S_{xx}}\mathbf{w} \\ \text{subject to} \quad \mathbf{w}^H\mathbf{v} = 1 \tag{37}$$

Owing to the distortionless response restriction, $\mathbf{w}^H\mathbf{v} = 1$, in the above optimization problem, $|\mathbf{w}^H\mathbf{v}|^2E[|s|^2]$ can not be modified, so, the minimization of $\mathbf{w}^H\mathbf{S_{xx}}\mathbf{w}$ leads to minimization of $\mathbf{w}^H\mathbf{S_{nn}}\mathbf{w}$, i.e., the noise variance at the array output. The optimal value of $\mathbf{w}$ is

$$\mathbf{w}^H = \frac{\mathbf{v}^H\mathbf{S_{xx}^{-1}}}{\mathbf{v}^H\mathbf{S_{xx}^{-1}}\mathbf{v}} \tag{38}$$

It is the minimum power distortionless response (MPDR) beamformer [1], [2], [3]. The MVDR beamformer is the same as in (38) with $\mathbf{S_{xx}^{-1}}$ is replaced by $\mathbf{S_{nn}^{-1}}$,

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H\mathbf{S_{nn}}\mathbf{w} \tag{39}$$

$$\text{subject to} \quad \mathbf{w}^H\mathbf{v} = 1$$

with a solution

$$\mathbf{w}^H = \frac{\mathbf{v}^H\mathbf{S_{nn}^{-1}}}{\mathbf{v}^H\mathbf{S_{nn}^{-1}}\mathbf{v}} \tag{40}$$

The spherical harmonic domain formulation of MPDR is as follows. In the spherical harmonic domain, (26) may be written as

$$\mathbf{x_{nm}} = \mathbf{p_{nm}} + \mathbf{n_{nm}} = \mathbf{v_{nm}}s + \mathbf{n_{nm}} \tag{41}$$

The MPDR optimization problem can be written as

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w_{nm}}^H\mathbf{S_{x_{nm}x_{nm}}}\mathbf{w_{nm}} \tag{42}$$

$$\text{subject to} \quad \mathbf{w_{nm}}^H\mathbf{v_{nm}} = 1$$

Similar to (38), a solution may be written for beamforming coefficients in the spherical harmonic domain as

$$\mathbf{w_{nm}}^H = \frac{\mathbf{v_{nm}}^H\mathbf{S_{x_{nm}x_{nm}}^{-1}}}{\mathbf{v_{nm}}^H\mathbf{S_{x_{nm}x_{nm}}^{-1}}\mathbf{v_{nm}}} \tag{43}$$

For the MVDR beamformer we have

$$\mathbf{w_{nm}}^H = \frac{\mathbf{v_{nm}}^H\mathbf{S_{n_{nm}n_{nm}}^{-1}}}{\mathbf{v_{nm}}^H\mathbf{S_{n_{nm}n_{nm}}^{-1}}\mathbf{v_{nm}}} \tag{44}$$

## The Proposed Method

In this section we propose a general approach for estimating the cross-spectrum matrix of noise signal. We assume that the amplitude of the desired signal is $s_0$, and the disturbance signals satisfy $s_i = A_is_0, i = 1,2,\dots,k$, where $A_i$ is a complex constant. We also assume that the desired signal propagates via a plane wave arriving from direction $(\theta_0,\phi_0)$, with variance of $\sigma_0^2 = E[|s_0|^2]$, and the disturbances are other plane waves with arrival direction $(\theta_i,\phi_i)$ with $\sigma_i^2 = |A_i|^2\sigma_0^2$. Specially uncorrelated sensor noise with variance $\sigma_n^2$ is also assumed.

Suppose that the noise at the array input is owing to the sensor noise. Using the discrete spherical Fourier transform, as in (14) the sensor noise in the spherical harmonic domain can be written as

$$\mathbf{n_{nm}} = \mathbf{Sn} \tag{45}$$

where matrix $\mathbf{S}$ depends on the sampling method as mentioned in the second section of the paper. This leads to

$$\mathbf{S_{n_{nm}n_{nm}}} = E[\mathbf{n_{nm}}\mathbf{n_{nm}}^H] = E[\mathbf{Snn}^H\mathbf{S}^H] \\ = \mathbf{S}E[\mathbf{nn}^H]\mathbf{S}^H = \sigma_n^2\mathbf{SS}^H \tag{46}$$

where we have assumed the IID noise. i.e, independent and identically distributed noise. As seen in (46), the spatial cross-spectrum of the noise depends on the sampling method.

