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Research paper

Influence of material and internal support on natural frequencies of thin-walled cylindrical tanks

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Abstract

Water storage tanks are amongst the essential infrastructures, and the study of their natural frequencies plays a pivotal role in predicting and detecting dynamic behavior. Therefore, it helps to the uninterrupted operation of an industrial plant and the use of tank water in emergencies. This paper studies the influence of different shell materials, including steel, aluminum, and laminated composites with three types of different fiber orientations, on the natural frequencies of thin-walled aboveground water storage tanks that have pinned boundary conditions at the base. Models investigated in this paper, either the roof is without an internal support structure or else a group of columns and radial beams are used for supporting it. These huge tanks had the height to diameter ratio of 0.4, and a water surface at 90% of the height of the tank's cylinder. The thicknesses of the cylindrical shells are tapered. The tanks without internal support included the vibrations that affect the cylinder mode shapes or the roof mode shapes or simultaneously both the cylinder and roof mode shapes. On the other hand, the mode shapes of the tanks with internal support affect predominantly only the cylinder. Among the studied tanks, the third type of composite tanks had the highest rigidity, and the first type of composite tanks had the lowest rigidity. The natural frequencies related to the first modes of vibrations for cylinder and roof shells with a wide range of circumferential wave numbers (n) and an axial half-wave (m) are studied.

1. Introduction

The dynamic of thin-walled cylindrical shells has been studied in recent decades. The major engineering industries require thin-walled tanks for the storage and transfer of liquids. In recent years, these studies, especially for composite shells, have been done more based on the theory of classical shells and for very small and unrealistic dimensions which, consequently makes their results unreliable for thin-walled huge tanks. Because most of the failures and fractures of these structures are caused by dynamic loads, the need to study and predict the dynamic behavior of thin-walled tanks and cylindrical shells becomes more significant than ever before.

Free vibration and natural frequencies of thin-walled cylindrical tanks are one of the most important and fundamental dynamical problems because it plays a considerable role in predicting seismic effects when applying seismic loads and determining the stiffness of the structure and, therefore, the elimination of possible defects. Especially considering that water storage tanks are not only used in the industry, and they help the environment in avoiding waste of water, but also, they play an important role in water shortages in emergencies. As a result, preservation of these structures against seismic effects should be considered. Also, the importance of the type of materials used in the thin-walled shells of these tanks is not only important for seismic subjects but also for economic issues. The main papers are mentioned in the literature review.

Mazuch et al. [1] studied the mode shapes and natural frequencies of cylindrical tanks partially filled with liquid using experiments in the technical literature. This study was performed by analytical and semi-analytical techniques [2, 3] and finite element methods [1, 4]. Goncalves and Ramos [4] evaluated the free vibration characteristics of a thin cylindrical shell, partially or completely filled with liquid, and under any variationally consistent group of boundary conditions on the lower and upper boundaries by a simple modal solution based on the underlying ideas of the hierarchical FE method.

A free vibration analysis was performed for cylindrical tanks partially filled with liquid with variable thicknesses by Han and Liu [3]. They obtained the natural frequencies for a tank with constant thickness and filled with water by the analytical method. The results are close to previous results published by Haroun and Tayel [5].

The transfer matrix approach was used by Han and Liu [3] for extending the procedure formulated for the constant thickness tank to the variable thickness case that was solved for the empty and partially filled with liquid tanks.

The natural frequencies for tanks that have their bottom plate resting on an elastic foundation and which are partially filled with liquid studied by Amabili [6] and Amabili et al. [7]. A pinned condition at the upper boundary of the cylinder were considered for the influence of the roof in both studies. Amabili [6] used an artificial spring method for the solution of the free vibrations of a tank, which is partially filled with liquid. A flexible bottom plate and a ring stiffener at the cylinder was considered for a tank by Amabili et al. [7]. A major effect on the mode shape of the shell and minor changes in the frequency was concluded by the use of the stiffener.

Haroun and Housner [8], and Veletsos et al. [9] considered the mass-spring analogy for modeling the impulsive and convective modes of vibration, which are made by hydrodynamic pressure of liquid during strong ground motion. The impulsive component is the liquid, which moves with the tank's wall coincidently, and contributes to the fundamental mode of vibration of the liquid tank with a very short vibration period. On the other hand, independent sloshing movement of the liquid in the upper part of the tank shell is represented by a convective component that is associated with a longer period of vibration and has an insignificant effect on the response of the tank. For the anchored cylindrical tank-liquid systems under horizontal motion, the fundamental impulsive modes of vibration were reported by Virella et al. [10]. They performed the analyses by a finite element program and considered the effect of the hydrostatic pressure and the self-weight on the natural periods and modes. They also studied the effect of a fixed roof on the natural periods of vibrations of thin-walled steel tanks, which have clamped boundary conditions at the base [11]. The theoretical background of a simplified seismic design method for cylindrical ground-supported tanks was provided by Malhotra et al. [12]. The method considers impulsive and convective (sloshing) actions of the liquid in concrete tanks or flexible steel fixed to rigid bases.

Balendra et al. [13] summarized the results of a numerical investigation of lateral free vibration of cylindrical storage tanks by an analytical method. Lam and Loy [14, 15] presented a straightforward procedure of analysis involving Ritz's procedure, and Love's first approximation theory is used to study the effect of boundary conditions and fiber orientation on the natural frequencies of thin orthotropic laminated cylindrical shells and then carried out a study on

