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Research paper

Non destructive damage severity estimation in beam using change in extended cross modal strain energy

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Abstract

This paper presents an extended cross modal strain energy change method to estimate the severity of damage associated with limited modal data in beam-like structures. This method takes in account the correlation between the analytical modal data and the measured incomplete modal data. A procedure was proposed and the analytical elemental stiffness of the damaged element after it is localized is included in quantification of the measured single damage extent. A three-dimensional numerical beam model with different damage cases is used to simulate the CMSE method application and to getting the bending displacements of the damaged element. An experimental modal analysis (EMA) on a cantilever beam with and without crack was carried out to evaluate the effectiveness of the extended CMSE method. The severity magnitude of the damage was predicted within an acceptable error range through the using validation process. Results reveal that the proposed damage estimation method successfully evaluates single damage severity in beam like structure and can be useful in maintenance technology and structural health monitoring system.

1. Introduction

Subjected continuously to static and dynamic loading, most engineering mechanical structures accumulate damages during their service life. These damages cause locally a negative stiffness that alters the modal properties such as the natural frequency and mode shape. Based on changes in these frequencies, and in these mode shapes, or in their combination, several structural damage identification techniques have been proposed for predicting damage location and severity [1-3].

Frequency-based methods are widely used and their applicability is relatively simple to monitor the structure on site [4] and to giving a useful existence indication of the damage [5-7]. However, the use of the natural frequencies change in structural damage identification cannot provide sufficient information to locate the damage, that because they mainly reflect the global structural response [8].

In the most studies in the literature, it is widely believed that a change in natural frequencies alone might not provide enough information for efficient damage identification [8].
Recently, many authors have demonstrated that the modal frequency is not an appropriate spatial index and it is very hard to locate the position of the damage [4, 9, 10], they noted their low sensitivity to estimate the damage severity in structures, they specified also that the identification methods require an advanced measurement instruments and accurate data analysis methods.
In order to overcome the insufficiency of the frequency change sensitivity to quantify the damage, mode shapes and the related product: Modal Strain Energy (MSE), have been widely applied as vibration based damage evaluation methods.
A method based on modal strain energy has been largely explored as an indicator of damage; the results demonstrate that, the majority of MSE methods are only capable of locating the damage, but do not provide an accurate estimate of the severity of the damaged structures [9, 11].
In order to remedy this deficiency, a new method for evaluating the extent of damage, called the "Cross Modal Strain Energy method" (CMSE), has been developed. The term "cross" means that the modal strain energy use the mixed products of the different mode shapes of the same model, either analytical or experimental of the healthy and damaged structure, or those coming from both models together [9]. While all modal strain energy methods available for assessing the extent of damage use an iterative procedure involving a gross presumption and substantial approximations, the CMSE method is a reliable method without an iterative approach [12].
Hu et al. were the pioneers who used the notion of the cross modal strain energy to assess the magnitude of damages [13]. Asgarian et al [14] have conducted a study of the successful application of the MSE method for damage localization and the CMSE method for estimating the severity of damage of an offshore platform.
The authors Yan et al. [15] used a combination of the CMSE method and an adaptive niche genetic algorithm to improve the ability to locate damage in mechanical structure.

In the present work, the study cross modal strain energy is considered to evaluate its practicability in experimental damaged beams as a non-destructive damage method; it is employed for quantifying damage in beam models after that is localized by modal strain energy method. The numerical study has been conducted and the results are also used to establish the validity of the proposed method.

2. Method for detecting and locating damage based on modal strain energy

The energy of modal strain concept to detect the crack in structure is firstly applied by Stubbs et al. who they developed a severitydamage estimation algorithms later in beam like structures [16, 17]. Seyedpoor et al. [18] and Huajun et al. [19] have developed this method to identify and quantify the deterioration of complex structural systems such as plate like structures and large structures such as bridge construction.
Ramesh et al. [20] have used successfully both methods damage index based in energy of modal strain and wavelet transform (WT) to locate and quantify a multi cracks in the aluminum beam.
This method is used to locate damage from a small number of modes by using the change of the MSE before and after the appearance of damage in each element of the structure [18, 20].
This approach is relatively simple and allows the precise detection of one or more structural damages, based on the assumption that the fraction of the elementary modal strain energy over the overall modal strain energy is the same for damaged and undamaged structures. Thus, the variation of the elementary modal strain energy for the jth element in the ith mode is given by the following expression:

$$\mathrm{MSEC}_{ij} = \Phi_i^{d\,T} K_j \Phi_i^d - \Phi_i^T K_j \Phi_i \qquad (1)$$

where K_j is jth elemental stiffness matrix and Φ_i and Φ_i^d denote respectively the ith modal eigenvector of the undamaged and damaged of the jth element beam.
The crack location index, known as the Energy Modal Strain Change Ratio (MSECR) can be either derived for a single mode such as mode i and element j in structure (Fig. 1) is given in Eq. (2) and the Average Normalized Modal Strain

Energy Change Ratio (ANMSECR) for the first modes shapes of the first "m" modes of the structural element j is shown in Eq. (3).

$$MSECR = \frac{\left|MSE_{ij}^{d} - MSE_{ij}\right|}{MSE_{ij}} \tag{2}$$

$$ANMSECR = \frac{1}{m}\sum_{i=1}^{m}\frac{MSECR_{ij}}{MSECR_{i,max}} \tag{3}$$

The index j and i indicate respectively the element number and mode number; MSE and MSE^{d} symbolize respectively the modal strain energy of undamaged and damaged structure.

3. Cross energy of modal strain method to to evaluate damage depths

3.1. Theoretical formulation

Once the element where the damage has occurred is situated among the most probably suspected beam elements by using the damage localization indicator, a size damage quantifying procedure is conducted for the damaged element employing a newly developed crack depth evaluation method, called Cross
Modal Strain Energy (CMSE) method.
Hu et al. [21] developed an approach based on the use of cross modal strain energy to quantify the damage; they give the stiffness matrix K^d, the i^{th} modal eigenvalue λ_i^d, and the i^{th} mode shape $ø_i^d$ of the damaged beam by the following expressions:

$$K^{d} = K + \sum_{j=1}^{L}\Delta K_j = K + \sum_{j=1}^{L}\alpha_j K_j \tag{4}$$

$$\lambda_i^{d} = \lambda_i + \Delta\lambda_i \tag{5}$$

$$\phi_i^{d} = \phi_i + \Delta\phi_i = \phi_i + \sum_{j=1}^{m} c_{ij}\,\phi_j \tag{6}$$

where α_j is a coefficient that describes a fractional reduction in the j^{th} elemental stiffness matrix K_j. This coefficient estimates the damage extent such as $-1 \leq \alpha_j \leq 0$, $\alpha j = 0$ means no damage and $\alpha_j = -1$ means totally damaged.
c_{ij} is the coefficient that defines a fractional change of the mode shape vector; and L is the total number of elements in the beam (Fig. 1).
The expression of stiffness matrix of cracked structure K^d is given by the equation:

$$K^{d} = K + \sum_{j=1}^{Nd}\alpha_n K_{ln} \tag{7}$$

where Nd is the cracked elements number; α_n and ln are respectively the extent of damage and the number of the j^{th} cracked element. K is the stiffness matrix of the intact structure.
Finally, the CMSE expression using the i^{th} mode of the healthy structure and the j^{th} mode of the cracked structure is as follows [21]:

$$C_{ij} = \phi_i^{T} K \phi_j^{d} \tag{8}$$

The related elementary CMSE of the K_{ln} matrix is given below:

$$C_{n,ij} = \phi_i^{T}\phi_j^{d} \tag{9}$$

The Eq. (7) can be rewritten as:

$$\sum_{n=1}^{Nd}\alpha_n C_{n,ij} = \left(\frac{\lambda_j^{d}}{\lambda_i} - 1\right)C_{ij} \tag{10}$$