The spatial spectrum matrix of the overall noise can be computed using the following theorem.

**Theorem 1:** The spatial spectrum matrix of the noise, including the contributions from the disturbances, is as

$$\mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{Overall} = \sigma_n^2\mathbf{SS}^H + \sum_{i=1}^{k}\sigma_i^2\mathbf{v_{nm}}_i\mathbf{v_{nm}}_i^H \\ + \sum_{i\neq j}\big(A_iA_j^*\sigma_0^2\mathbf{v_{nm}}_i\mathbf{v_{nm}}_j^H \\ + A_i^*A_j\sigma_0^2\mathbf{v_{nm}}_j\mathbf{v_{nm}}_i^H\big) \tag{47}$$

where

$\mathbf{v_{nm}}_i = [v_{00}(k,r), v_{1(-1)}(k,r), v_{10}(k,r), \dots, v_{NN}(k,r)]^T$, $v_{nm}(k,r) = b_n(kr)[Y_n^m(\theta_i,\phi_i)]^*$ and $(\theta_i,\phi_i)$ is the arrival direction of $i$th disturbance.

**Proof:** The array input in the spherical harmonic domain is as

$$\mathbf{x_{nm}} = \mathbf{p_{nm0}} + \mathbf{n_{nm}} + \sum_{i=1}^{k}\mathbf{p_{nmi}} \\ = \mathbf{v_{nm0}}\mathbf{s_0} + \mathbf{n_{nm}} + \sum_{i=1}^{k}\mathbf{v_{nmi}}\mathbf{s_i} \tag{48}$$

where $\mathbf{p}_{\mathbf{nm}i}, i = 1,2, \dots, k$ is the spherical Fourier transform of pressure due to the $i$th disturbance, and $\mathbf{p}_{\mathbf{nm0}}$ is the spherical Fourier transform of pressure due to the desired signal.

Now, the overall noise is $\mathbf{n}_{\mathbf{nm}} + \sum_{i=1}^{k} \mathbf{v}_{\mathbf{nm}i} s_i$, so the spatial spectrum matrix of the noise is

$$\mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{Overall} = E\left[\left(\mathbf{n}_{\mathbf{nm}} + \sum_{i=1}^{k} \mathbf{v}_{\mathbf{nm}i} s_i\right)\left(\mathbf{n}_{\mathbf{nm}} + \sum_{i=1}^{k} \mathbf{v}_{\mathbf{nm}i} s_i\right)^H\right] = E[\mathbf{n}_{\mathbf{nm}}\mathbf{n}_{\mathbf{nm}}{}^H] + E\left[\left(\sum_{i=1}^{k} \mathbf{v}_{\mathbf{nm}i} s_i\right)\left(\sum_{j=1}^{k} \mathbf{v}_{\mathbf{nm}j}{}^H s_j^*\right)\right] = \sigma_n^2 \mathbf{S}\mathbf{S}^H + \sum_{i=1}^{k} \sigma_i^2 \mathbf{v}_{\mathbf{nm}i} \mathbf{v}_{\mathbf{nm}i}{}^H + \sum_{i \neq j} (A_i A_j^* \sigma_0^2 \mathbf{v}_{\mathbf{nm}i} \mathbf{v}_{\mathbf{nm}j}{}^H + A_i^* A_j \sigma_0^2 \mathbf{v}_{\mathbf{nm}j} \mathbf{v}_{\mathbf{nm}i}{}^H) \quad (49)$$

■

For computing $\mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{Overall}$, in the situation of uncorrelated disturbance, it is adequate to record the input signal during the time intervals when the disturbance is active but the desired signal is not active. However, in the case of correlated disturbance, both the disturbance and the desired signal appear and disappear cohesively. So, we need to have a mechanism to extract information for computing $\mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{Overall}$ from the array input, i.e., $\mathbf{x}$. This information includes $A_i, i = 1,2, \dots, k$ and $s_0$. The following theorem can be used for this purpose.

**Theorem 2.** The amplitude of the disturbance plane waves can be computed using an axis-symmetric beamformer with $d_n(k) = 1$.