the effect of boundary conditions for a thin laminated rotating cylindrical shell. The analysis was performed using the Love-type shell theory and solved by Galerkin's method.
The free vibration characteristics of fluid-filled cylindrical shells on elastic bases were reported by a semi-analytical FE method by Gunawan et al. [16] that the fluid domain was defined by the potential flow theory. The hydrodynamic pressure acting on shells was derived from the condition for dynamic coupling of the fluid-structure. Taut et al. [17] reviewed most of the research performed in past years (2000-2009) on the dynamic behavior containing vibration of composite shells and also studied on the advances in the dynamic behavior of shells and free vibration behavior of isotropic and composite shell panels [18, 19].
Free vibrations of the laminated composite cylindrical shells with clamped boundaries were studied by Lopatin and Morozov [20]. The calculations were verified by comparison with a FE solution. It was demonstrated that the analytical described formula provides an efficient means for rapid and reliable calculation of the fundamental frequency, which can be used for the evaluation of the structural stiffness of the shells in the design analysis. Liu et al. [21] provided an analytical method and closed-form vibration solutions that analytically determined coefficients for orthotropic circular cylindrical shells having classical boundary conditions based upon the simplest thin shell theory.
Guoyong Jin et al. [22, 23] applied a simple precise solution method according to the Haar wavelet discretization method to the free vibration analysis of composite laminated cylindrical shells under various boundary conditions. They discussed the influences of several important aspects containing boundary conditions, length to radius ratios, lamination schemes, and elastic modulus ratios on natural frequencies, and also focused on the free vibration analysis of composite laminated conical, cylindrical shells, and annular plates with different boundary conditions, according to the first-order shear deformation theory by the Haar wavelet discretization method. Kumar et al. [24, 25] provided the reviews concerned with recent progress in describing the intricacies of mechanical and thermal properties of graphene composites. In their research, the utility of the dynamic mechanical analysis and thermogravimetric analysis applied for thermal characterization that has been presented by various researchers was analyzed. They highlighted the improvements in properties of two- and three-phase composites, due to the addition of graphene/ CNT, and focused on the comparison of various properties of CNT- and graphene-reinforced composites in the review.
In this paper, a case study on water storage tanks is presented to highlight the influences of shell materials with different mechanical properties on the natural frequencies and mode shapes of vibrations. Considering that in most studies in this field, either only structural issues such as different ratios of length to width of empty reservoirs have been investigated or only the discussion of various mechanical properties of composite shells in unrealistic and small dimensions; therefore, this paper is distinct from other researches because it has considered huge tanks with both structural subjects and mechanical issues, which are the comparison of various types of materials of thin-walled shells with different mechanical properties.

2. Tank models

In this paper, a pinned condition at the base is considered for the cylindrical tanks, and the materials of tanks include a laminated composite of graphite-epoxy with different types of fiber's rotation angles, steel, and aluminum, with a cone roof which either it has the internal support, as shown in Fig. 1, or else it is without internal support. The columns are fixed to both bottom of the tank and ring beams, and the radial beams are connected to the cylinder of the tank directly. Also, all of the beams and columns are steel.
The geometry considered in this work has an aspect ratio H/D = 0.4, where H and D are shown in Fig. 2. The water height is assumed to be $H_L = 0.9H$. The thicknesses of the cylindrical shell are tapered as indicated in Fig. 2. The section of beams and columns are considered of the IPE section with dimensions of 0.311m, 0.165m, and 0.0095m in height, width, and thickness, respectively. The finite element package ABAQUS [26] is used to model and perform the computations. Virella et al. [10] used this model in their research. Each tank has a different shell materials. The differences between

graphite-epoxy laminated composites, which have different fibers angles, steel, and aluminum are investigated in this study. Details of all materials are given in Tables 1, 2, and 3.

2.1. Tank-water models

Only those modes corresponding to the impulsive mode in which there is a coupling action between the tank and water are considered in this paper. The liquid is represented with acoustic 3D finite elements based on linear theory. In addition, an inviscid liquid is considered for the present research. Between the surfaces of water and the tank cylinder and the bottom, the surface tied normal contact is assumed which this contact formulation is based on a master–slave technique that is both surfaces stay in contact throughout the simulation and transmit the normal forces between them. No sloshing waves are considered in this study, and thus no pressure was exerted to the nodes at the free water surface.

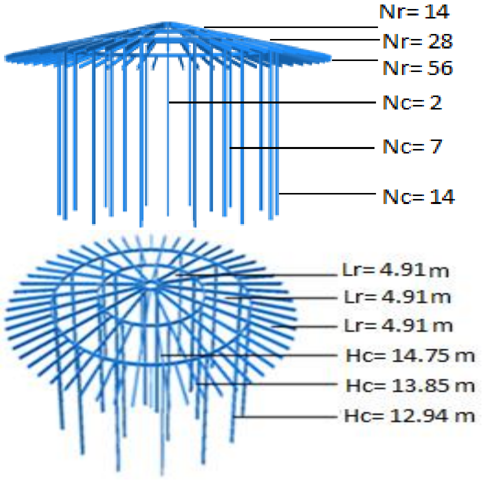

Fig. 1. Configuration of the internal support of the tank (Lr= the length of rafters, Hc= the height of columns, Nr= the number of rafters, and Nc= the number of columns).

Table 1. Mechanical properties of steel tank, aluminum tank, rafters, and liquid.

Materials properties	Steel	Aluminum	Fluid (water)
Density (kg/m³)	7900	2700	983
Poisson's ratio	0.3	0.33	
Young's modulus (GPa)	200	70	
Bulk modulus (GPa)	-	-	2.07

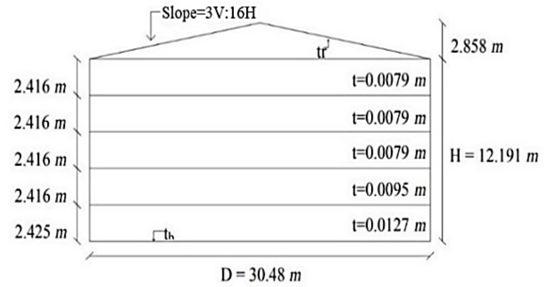

Fig. 2. Tank model with cone roof: t = cylinder shell thickness; t_r = roof shell thickness; t_r = 0.0127 m (roof without internal support); t_r = 0.00635 m (roof with internal support); t_b = bottom shell thickness; t_b = 0.012 m.

Table 2. Mechanical properties of composite tanks.

Materials properties	Graphite/epoxy
Density (kg/m³)	1586
Major poisson's ratio	0.3
Longitudinal elastic modulus (Gpa)	153
Transverse elastic modulus (Gpa)	10.9
Shear modulus (GPa)	5.6

Table 3. Fiber's rotation angles of composite tanks**.**

Shell's thickness (mm)	Layer's number	Composite	Composite3
12.7	10	Composite1	$[0/90/45/-45/45]_s$
12.0	10	$[45/-45]_s$	$[0/90/45/-45/45]_s$
9.50	8		$[0/90/45/-45]_s$
7.90	7	Composite2	[0/90/45/-45/45/90/0]
6.35	6	$[0/90]_s$	$[0/90/45]_s$

3. Theoretical formulation for composite tanks

3.1. Energy of shell

The equations of motion of the laminated cylindrical shell have been derived based on the first-order shear theory. In this theory, the middle surface displacement is referenced, and the displacements of the other points of the shell are related to the middle surface displacement by [14]:

$$u = u_o(x,\theta) + z\psi_x(x,\theta) \quad (1a)$$

$$v = v_o(x,\theta) + z\psi_\theta(x,\theta) \quad (1b)$$

$$w = w_o(x,\theta) \quad (1c)$$

where u_O, v_O, and w_O are the middle surface displacements in the three directions x, θ, and z, respectively, as shown in Fig. 3, ψ_x and ψ_θ are the rotations of the middle surface along x and θ, and z is the distance of each point from the shell to the middle surface.