Therefore, the coefficient α_n to estimate the damage extent is calculated by solving the Eq. (10). For the beam case with single crack in nth element, the fractional reduction of the n^{th} elemental stiffness α_j is given in the expression:

$$K^{d} = K + \alpha_n K_n \tag{11}$$

Then, the damage extent αn is estimated by:

$$\alpha_n = \frac{K^{d} - K}{K_n} \tag{12}$$

The parameter α_n calculated by CMSE method for the case of single crack scenario, is given by

$$\alpha_n = \frac{\left(\frac{\lambda_j^{d}}{\lambda_i} - 1\right)C_{ij}}{C_{n,ij}} = \frac{\left(\frac{\lambda_j^{d}}{\lambda_i} - 1\right)\phi_i^{T} K \phi_j^{d}}{\phi_i^{T} K_{ln} \phi_j^{d}} \tag{13}$$

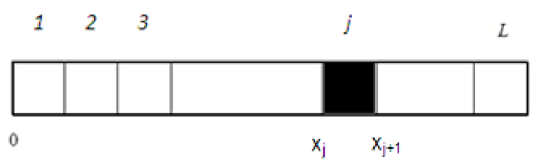

Fig. 1. J^{th} damaged element in beam.

It should be noted that negative value for α denotes the percentage of stiffness reduction in damaged element and other values denote that no damage is occur in element.

3.2. Experimental study

The modal analysis experiment on beam has been carried out to verify the finite element model results and to provide eigenfrequency and mode shapes data that were employed in the application of the CMSE method as a technique of Non destructive damage severity estimation.

To carry out the modal experiments, a test protocol was used to demonstrate that the data and modal parameters meet the reliability requirements. The experimentation plan consists of an implementation and preparation phase, acquisition and analysis phases. The first phase includes preliminary checks and preparation steps that include instrumentation calibration, noise effect and frequency range determination, bench adjustment, specimen fixation and notch cutting in crack position. The second phase contains the acquisition and analysis of the frequency response functions (FRFs) and the identification of influencing parameters in the applied CMSE method.

To make the experiment, steel cantilever beam of square cross-section (0.016x0.016m) and length of l=0.7m (Fig. 2), with a controlled crack is subjected to number of experimentation by the use of experimental set up shown in Fig. 3. The beam dimensions meet the requirements of the Bernoulli-Euler model. A transverse crack (controlled notch) is performed at the location of c=150mm (c/l=0.214, c/l is damage position to the beam length ratio) from of the clamped end of the beam using a hand saw [22].

In order to get the mode shapes of beam with and without crack, the FRF data have been measured in seven positions of the cantilever beam by moving the accelerometer (type Brüel & Kjær 4384) from the extremity to other position lengthwise of the beam (Fig. 4).

The accelerometer type 4384 (weight of 11g, sensitivity is 1 pC/ms-2 and frequency range: 0.1 to 12600 Hz) is selected in order to cover main specimen eigenvalues (0 to 1600 Hz) and to get the best resolution possible. Its design ensures a high ratio of sensitivity to mass, a relatively high resonance frequency and a good isolation from base strains and temperature transients.

The shaker (B&K 4812) and the force sensor (B&K 8200) have been used to excite the beam with a white noise signal. This signal is used as an excitation force to make the effect of noise insignificant. The shaker was placed constantly at the free end of the beam (Fig. 2).

The FRFs are taken at each measurement point by means of a Personal Computer Memory Card International Association (PCMCIA) as interface acquisition data card and computer accessories based FFT analyzer (Fig. 3). The FRFs are smoothed with Lagrange polynomial algorithm by using the DAQ700 Software.

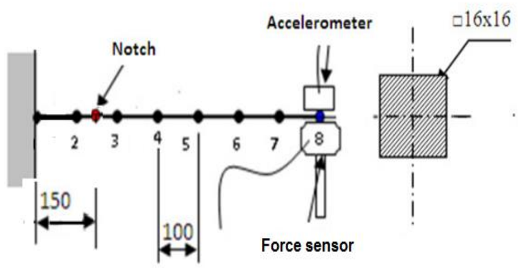

Fig. 2. Damaged beam dimensions and measuring positions.