**Proof:** We consider the axis-symmetric beamformers. As mentioned in the second section, axis-symmertic beamformers are beamformers with the following weights [30]

$$w_{nm}^*(k) = \frac{d_n(k)}{b_n(kr)} Y_n^m(\theta_l, \phi_l) \quad (50)$$

If we set $d_n(k)$ to unity, we have

$$w_{nm}^*(k) = \frac{1}{b_n(kr)} Y_n^m(\theta_l, \phi_l) \quad (51)$$

So, (15) can be rewritten for this case as

$$y = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} w_{nm}^*(k) p_{nm}(k, r) = Y_n^m(\theta_l, \phi_l) \frac{p_{nm}(k, r)}{b_n(kr)} \quad (52)$$

From (2) we have

$$\frac{p_{nm}(k, r)}{b_n(kr)} = a_{nm}(k) \quad (53)$$

So, we have

$$y = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm}(k)\, Y_n^m(\theta_l, \phi_l) = a(k, \theta_l, \phi_l) = A_l \quad (54)$$

■

Equation (54) shows that y, as a function of look direction, gives $A_l$. Using these values we can compute $\mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{Overall}$, and finally the beamforming weight can be computed as

$$\mathbf{w}_{\mathbf{nm}}{}^H = \frac{\mathbf{v}_{\mathbf{nm}}{}^H \mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{overall\ -1}}{\mathbf{v}_{\mathbf{nm}}{}^H \mathbf{S}_{\mathbf{n_{nm}n_{nm}}}^{overall\ -1} \mathbf{v}_{\mathbf{nm}}} \quad (55)$$

Finally, the beam pattern is computed using $\mathbf{w}_{\mathbf{nm}}$ and (22) as

$$y(\theta, \phi) = \mathbf{w}_{\mathbf{nm}}{}^H \mathbf{v}_{\mathbf{nm}}(\theta, \phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} w_{nm}^*(k) b_n(kr) [Y_n^m(\theta, \phi)]^* \quad (56)$$

## Simulation Examples

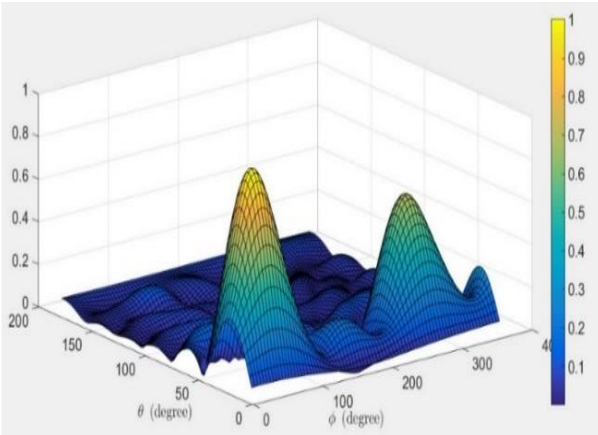
Examples of beam patterns designed using the proposed method are presented in this section. A spherical microphone array around a rigid sphere, operating at $kr = N$, with $N = 4$ is assumed. We also consider $Q = 36$ nearly-uniformly arranged microphones, with spatially uncorrelated sensor noise and with variance $\sigma_n^2 = 0.1$. So, $\mathbf{S}_{\mathbf{n_{nm}n_{nm}}}$ due to sensor noise can be written as

$$\mathbf{S}_{\mathbf{n_{nm}n_{nm}}} = \sigma_n^2 \mathbf{S}\mathbf{S}^H = \sigma_n^2 \frac{4\pi}{Q} \mathbf{I} \quad (57)$$