By using strain-displacement formulas in the cylindrical system, the strain relations are obtained, and the strain vector including the middle layer strains, the middle layer curves, and the transverse shear values are attained [27]. By the stiffness matrix and the rotation matrix, the shell strain potential energy is obtained, where L is the cylinder length, ε is the strain vector and S is the stiffness matrix [14]:

$$U_{shell} = \frac{1}{2}\int_0^l\int_0^{2\pi} \varepsilon^T [S]\varepsilon R d\theta dx \quad (2)$$

The kinetic energy of the shell is calculated using the following expression [14].

$$T = \frac{1}{2}\int_0^L\int_0^{2\pi} \rho\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] R d\theta dx \quad (3)$$

where [S] and ε are the stiffness matrix and strain vector, respectively, and for a laminated cylindrical shell, ρ can be written as:

$$\rho_T = \sum_{k=1}^{N} \rho_k \left(h_k - h_{k-1}\right) \quad (4)$$

where ρ_k is the density per unit length of the k^{th} laminate and N is the number of layers.

3.2. The influence of liquid-structure interaction on the composite shell vibrations

To investigate the effect of liquid in the tank on the vibration of the shell, a mathematical model is used to analyze the interaction of solid and liquid at their contact surface, which is based on the following assumptions [6]: liquid flow is a potential flow type, inviscid and incompressible. In this section of the study, the linear wave theory is applied to represent water. The liquid velocity along the cylinder axis is zero.

The effect of liquid surface waves and hydrostatic pressure is not taken into account. The kinetic energy of the liquid resulting from the movement of the liquid due to the displacement of the shell surface is defined as:

$$T_{fl} = \frac{1}{2}\rho_{fl}\int_0^R\int_0^{2\pi}\int_0^l r v^2 dx d\theta dr \quad (5)$$

where v is the velocity, and ρ_{fl} is the fluid density. By considering the incompressible liquid as well as ignoring the effects of surface waves, the potential energy of the liquid will be zero.

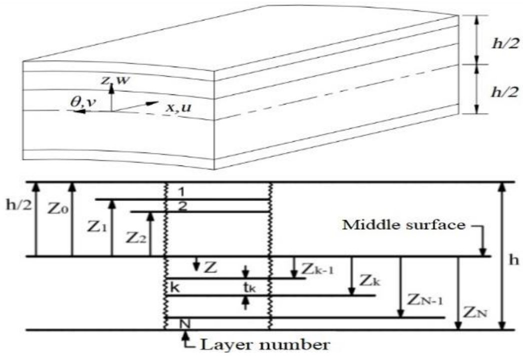

Fig. 3. Layout of laminated shell layers

3.3. Energy potential function and natural frequency

By the kinetic and potential energies of the shell and liquid, the energy potential function is formed as follows:

$$F = T_{fl} + T_{shell} - U_{shell} \quad (6)$$

where T_{fl} is the kinetic energy of the liquid, T_{shell} is the kinetic energy of the shell, and U_{shell} is the shell strain potential energy. The Rayleigh-Ritz method is used to find natural frequencies and mode shapes [14, 27]. This method is based on the principle of minimum potential energy.

According to this method, in order to minimize the potential energy as a function of the coefficients A, B, C, D, E, the derivatives of the total potential energy must be zero relative to the coefficients used in the displacement field and is calculated using the following expression:

$$\left[[K] - \omega^2[M]\right]\begin{Bmatrix} A \\ B \\ C \\ D \\ E \end{Bmatrix} = 0 \quad (7)$$

which K is the structural stiffness matrix and M is the mass matrix. The K matrix contains the geometrical dimensions and physical properties of the structures [15].

3.4. Verification of FE model

For verifying results of natural period of vibration gained from FE method with the natural period introduced in Eurocode-8 [28], IITK (Indian Institute of Technology Kanpur) [29], and NZSEE (New Zealand Society for Earthquake Engineering) [30] guideline for seismic design of liquid storage tanks, modal analysis is executed on the model. For the aforementioned codes, the analytical solution for obtaining the fundamental natural period of impulsive mode is as follows:

Eurocode-8:

$$T_{imp} = \frac{C_i H}{\sqrt{s}} \sqrt{\frac{\rho R}{E}} \tag{8}$$

The coefficient C_i is given in Table B.1 reported in Eurocode-8.

IITK GSDMA:

$$T_i = C_i \frac{H}{20\sqrt{5tu}} \sqrt{\frac{\rho D}{E}} \tag{9}$$

The following equation is used to determine Ci:

$$C_i = \left(\frac{20\sqrt{5D}}{\sqrt{H}\left(0.46 - 0.3\frac{H_L}{D} + 0.067\left(\frac{H_L}{D}\right)^2\right)} \right) \tag{10}$$

NZSEE:

$$T_i = \frac{5.61\pi H_L}{k_h} \sqrt{\frac{\rho R}{E}} \tag{11}$$

T_i is the fundamental mode of impulsive period (*sec*), D is the nominal tank diameter (*m*), H is the maximum design product level (*m*), t_u is the equivalent uniform thickness of the tank shell (*mm*), ρ is the density (kg/m^3), E is the Young's modulus of elasticity of tank material (Mpa), and k_h is the period coefficient, which relies on the ratio of the height to the radius of the tank. [30].

In Table 4, the result of FEM computed by eigenvalue modal analysis for the case of a steel tank is compared with the analytical solutions calculated by different codes and numerical result computed by virella et al. [11]. The natural period obtaned by FEM is obviosly in acceptable agreement with analytical formulations and numerical result.

3.5. Validation of finite element mesh

Meshing FE modeling verified to the optimum state. The FE package ABAQUS [26] is used to perform the computations using S3R triangular elements and S4R quadrilateral elements. The S4R is a shell element, which is a 4-nodes and doubly curved with hourglass control, reduced integration, and finite membrane strain formulation.
A degenerated version of the S4R and a 3-nodes with finite membrane strain formulation is S3R. High mesh refinement is needed for modeling pure bending situations because the element S3R has constant bending and membrane strain approximations. The triangular elements (S3R) are applied in the roof shell, and the quadrilateral (S4R) and triangular (S3R) elements are used in the cylinder shell and bottom shell for the FE model of the tank. The number of shell elements used in the FE meshes for the Model is 12170 elements.
A 3D view of one of the tank models with its corresponding FE mesh is illustrated in Fig. 4. The presented water elements in ABAQUS are AC3D8R. They are solid, 8-nodes, and linear acoustic brick elements. In the model, the water mesh has 46278 acoustic elements. The boundary conditions are illustrated in Fig. 5.