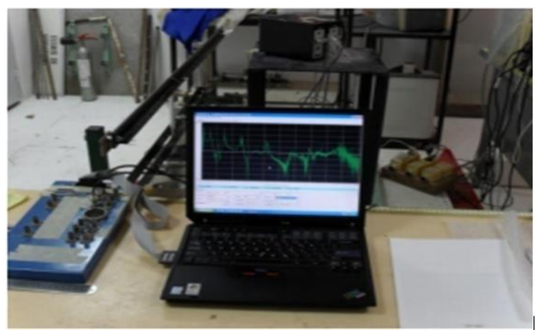

Fig. 3. Test bench and system acquisition.

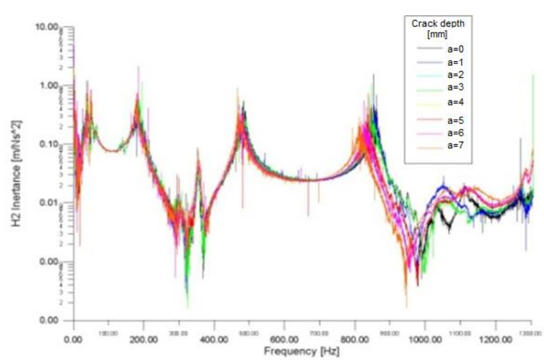

Fig. 4. Experimental FRFs of intact and damaged beam.

3.3. Numerical simulation

The main objective of the numerical analysis is to demonstrate the effectiveness of the CMSE method to estimate the damage extent; this numerical study is investigated by using data generated from the finite element models associated with an experimental analysis of beam with different damage scenarios.

The 3D Finite Element Model using ANSYS code consisted of approximately 8628 elements type Solid186 and 16021 nodes. The beam material is considered linear and isotropic and the Young's Modulus equals 2.1×10^5 MPa; Poisson's ratio is 0.3 [22].

The FRFs of the cracked beam cases were used to obtain the first seven eigenfrequencies and the bending mode shapes. To model the change in stiffness of the cracked beam, a parallelepiped with a thickness equal to the depth of the crack was subtracted from the beam (Fig. 5). The position of the crack is located at 150 mm and the depth ratio a/h varies from 0% to 62.5% with a percentage increase Δa/h equals to 6.25% (a/h is a ratio of the depth of the crack to the beam section height (h)).

The Comparison of the FEM results obtained with those of the experiment shows a good agreement between the two sets.

4. Results and discussion

4.1. Damage localization

The localization experimental results are summarized and given in the Fig. 6; this figure gives the Average Normalized Modal Strain Energy Change Ratio (ANMSECR) distribution obtained from the summation of unit index of the first seven modes; this indicator parameter ANMSECR is used to intensify the signal caused by the presence of damage while reducing the effect of noise on the measured modal parameters.

The Fig. 6 indicates that the Average indicator ANMSECR has a maximum change (peak) in the vicinity of the local area of the crack and that for crack depth ranging from a/h = 6.25 % to a/h = 37.5%, the error localization is about ±7%. The localization by this index becomes less accurate when the a/h exceeds 37.5%, and the localization uncertainty becomes ±14% for the case of size crack equal to a/h = 42.5%.

It should be noted that the ANMSECR index gives a good estimate than the method using the Modal Strain Energy Change Ratio (MSECR), the ANMSECR method detects early the crack presence (for a/h smaller than 10%). It shows the crack position with an error close to ±7%. The experimental results of localization are less accurate than the numerical results; this is due to the noise that was unavoidable when measuring the output signal; and to the reduced number of measurement points. Therefore the localization uncertainty can be reduced by using more sampling measured points.

4.2. Estimation of the correlation between the damage extent α_n and the damage depth

The method of extended cross modal strain energy method was developed with the assumption that the presence of damage only changes the stiffness matrix, but not the mass matrix of the beam. From the Eq. (13), the following derivation as λ_i, λ_j^d, K, ϕ_i , ϕ_j^d and K are presumably known, the unknown term is K_{ln}; the first step in the estimation of this term is to measure the displacement of nodes of the damaged element; after that the stiffness K_{ln} is estimated for the beam case with single crack located in one element with the following equation.