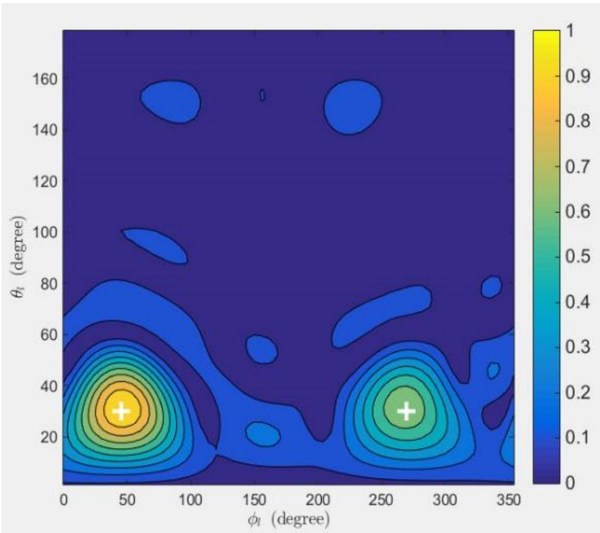
### *A. The First Example*

In the first example we assume that the desired signal propagates with a plane wave coming from direction $(\theta_0, \phi_0) = (30,45)$, with variance of $\sigma_0^2 = 1$. A disturbance signal is included that has correlation with the desired signal and propagates with another plane wave coming from direction $(\theta_1, \phi_1) = (30,270)$, with $\sigma_1^2 = |A_1|^2 \sigma_0^2$ and $A_1 = 0.7e^{-i\pi/3}$.

In the first step we must approximate $s_0$ and $A_1$. For that we compute $y(\theta_l, \phi_l)$ using (54). An equal angle of $60 \times 60$ points was utilized to create $(\theta_l, \phi_l)$. Figure 1 shows the normalized magnitude of $y(\theta_l, \phi_l)$.



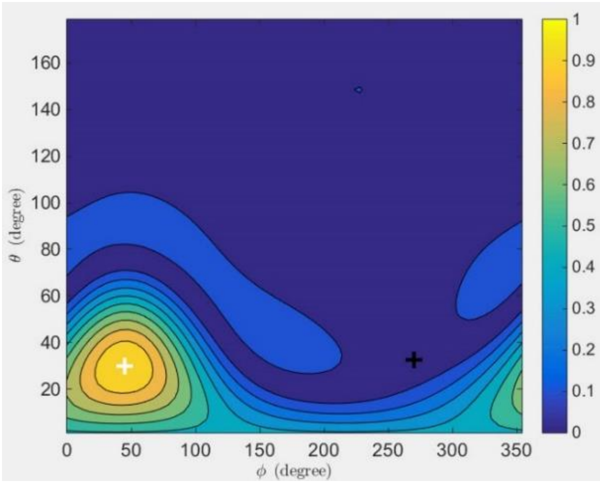

(a)



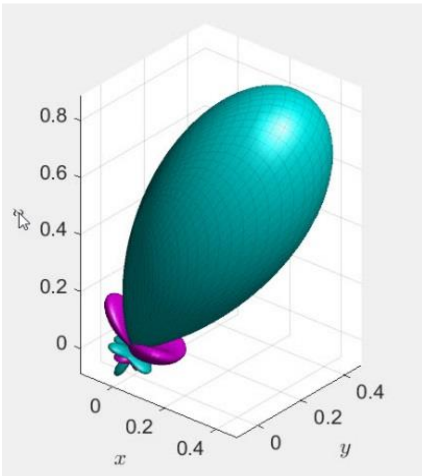

(b)

Fig. 1: Normalized magnitude of $y(\theta_l, \phi_l)$. $(a)$ surface plot. $(b)$ contour plot. The arrival directions of the two plane waves are indicated with *white* "+".

The peaks in these plots indicates $a(k, \theta_l, \phi_l)$, i.e., the plane wave amplitude density that is identical to $|A_l|$. In this example we obtain amplitude density of $1e^{-i0.025}$ and $0.682e^{-i1.029}$, and also arrival directions of $(30.375{,}45)$ and $(32.625{,}270)$ for the desired signal and disturbance signal respectively. Using these values as $s_0$, $A_1$, $(\theta_0, \phi_0)$ and $(\theta_1, \phi_1)$, we can compute $\sigma_0^2 = E[|s_0|^2]$ and $\sigma_1^2 = |A_1|^2\sigma_0^2$. Finally, according to (49) we compute $\mathbf{S}_{\mathbf{n}_{\mathbf{nm}}\mathbf{n}_{\mathbf{nm}}}^{Overall}$, and then the spherical harmonic domain beamforming weights may be computed from (55). The resulting beam pattern is then computed using (56). The magnitude of the beam pattern has been shown in Fig. 2. As we can see in this figure, the first side lobe has a null in the disturbance signal arrival direction. So, the proposed method has the ability to shape the beam pattern to consider the correlated disturbances in the sound field.