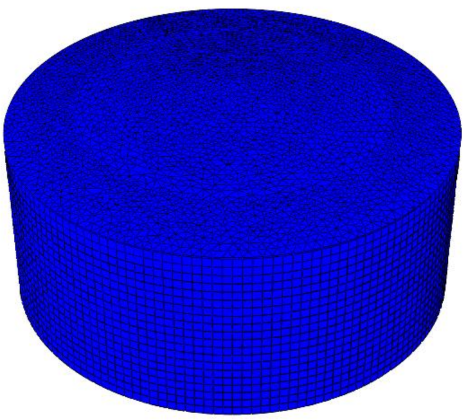

Fig. 4. Typical FE mesh for the model.

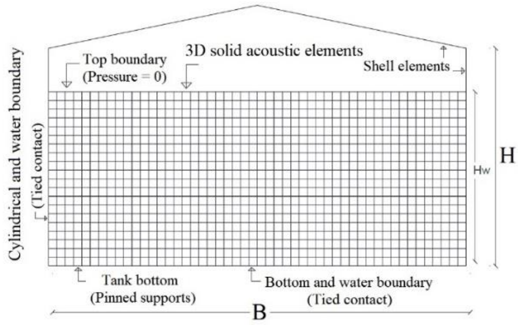

Fig. 5. Conditions assumed for the 3D tank-water finite element model.

Table 4. Comparison between fundamental period of vibration obtained from different methods

Different methods	Fundamental period (s)	Difference (%)
Eurocode-8	0.2140	4.95
IITK GSDMA	0.2030	0.19
NZSEE	0.2070	1.73
Virella et al.	0.2100	3.14
Present study	0.2034	-

4. Numerical results

4.1. Tanks without internal support

The first 40 natural frequencies for the models without internal support tanks, listed in Tables 5, are in the ranges of 2.80-5.57, 2.85-5.70, 2.77-5.61, 2.83-6.26, and 3.06-6.81, respectively, for steel tank, aluminum tank, first composite tank, second composite tank, and third composite tank. For duplicated symmetrical natural frequencies obtained, merely the odd natural frequencies presented in the table. As shown in Table 5, in all tanks without internal support and most of the early modes, only roof mode shapes can be seen. In the steel tank in 7 modes between modes 23 to 35, cylinder modes are predominant and in modes 37 and 40, both roof and cylinder mode shapes can be seen simultaneously.

In the aluminum tank, in the 6 modes between 25 and 33 and 40^{th} mode, only the cylinder mode and the modes 35 and 37 both the roof and cylinder modes are dominant. In the first type of composite tank, there is cylinder mode between modes 23 to 40 except for mode 35. In the composite2 tank, in the modes 25, 27, 31, 37, and 40, the cylinder mode is dominant and in the two modes, 33 and 35 both the cylinder and the roof mode shapes are observed. In the third type of composite tank, in the modes 27, 29, and 35, the cylinder mode predominant and in modes 31, 33, and 37 both cylinder and roof modes are observed.

According to Table 5, in tanks without internal supports, in the lower modes, the composite1 tank vibrates at a lower frequency than the other tanks and thus has less rigidity. Then the steel tank, the second type of composite tank, the aluminum tank, and the composite3 tank, vibrate at higher frequencies respectively, and thus the rigidity of the tanks is as mentioned. But at higher modes, after the first type of composite tank, respectively, the steel tank, the aluminum tank, the composite2 tank, and the composite3 tank, vibrate at higher frequencies, which means that at the higher modes, the composite2 tank is more rigid than the aluminum tank. For the steel tank without internal support, the natural frequencies ranged from 2.8-5.57 Hz.

Natural frequencies from 2.85-5.7 Hz, 2.77-5.61 Hz, 2.83-6.26 Hz, and 3.1-6.81 Hz are obtained respectively for the aluminum, composit1, composit2, and composit3. For the models without internal support, the cylinder modes and the roof modes are specified by a wave pattern, as shown in Fig. 8. The defined modes as a circumferential wave pattern meet the cos(nθ) classification, in which "θ" is the angular coordinate in the circumferential direction, and "n" is the number of circumferential waves, described by Amabili. [31] and Williston and Haroun [32]. The variations of the natural frequencies corresponding to the first cylinder and roof modes for the tanks without internal support and rafters-supported roofs are presented in Fig. 6.

4.2. Tanks with internal support

The natural frequencies of the first 40 modes for tanks with internal support are presented in Tables 6, which are in the ranges of 4.99-9.42, 5.65-9.72, 4.77-9.19, 5.08-9.58, and 6.56-11.42, respectively, for steel tank, aluminum tank, composite1 tank, composite2 tank, and composite3 tank. In all of them, only the cylindrical shell modes are dominant, resulting from the presence of internal supports, so that the stiffness of the roof is increased to such an extent that the cylindrical shell is the most flexible part of the tank.

Table 5. Natural frequency for tanks model without internal support (Hz).

Mode	Steel	Al	Composite 1	Composite 2	Composite 3
1	2.7997[b]	2.8469[b]	2.7747[b]	2.8301[b]	3.0557[b]
3	2.8352[b]	2.8854[b]	2.8592[b]	2.8749[b]	3.0747[b]
5	2.9780[b]	3.0250[b]	2.8713[b]	2.9231[b]	3.4556[b]
7	2.9928[b]	3.0468[b]	3.0382[b]	3.0384[b]	3.4701[b]
9	3.2437[b]	3.3048[b]	3.3670[b]	3.2714[b]	3.7515[b]
11	3.4772[b]	3.5493[b]	3.4390[b]	3.5071[b]	4.0649[b]
13	3.5819[b]	3.6506[b]	3.5162[b]	3.8209[b]	4.1672[b]
15	3.6399[b]	3.6903[b]	3.7795[b]	4.1173[b]	4.3038[b]
17	3.9702[b]	4.0481[b]	4.2003[b]	4.2556[b]	4.5146[b]
19	4.4190[b]	4.5072[b]	4.6695[b]	4.7285[b]	4.9899[b]
21	4.8192[b]	4.9170[b]	4.6902[b]	5.2532[b]	5.5238[b]
23	4.9155	5.0158[b]	4.7437	5.5676[b]	6.0843[b]
25	4.9312	5.0466	4.7582	5.6339	6.1912[b]
27	4.9330	5.0490	4.8372	5.6372	6.5410
29	5.0264	5.1449	4.8962	5.6865[b]	6.5426
31	5.0346	5.3265	5.0209	5.7411	6.6709[c]
33	5.2044	5.3647	5.1790	5.7534[c]	6.6791[c]
35	5.2484	5.5806[c]	5.1970[b]	5.8174[c]	6.6824
37	5.4558[c]	5.5856[c]	5.2812	5.9505	6.7725[c]
40	5.5663[c]	5.7001	5.6073	6.2553	6.8097[b]

[a] The natural frequencies indicate cylinder mode shapes unless otherwise demonstrated.
[b] The natural frequencies indicate roof mode shapes.
[c] The natural frequencies indicate cylinder and roof mode shapes simultaneously.