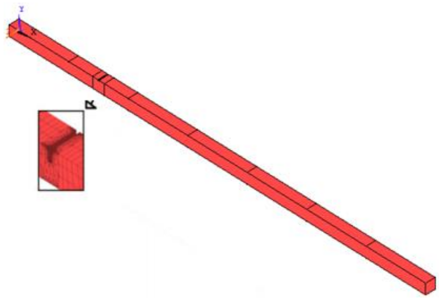

Fig. 5. Model 3D of the cracked beam.

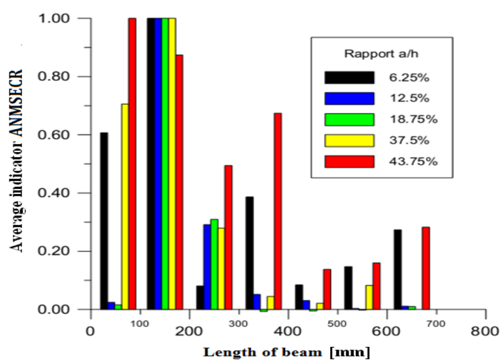

Fig. 6. Experimental ANMSECR distribution of all damage cases.

$$K_n = \frac{F_n}{d_{n2} - d_{n1}} \tag{15}$$

where d_{n1} and d_{n2} are displacements of the two nodes of the suspected element and Fn is applied force, it is taken equal to 50 N.

The Fig. 7 shows the numerical flexural displacement trend of the damaged beam for six scenarios corresponding to the depth crack varying from 0 to a/h=0.625.

An extended modal strain energy approach is proposed to estimate the measured and the analytical damage extent through the Eq. (13) by introducing the numerical displacements of the dominant modes of the damaged element in the stiffness K_{ln}. This procedure to complete the set of displacement is justified by the limited number of measured ones due to the limited number of displacement sensors and to the difficulty in measuring modal displacement.

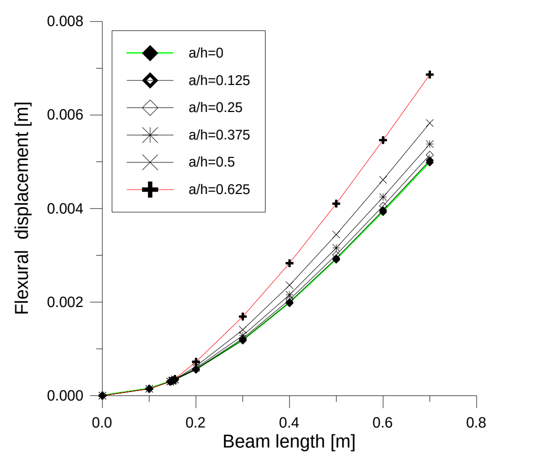

Fig. 7. Stiffness loss of intact and damaged beam.

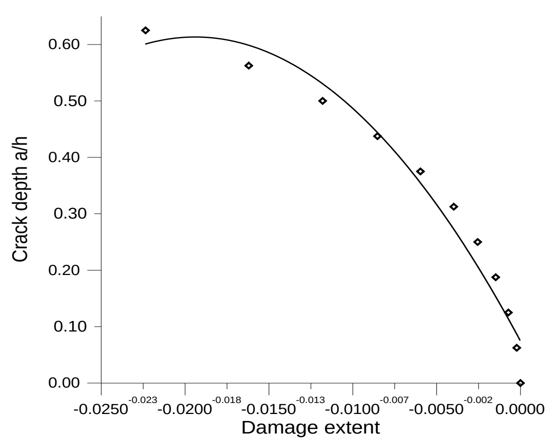

Fig. 8. Damage extent α_n and the damage severity.

The Fig. 8 shows the curve that gives the correlation between the crack depth and the value of the crack extent α_n established by using the Eq. (12).

It is important to highlight that the numerical damaged beam 3D model is calibrated by using experimental model with controlled damage and the use of the lowest measured frequency of a damaged structure is very desirable because that it permit to get an improved and more accurate estimation.

The procedure developed is proposed in two steps: firstly, the local stiffness K_{23} of the damaged element (e.g. element 2 of beam (Fig. 1)) is estimated numerically and it is introduced in the equation of CMSE through the Eq. (13); after that, the procedure is validated and the value of damage extent is compared to that given by the Eq. (12) and presented in Fig. 8.