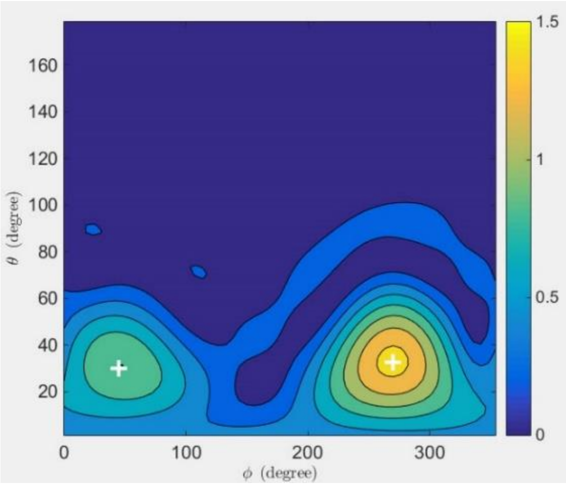

(a)



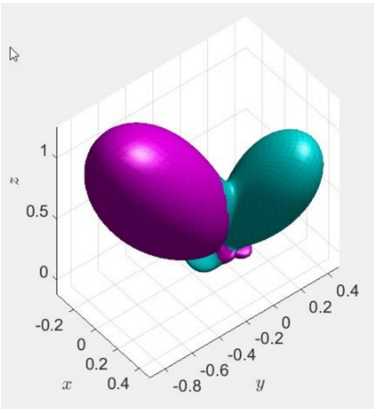

(b)

Fig. 2: $|y(\theta, \phi)|$ for MVDR beamformer with the cross-spectrum matrix of noise obtained by the proposed method. $(a)$ contour plot, arrival direction of the desired plane wave is indicated by the *white* "+" and the arrival direction of the disturbance plane wave is indicated by the *black* "+". $(b)$ balloon plot. In this plot *cyan* color represents positive values of $\mathrm{Re}\{y(\theta, \phi)\}$, and *magenta* color represents negative values of $\mathrm{Re}\{y(\theta, \phi)\}$.

Without the proposed method, $s_0$ and $A_1$ are not available. So, we have to use MPDR beamformer as expressed in (38). For this, we calculate the spatial spectrum matrix of the overall input signal, i.e., $\mathbf{x}$, using (35) and then using (38) and (22) we can calculate the corresponding beam pattern. This beam pattern has been shown in Fig. 3 in which we can see the signal cancellation phenomenon clearly.




(a)

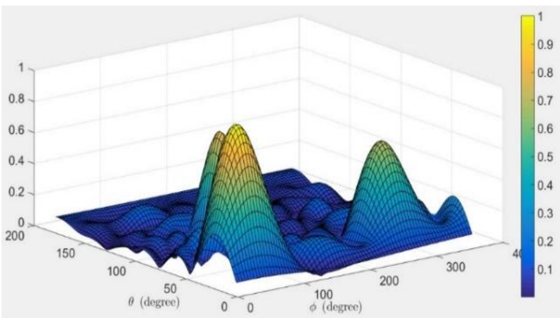



(b)

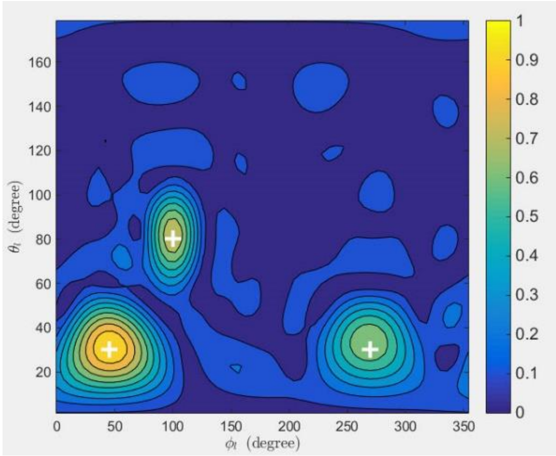
Fig. 3: $|y(\theta,\phi)|$ for MPDR beamformer. $(a)$ contour plot, arrival direction of the disturbance plane wave and desired plane wave are indicated by the white "+". $(b)$ balloon plot.