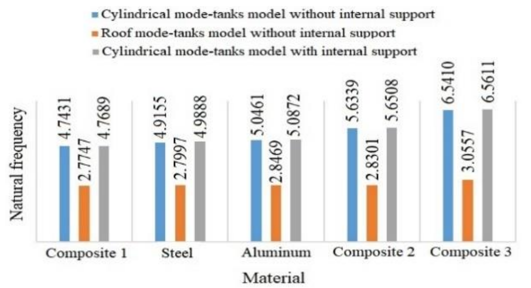

Fig. 6. The natural frequencies corresponding to the first cylinder and roof modes, for without internal support and rafters-supported tanks.

As shown in Fig. 6, the internal supports have almost no effect on the value of the fundamental frequencies of the cylindrical shell [11], while the internal supports have a considerable effect on the roof's stiffness, so that no roof vibration modes are observed in the tanks with the internal support. Although at modes much higher than the modes investigated in this study, the vibration of the roof somewhat can be seen, but in tanks without internal support which roof shell thickness is twice that of the tanks with internal support, from the first modes, the roof mode shapes can be seen. This indicates that the internal support causes the dominant modes to be cylinder modes, and the roof modes to be predominant in the tanks without internal support. Thus, internal supports have a significant role in the overall dynamic behavior of the tanks.

According to Table 6, in tanks with internal support, composite tank1 vibrates at a lower frequency than other tanks, which means that it has the least rigidity among the tanks. Then steel, aluminum, composite2, and third composite tank vibrate at higher frequencies, which means the third composite tank has the highest rigidity among these tanks. The mode shapes for the cylinder in the rafter-supported tank, and for the roof in the tank without internal support present with a wave pattern around the circumference, as shown in Figs. 7 and 8.

The lateral vibration modes of cylindrical shells were classified into two types by Amibili [26], Williston, and Haroun [27]. The first type was recognized as "cos (θ)" for that a single cosine wave of deflection is found, which "θ" is the angular coordinate in the circumferential direction. The second type was identified as "cos (nθ)" for that the deformation of the shell corresponds to the number (n) of circumferential waves. As shown in Fig. 7, the typical circumferential pattern of displacements was illustrated here for the rafters-supported tanks, which is corresponds to the cos (nθ) classification determined in references [26, 27].

Table 6. Natural frequency for tanks model with internal support (Hz).

Mode	Steel	Al	Composite 1	Composite 2	Composite 3
1	4.9888	5.0782	4.7689	5.6508	6.5611
3	4.9901	5.0823	4.7852	5.6563	6.5617
5	5.0788	5.1778	4.8630	5.7577	6.7017
7	5.0976	5.1809	4.9213	5.7799	6.7071
9	5.2516	5.3591	5.0452	5.9665	6.9632
11	5.3100	5.3951	5.2041	6.0134	7.0179
13	5.4989	5.6151	5.3026	6.2692	7.3332
15	5.6429	5.7240	5.6237	6.3778	7.4875
17	5.8131	5.9372	5.6606	6.6609	7.7987
19	6.1122	6.1921	6.0058	6.8704	8.1679
21	6.1830	6.3176	6.2819	7.1328	8.3296
23	6.6088	6.7536	6.4312	7.5130	8.3501
25	6.6971	6.7813	6.9048	7.5176	8.9773
27	7.0805	7.2378	7.1426	7.6789	9.0213
29	7.4469	7.5352	7.4173	8.2961	9.6699
31	7.5992	7.7689	7.9670	8.3374	10.0940
33	8.1863	8.3186	7.9701	8.3446	10.1070
35	8.7692	8.4671	8.2793	8.9791	10.4290
37	8.7738	8.9680	8.5271	9.3713	11.2510
40	9.4212	9.5812	9.1925	9.7247	11.4190

5. Comparison between different materials of tank

Among the tanks with different materials and considering the first impulsive mode of cylindrical tanks, in both types of tanks with internal support and without internal support, according to Table 7 and Fig. 6, the first type of composite tank has the lowest natural frequency. The natural frequencies of the first impulsive mode of the cylindrical shell of the steel tank increase about 5%, aluminum tank 6%, second type of composite tank 19%, and third type of composite tank 38% compared to the composite1 tank.

As a result, the third type of composite tank is considerably more rigid than the other types. Since graphite/epoxy is used in all composite tanks, so they have the same mass and differ only in the angles of the fibers. Therefore, the angles of the fibers considerably influence the stiffness of the composite tanks.

The difference in the frequencies of the impulsive mode in the laminated composites indicates the importance of fiber orientation in dynamic problems. For the reason that the laminated composite mass is lower than steel mass, it can be expected that the first type of composite tank will vibrate at a higher natural frequency. However, it is observed that due to the angles of the fibers in the first type of composite tank, the stiffness reduces more than in other tanks. Therefore, the first type of composite tank has a lower impulsive natural frequency than the steel tank.

According to the natural frequency formula, which has a direct relation to stiffness and an inverse relation to mass, this ratio for steel is larger than the first type of composite. It is due to the dominance of stiffness over mass. Also, this ratio is higher for the second and third types of composites compared to the first type of composite. Considering that composites have the same mass but differ in stiffness, so it proves the importance of fibers orientation in the composites. Furthermore, this ratio for steel shows the fact that mass has a dominant role over its high stiffness.

As shown in Fig. 6 and Table 7, aluminum has a higher frequency than the first type of composite.

Table 7. Lowest natural frequency (Hz), for the different materials.