4.3. Damage severity estimation by the extended CMSE method

In case of our beam model, after the position of the suspected damaged element is determined, it is the element number 2, the damage extent will be estimated by introducing the value of $K_{ln} = K_{23}$; and the values of λ_i, λ_j^d, K, ϕ_i , ϕ_j^d for the first vibration modes in the Cross Mode Strain Energy method by means the Eq. (13).

The first four vibration modes of the cracked and intact beam are considered for both numerical and experimental studies. Many combinations are performed to estimate the damage extent by this method; the results are summarized in Table 1.

From the obtained numerical results, we can observe that the use of the same mode order (i= j=3) and (i=j=4) for both undamaged and damaged beam always gives a good estimation for all damages cases, the error estimation is smaller as revealed in Table 1.

And when the mode order of the undamaged structure is different from the mode order of damage structure i.e. (i=4, j=3), (i=2, j=1), (i=3, j=1); the estimate damage severity is underestimate or overestimate the reel damage depth, so the crack extent is predicted within an inacceptable error range.

Table 1. Severity of damage estimation.

	Intact mode i	Damage d mode j	Damage extent estimated α_n Applying Eq. (13)	Estimate severity damage [mm] from Fig. 8	True severity damage [mm]
FEA	3	3	-0.000377 -0,001395 -0.011036	1~2 3~4 ≈ 8	2 4 10
	4	4	-0.000925 -0.00335 -0.027823	2~3 4~5 10~11	2 4 10
EMA	2	2	-0.000349 -0.003538 -0.001446 -0.004608 -0.006098	1~ 2 4~5 3~4 5~6 6~7	1 2 3 5 7
	3	3	-9.4531E-06 -5.9653E-05 -2.8261E-05 -0.000233 -0.002866	~ 0 ~ 0 ~ 0 ~ 1 4 ~ 5	1 2 3 5 7
	4	4	-0.000598 -0.008098 -0.040393 -0.053718 -0.365354	1~2 2~3 ~8 >10 >10	1 2 3 5 7

The combination of modes (i=1, j=1) and modes (i=2, j=2) does not given a correct severity estimate for all damage cases.
It should be mentioned that the severity estimate is not obtained for damage case by using the average of the summation of CMSE for the four first modes.
From the numerical simulation, it is observed that the absolute changes in extended cross modal strain energy are quantifying the damage in the damage region with acceptable error, and hence it can be used to characterize the damage in a real beam like structure. And also, the use of the method is found successful for quantifying the damage in the beam models with different boundary conditions.
Once a modal expansion is adopted to match the displacements of some points in beam between the numerical and experimental models; the using of the combination of different modes in the experimental results reveals that the proposed method is capable to estimate the damage severity only with the employ of combination of modes (i=2,2) and modes (4,4); the error estimation damage size is acceptable only for the depth crack smaller than 3mm (a/h=0,1875) as shown in Table 1. This deviation is due to the effect of the crack growth on the local flexibility of damaged element; this effect becomes important with the severity crack increase [23], whereas it is less important in the analytical local flexibility K_{ln} taken into account in the Eq. (13).

5. Conclusions

This paper investigates the experimental feasibility of a developed method based on cross modal strain energy for the non destructive quantification of crack severity in beam-like structures.
Results of the cases of a cantilever beam with single-crack are indicated that the MSECR index was applied successfully on both numerical and experimental data in particular for the low damage severity and for the low frequency modes.
The Average Normalized MSECR index has been used in order to reduce the noise effect in the measured modal parameters; the application of this indicator showed that the crack with a

depth ratio close to 7% was detected and was located with an uncertainty equal to ±7%.
By using extended cross modal strain energy method, the crack size estimation is performed considering combination of the first four bending modes of the intact and cracked beam. The damage extent obtained by the ECMSE method is adjusted by using bending stiffness data from damaged 3D beam model validated by experimental beam with controlled damage. From the numerical results; some observations are noted and used in the experimental setting. From the experimental results, it was concluded that the using of the same modes order for both undamaged and damaged beam by the extended CMSE method always gives a better estimate especially for a smaller crack size.

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