*B. The Second Example*

In the second example, the first example is further extended to include another disturbance propagates with a plane wave with arrival direction $(\theta_2,\phi_2) = (80,100)$, with $\sigma_2^2 = |A_2|^2\sigma_0^2$ and $A_2 = 0.8e^{-i\pi/2}$. Fig. 4, Fig. 5, and Fig. 6 show the simulation results. We can see again the ability of the proposed method in solving the signal cancellation problem.

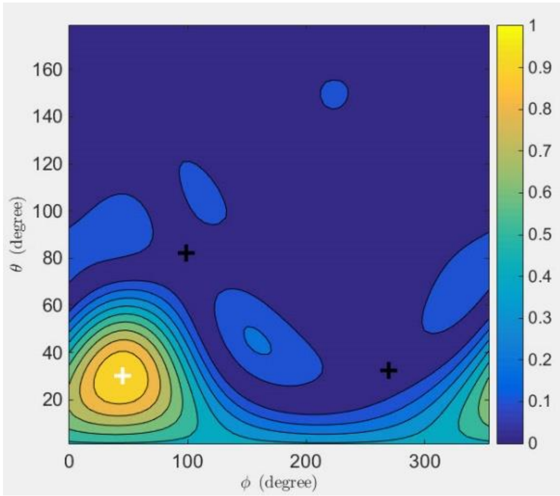



(a)

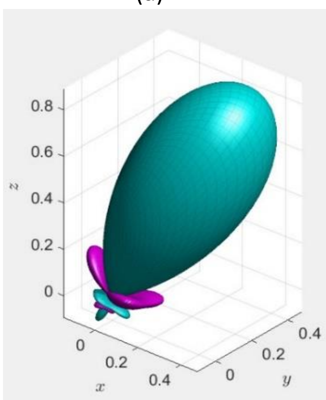



(b)

Fig. 4: Normalized magnitude of $y(\theta_l,\phi_l)$. $(a)$ surface plot. $(b)$ contour plot. The arrival directions of the three plane waves are indicated by the white "+".

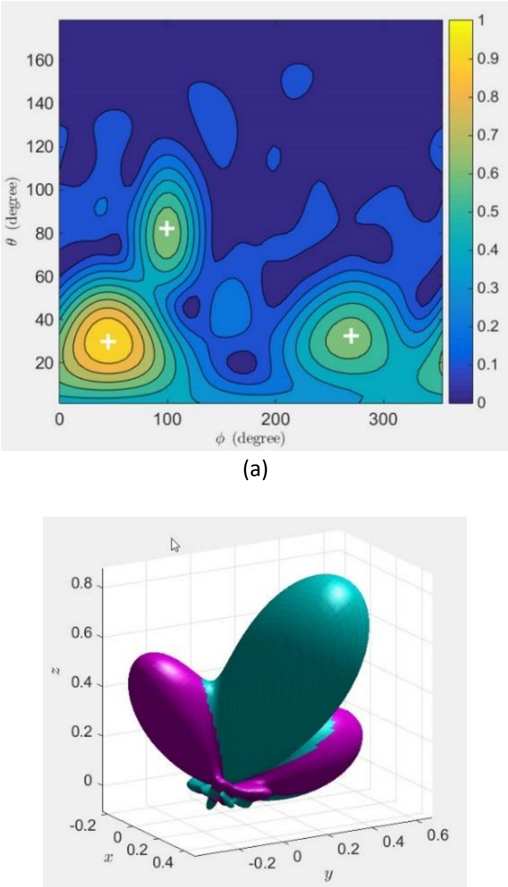



(a)




(b)

Fig. 5: $|y(\theta,\phi)|$ for MVDR beamformer which use the cross-spectrum matrix of noise obtained by the proposed method. $(a)$ contour plot, arrival direction of the disturbance plane wave is indicated by the *black* "+", and the arrival direction of the desired plane wave is indicated by the *white* "+". $(b)$ balloon plot.