Associated with tank cylinder vibration		
Roof condition		
Material	Without support roof	Rafters-support roof
Steel	4.9155	4.9888
Aluminum	5.0461	5.0782
Composite 1	4.7431	4.7689
Composite 2	5.6339	5.6508
Composite 3	6.541	6.5611
Associated with tank roof vibration		
Roof condition		
Material	Without support roof	Rafters-support roof
Steel	2.7997	Not found
Aluminum	2.8469	Not found
Composite 1	2.7747	Not found
Composite 2	2.8301	Not found
Composite 3	3.0557	Not found

Therefore, it has a higher stiffness-to-mass ratio. Regarding that, aluminum has a greater mass than composite, so it should have higher stiffness. Due to the composites having less mass and more modulus of elasticity, it can be expected that as the second and third types of composites, the first type of composite have been a higher frequency than aluminum, but the only reason for reducing the frequency and stiffness is the angles of the fibers in the composite1. In the comparison between aluminum and steel, aluminum has a lower mass and stiffness than steel. However, it has a higher stiffness-to-mass ratio, so the natural frequency for aluminum is greater than steel. Similarly, in the comparison between aluminum and the second and third types of composites, the composites have lower mass and a larger modulus of elasticity. Also, unlike composite 1, they have appropriate fibers orientation that increases their stiffness, so they have a higher natural frequency than aluminum. First vibration mode for cylinder tank with/no internal supports illustrated in Figs. 7 and 8.

As shown in Figs. 9 and 10, for the cylindrical modes, at the circumferential waves corresponding to the fundamental frequencies in each of the tanks, the effect of the shell material on the natural frequency of the first vibrational mode of the cylinder is less because the diagrams are closer together.

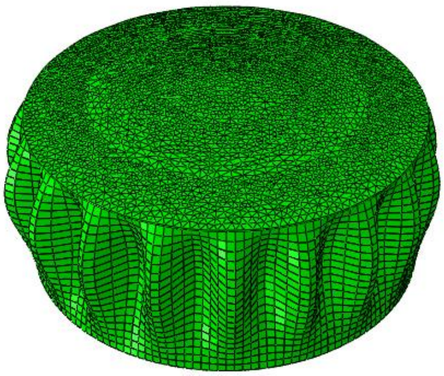

Fig. 7. First vibration mode of the cylinder for the internal supported tank.

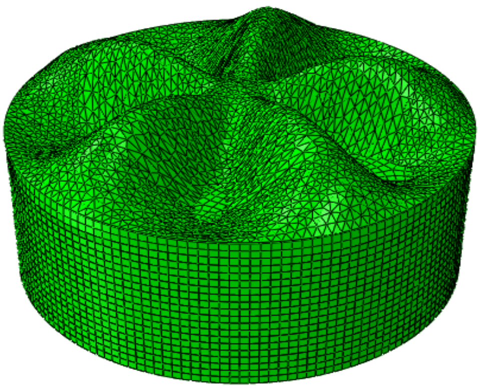

Fig. 8. First tank roof vibration mode for non-internal supported tank.

However, as shown in Table 7, the composite3 tank has the highest fundamental frequency and the composite1 tank has the lowest fundamental frequency among all the tanks.

With increasing the circumferential waves number, the effect of shell material on the second and third types of composite tanks increases as compared to steel, aluminum, and the first type composite tanks. And, the diagrams of the composites 2 and 3 tanks, which are nearly identical at higher circumferential waves, compared to the diagrams of steel, aluminum, and the composite1 tanks, which are nearly identical, there is a significant difference in frequencies corresponding to the first vibrational mode of the cylindrical part (cylinder mode). As the circumferential wave declines, the difference of this effect (shell material) between the composite 2 and 3 tanks is also observed. As observed in Fig. 10, the composite3 tank has a higher vibrational first mode natural frequency, approximately in all of the circumferential waves investigated. By reducing the circumferential waves, the composite2 tank has the lowest natural frequency compared to other tanks. It can be observed in Fig. 9 that in the first composite tank, with decreasing circumferential wave, the natural frequencies of the first cylinder mode are similar to that of the composite3 tank, and with the increasing circumferential wave, the difference between the frequencies of these two tanks increases.

Aluminum and steel tanks have approximately similar frequencies at all circumferential waves, as shown in Fig. 9, their diagrams are nearly identical over the whole path. Also, as illustrated in Fig. 11 for the roof modes, in the circumferential waves corresponding to the fundamental modes, similar to the result obtained for the cylindrical shell, the effect of the shell material on the natural frequency of the first vibrational mode of the roof is less.

Fig. 12 illustrates the angular orientation effect of fibers of each layer on the vibrational behavior of the laminated cylindrical shells. In circumferential waves corresponding to the fundamental frequencies, as summarized in Table 7, the third composite tank has the highest natural frequency, and the composite1 tank has the lowest natural frequency of the roof modes. In all circumferential waves, the composite3 tank has the highest natural frequency of the first roof vibrational mode.

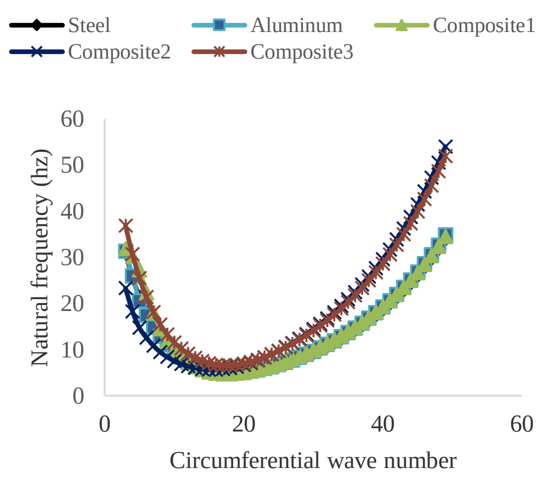

Fig. 9. The natural frequency variation of cylinder modes versus the number of circumferential wave n and axial half-wave number m = 1 for tanks without internal support.

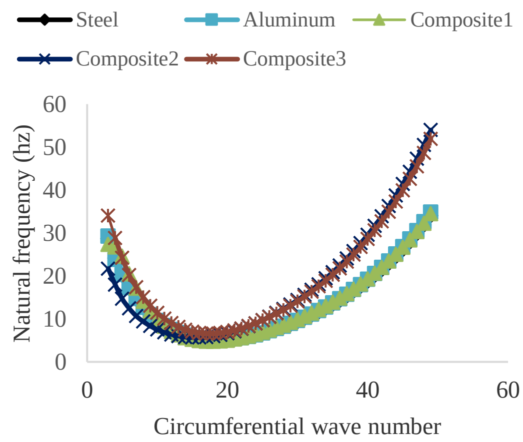

Fig. 10. The natural frequency variation of cylinder modes versus the number of circumferential wave n and axial half-wave number *m* = 1 for tanks with internal support.

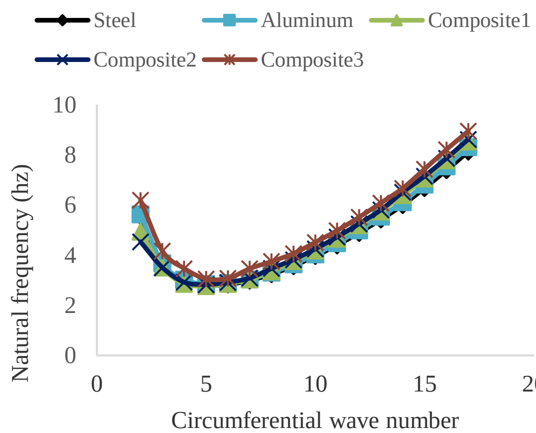

Fig. 11. The natural frequency variation of roof modes versus the number of circumferential wave n and axial half-wave number *m* = 1 for tanks without internal support.

With the increase of the circumferential wave, the steel tank has the lowest natural frequency and with the decrease of the circumferential wave, initially, the composite1 tank and then the composite2 tank have the lowest natural frequency of the first roof vibrational mode.

It can be seen from Figs. 9 and 10 that the internal support has approximately no effect on the value of the first vibration modes of the cylindrical shell corresponding to a wide range of the different number of circumferential waves. For all tanks, this difference is less than 2%, which is higher in tanks with internal support. Thus, the internal support adds less than 2% to the rigidity of the cylindrical shell. However, it can be seen from Tables 8 and 9 and Fig. 13 that the effect of the internal supports on the cylindrical shell vibrations is quite evident in the small range of the smallest number of circumferential waves, in order that the difference in natural frequencies between tanks without internal support and tanks with internal support in the lowest number of circumferential waves (*n*=3) for steel, aluminum, and the composite3 tanks is about 8%, for the composite2 tank is 7% and for the composite1 tank is about 16%.

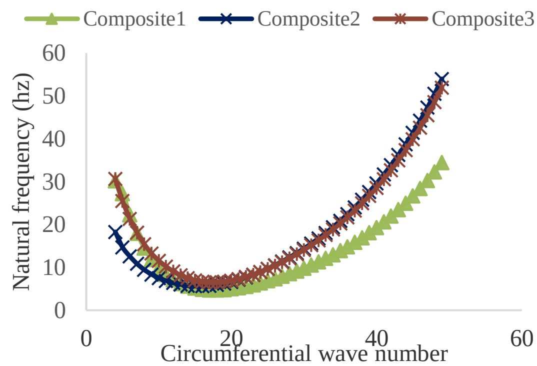

Fig. 12. The natural frequency variation of cylinder modes versus the number of circumferential wave n and axial half-wave number *m* = 1 for laminated composite tanks.

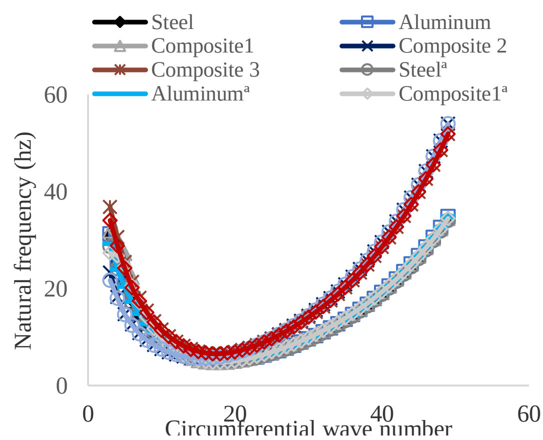

Fig. 13. Comparison of the natural frequency of cylinder modes with the number of circumferential wave n and axial half-wave number *m* = 1 for tanks with internal support and tanks without internal support.

[a]Legend: related to tanks with internal support.

Table 8. Comparison of the natural frequency of cylinder modes with the number of circumferential wave *n* and axial half-wave number *m* = 1 for tanks without internal support.

Tanks with self-supported roofs					
n	Steel	Al	Composite1	Composite2	Composite3
3	30.998	31.298	31.855	23.289	36.856
4	25.674	25.929	30.226	18.277	30.641
5	21.302	20.568	27.262	14.689	25.494
6	17.784	17.418	22.305	12.591	21.366
7	14.750	14.861	17.968	10.841	18.098
8	12.809	12.922	14.540	9.4353	15.422
9	10.995	11.092	11.836	8.3756	13.237
10	9.5204	9.6315	9.8504	7.5157	11.558
11	8.3740	8.5105	8.3155	6.8824	10.192
12	7.4156	7.5459	7.1632	6.3709	9.0679
13	6.6530	6.7786	6.2808	6.0318	8.1851
14	6.0000	6.1210	5.6268	5.7527	7.5062
15	5.5818	5.7001	5.1790	5.6372	7.0057
16	5.2478	5.3640	4.8953	5.6339	6.6709
17	5.0335	5.1486	4.7565	5.7398	6.5419
18	4.9307	5.0461	4.7431	5.9505	6.5410
19	4.9155	5.0486	4.8367	6.2553	6.6821
20	5.0259	5.1446	5.0209	6.6483	6.9450
21	5.2039	5.3259	5.2812	7.1217	7.3158
22	5.4558	5.5806	5.6064	7.6698	7.7830
23	5.7739	5.9067	5.9875	8.2875	8.3347
24	6.1497	6.2898	6.4175	8.9714	8.9619
25	6.5777	6.7264	6.8908	9.7190	9.6579
26	7.0539	7.2127	7.4059	10.530	10.419
27	7.5758	7.7458	7.9607	11.404	11.244
28	8.1415	8.3240	8.5545	12.341	12.132
29	8.7508	8.9470	9.1876	13.344	13.082
30	9.4042	9.6163	9.8616	14.413	14.097
31	10.103	10.326	10.577	15.553	15.181
32	10.848	11.089	11.337	16.765	16.334
33	11.643	11.902	12.143	18.053	17.562
34	12.488	12.767	12.997	19.421	18.867
35	13.387	13.687	13.904	20.873	20.254
36	14.344	14.665	14.865	22.415	21.728
37	15.352	15.697	15.849	24.052	23.293
38	16.433	16.800	16.931	25.789	24.956
39	17.582	17.976	18.114	27.634	26.722
40	18.802	19.226	19.332	29.593	28.599
41	20.101	20.553	20.626	31.675	30.593
42	21.482	21.966	22.001	33.887	32.712
43	22.952	23.469	23.463	36.240	34.967
44	24.518	25.071	25.018	38.744	37.365
45	26.186	26.776	26.673	41.411	39.917
46	27.964	28.596	28.436	44.253	42.635
47	29.864	30.538	30.316	47.283	45.530
48	31.894	32.614	32.323	50.512	48.615
49	34.064	34.834	34.465	53.971	51.903

In contrast to higher circumferential wave numbers, the natural frequencies at lower circumferential waves in tanks without internal support are higher than in tanks with internal support.