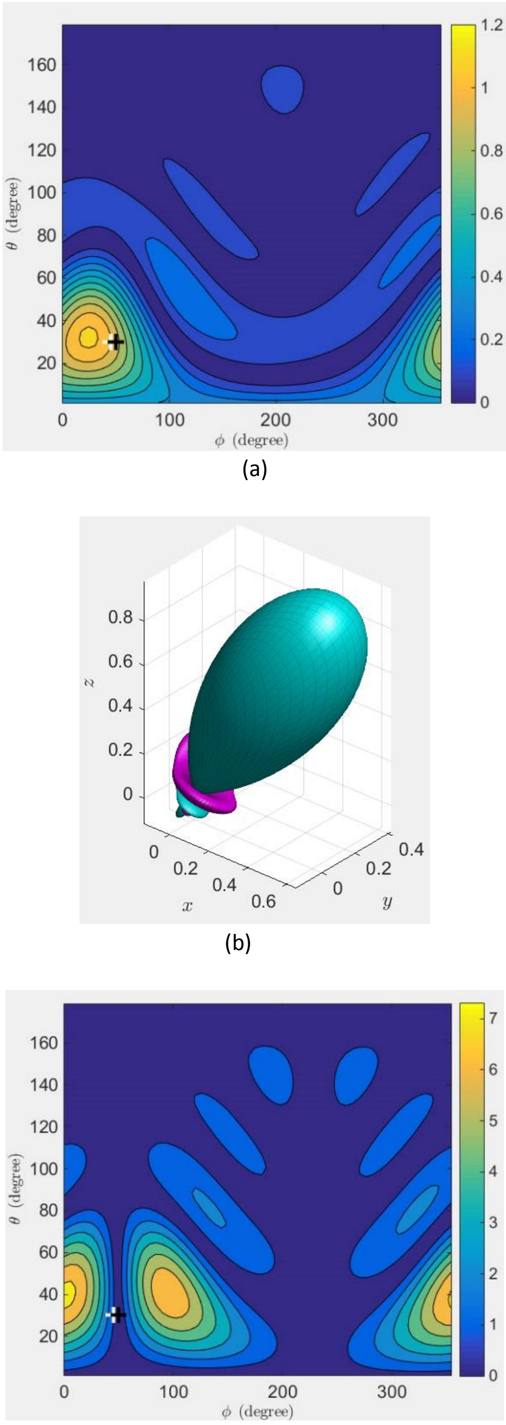

(a)

(b)

Fig. 6: $|y(\theta,\phi)|$ for MPDR beamformer. $(a)$ contour plot, arrival direction of the disturbance plane wave and desired plane wave are indicated by the *white* "+". $(b)$ balloon plot.

### *C. Comparison with BF-LCMV method*

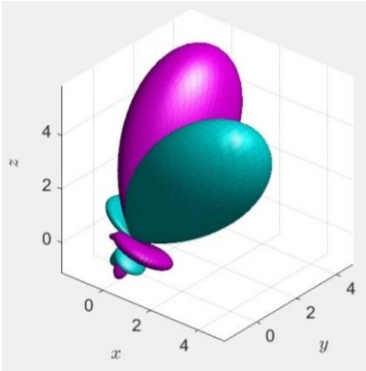
In this section we compare the proposed method with the rival method presented in [16] in which the signal cancellation problem has been solved using direction of arrival (DOA) estimation and the LCMV beamformer. For this purpose two performance measures have been considered: directivity factor (DF) which is the ratio between the array response in the look direction and the average response over all directions, and white noise gain (WNG) which is a general measure for array robustness and is defined as the improvement in signal-to-noise ratio (SNR) at the array output relative to the array input. Experimental results show that two methods have similar performance in term of these two performance measures in almost all cases. For example with simulation setup as in the first example, the two methods gain DF of 23.97 and WNG of 39.91 dB. But, in the case of disturbance arriving from direction that is near the look direction and with amplitude that is a small fraction of the desired signal, the proposed method is superior as shown in Fig. 7. The parameters in this simulation is as in the first example except the disturbance signal parameters which are $(\theta_1,\phi_1)=(30,50)$ and $A_1=0.2e^{-i\pi/3}$. In this case, the DF and WNG for the proposed method are 19.19 and 37.5 dB respectively, and for the BF-LCMV method, these objective measures are 0.275 and 5.23 dB. This phenomenon is probably due to the role that the amplitude density characteristic of the disturbance signal plays in this specific case. The proposed method considers this characteristic but the BF-LCMV method only takes into account the arriving direction of the disturbance.




(a)

(b)

(c)




(d)

Fig. 7: $(a)$ and $(b)$, $|y(\theta,\phi)|$ for MVDR beamformer which use the cross-spectrum matrix of noise obtained by the proposed method. $(c)$ and $(d)$, $|y(\theta,\phi)|$ for RF-LCMV method.

## Results and Discussion

The simulation results show that the proposed method has the ability to shape the beam pattern to consider the correlated disturbances in the sound field and consequently to solve the signal cancellation problem. The comparisons with the rival method show that both methods have the same performance in almost all cases except in the case of disturbance arriving from direction that is near the look direction and with amplitude that is a small fraction of the desired signal, in which the proposed method is superior.

## Conclusion

The signal cancellation problem is a major problem in the MPDR beanformer. It occurs whenever the disturbance signals include at least one signal that is correlated with the desired signal. Using the MVDR beamformer, we can avoid this problem, but this approach requires that the cross-spectrum matrix of the noise signal is available. A common belief is that in the case of correlated disturbance, the estimation of this matrix is not possible. In this paper we showed that this common belief is a fault. We proposed a general approach for estimating the cross-spectrum matrix of noise signal that is applicable even in the case of correlated disturbance. Simulation examples show that using the proposed method along with the MVDR beamformer, we can bypass the signal cancellation problem completely.

## Author Contributions

M. Kalantari has written the whole paper without participation of anybody. All parts of this work have been accomplished by the author as the single author and the corresponding author of the paper.

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## Conflict of Interest

The author declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

## Abbreviations

| | |
|---|---|
| LCMV | Linearly constrained minimum variance |
| MPDR | Minimum power distortionless response |
| MVDR | Minimum variance distortionless response |
| SNR | Signal-to-noise ratio |
| WNG | White noise gain |
| $\alpha_q$ | Sampling weights |
| $\boldsymbol{\alpha}$ | Vector of sampling weights |
| $\theta$ | Elevation angle |
| $\phi$ | Azimuth angle |
| $a(\cdot)$ | Plane-wave decomposition in the space domain |
| $a_{nm}$ | Plane-wave decomposition in the spherical-harmonics domain |
| $b_n(\cdot)$ | Function relating pressure to plane-wave decomposition |
| DF | Directivity factor |
| $d_n$ | Axis-symmetric beamforming weighting function |
| $j_n(\cdot)$ | Spherical Bessel function of the first kind |
| k | Wave number |
| $\mathbf{k}$ | Wave vector denoting propagation direction |
| $\tilde{\boldsymbol{k}}$ | Wave vector denoting arrival direction |
| $N$ | Order of spherical harmonics |
| $\mathbf{n}$ | Noise vector in the space domain |
| $\mathbf{n_{nm}}$ | Noise vector in the spherical harmonics domain |
| $p$ | Sound pressure in the space domain |
| $p_{nm}$ | Sound pressure in the spherical harmonics domain |
| $\mathbf{p}$ | Sound pressure vector in the space domain |
| $\mathbf{p_{nm}}$ | Sound pressure vector in the spherical harmonics domain |
| $Q$ | Number of samples or microphones |
| $\mathbf{r}$ | Vector of spherical coordinates |
| $\mathbf{S}$ | Spherical Fourier transform matrix |
| $\mathbf{S_{xx}}$ | Cross-spectrum matrix in the space domain |
| $\mathbf{S_{x_{nm}x_{nm}}}$ | Cross-spectrum matrix in the spherical harmonics domain |
| $\mathbf{S_{nn}}$ | Noise cross-spectrum matrix in the space domain |

| | |
|---|---|
| $\mathbf{S}_{\mathbf{n}_{nm}\mathbf{n}_{nm}}$ | Noise cross-spectrum matrix in the spherical harmonics domain |
| $\mathbf{v}$ | Steering vector in the space domain |
| $\mathbf{v}_{\mathbf{nm}}$ | Steering vector in the spherical harmonics domain |
| $w(\cdot)$ | Beamforming weighting function in the space domain |
| $w_{nm}$ | Beamforming weighting function in the spherical harmonics domain |
| $\mathbf{w}$ | Beamforming weighting vector in the space domain |
| $\mathbf{w}_{\mathbf{nm}}$ | Beamforming weighting vector in the spherical harmonics domain |
| $Y_n^m(\cdot)$ | Spherical harmonics |
| $\mathbf{Y}$ | Matrix of Spherical harmonics |

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## Biography

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