It can be concluded that since the circumferential waves are less than the number corresponding to the fundamental mode, the amount of natural frequencies is gradually increased in the tanks without internal support, thus in this range of the number of circumferential waves, unlike the number of circumferential waves corresponding to the fundamental mode and the range of number of circumferential waves greater than that, the cylindrical shell rigidity of the tanks without the internal support is greater.

Table 9. Comparison of the natural frequency of cylinder modes with the number of circumferential wave *n* and axial half-wave number *m* = 1 for tanks with internal support.

Tanks with rafters supported roofs					
n	Steel	Al	Composite1	Composite2	Composite3
3	28.683	29.276	27.318	21.715	34.063
4	24.154	23.872	26.743	17.992	28.866
5	20.214	20.38	24.56	14.684	24.215
6	17.122	17.321	20.501	12.434	20.182
7	14.636	14.77	17.177	10.669	17.404
8	12.585	12.707	14.114	9.3713	15.059
9	10.885	10.99	11.765	8.3374	13.056
10	9.5042	9.6094	9.7486	7.513	11.412
11	8.3716	8.4671	8.2764	6.8704	10.099
12	7.4446	7.5319	7.1379	6.374	9.0213
13	6.6969	6.7813	6.2811	6.013	8.1651
14	6.0929	6.1763	5.6443	5.7713	7.4875
15	5.6423	5.724	5.2034	5.6563	7.0179
16	5.31	5.3911	4.9213	5.6508	6.7071
17	5.0956	5.1809	4.7852	5.7575	6.5611
18	4.9901	5.0782	4.7689	5.9649	6.5617
19	4.9888	5.0818	4.8621	6.2692	6.7017
20	5.0779	5.1778	5.0452	6.6609	6.9632
21	5.2516	5.3583	5.3026	7.1328	7.3332
22	5.4989	5.6137	5.6224	7.6789	7.797
23	5.8121	5.9372	6.0058	8.2961	8.3501
24	6.183	6.3176	6.4312	8.9775	8.9733
25	6.6079	6.7536	6.9043	9.7245	9.6693
26	7.0805	7.2378	7.4158	10.534	10.429
27	7.5992	7.7682	7.967	11.407	11.251
28	8.1403	8.3186	8.5271	12.334	12.112
29	8.7704	8.9675	9.1925	13.346	13.078
30	9.4204	9.6318	9.8576	14.411	14.104
31	10.123	10.352	10.584	15.554	15.184
32	10.864	11.109	11.337	16.763	16.333
33	11.658	11.923	12.144	18.054	17.558
34	12.502	12.785	12.983	19.421	18.874
35	13.4	13.701	13.902	20.872	20.255
36	14.355	14.681	14.861	22.418	21.727
37	15.37	15.718	15.88	24.052	23.289
38	16.448	16.821	16.96	25.788	24.952
39	17.596	17.994	18.107	27.632	26.715
40	18.815	19.242	19.324	29.593	28.617
41	20.112	20.568	20.617	31.674	30.597
42	21.489	21.976	21.986	33.877	32.712
43	22.96	23.48	23.451	36.24	34.963
44	24.52	25.078	25.003	38.748	37.349
45	26.18	26.789	26.661	41.41	39.916
46	27.964	28.605	28.422	44.251	42.633
47	29.873	30.547	30.303	47.283	45.526
48	31.908	32.617	32.29	50.511	48.598
49	34.071	34.841	34.474	53.971	51.902

However, the major impact of internal support is only on the rigidity of the roof. As shown in Tables 8 and 9 and Fig. 13, it is possible to compare the natural frequencies for tanks without internal support and tanks with internal support versus the number of circumferential waves for a significant number of vibrating modes with the condition $m = 1$.

6. Conclousions

In this study, free vibration and natural frequencies were investigated in steel, aluminum, and laminated composite tanks with different fibers orientation in the tanks with internal support and without internal support. For all models, the tapered thicknesses of the cylindrical shells were considered. In tanks without internal support, by examining the first 40 modes of vibrations for the tanks, it was found that in the first modes of this range of modes, there were only roof modes and then in the later modes, there were cylinder modes of vibrations. Also, in some cases, simultaneous vibrations of the roof and the cylinder were seen.

In tanks without internal support in the first modes, the composite1, steel, composite2, aluminum, and the composite3 tanks, respectively, were vibrated at higher frequencies. But at higher modes, after the composite1 tank, steel, aluminum, the composite2, and the composite3 tanks vibrated at higher frequencies, respectively.

In the tanks with internal support, only the cylindrical shell modes are dominant; in fact, the internal beams and columns have increased the stiffness of the roof to an extent that the cylinder shell was the most flexible part of the tank and they had approximately no effect on the natural frequencies of the cylindrical shell. While they had an important influence on the stiffness of the roof and no roof vibration modes were seen for tanks with internal support in the studied range of frequencies. However, it can occur at higher frequencies. The internal support only affected the type of modes of vibration. For the tanks with internal support, the dominant modes were cylinder modes, while for the tanks without internal support, the roof modes were dominant. Thus, internal supports have an important role in the overall dynamic behavior of the tank. In the tanks with internal support, the composite1 tank had the lowest rigidity and natural frequency among the tanks. The natural frequencies of the first mode of vibration of the cylindrical shell of the steel tank increased by about 5%, aluminum tank 6%, composite2 tank 19%, and the composite3 tank 38% compared to the composite1 tank. As a result, the composite3 tank had significantly higher rigidity than other tanks, and this result holds for both types of tanks with and without internal support. Given that graphite/epoxy was used in all three types of composite tanks, so they had the same mass. They were different only in fibers orientation. Thus, the significance of the effect of fibers orientation on the stiffness of laminated composites as well as dynamic problems can be resulted.

In the circumferential waves corresponding to the fundamental frequencies for the cylinder modes, the effect of the shell material on the natural frequency of the first vibration mode of the cylinder was less. As the number of cylinder circumferential waves increased, the effect of shell material on natural frequencies increased for the second and third types of laminated composite tanks compared to steel, aluminum, and the composite1 tanks. Besides, with the decrease in the number of circumferential waves, the effect of shell material on natural frequencies between the second type and third type of laminated composite tanks was also observed. Furthermore, at circumferential waves corresponding to the fundamental frequencies for the roof modes, the influence of the shell material on the natural frequency of the first vibration mode of the roof was less.

With increasing the number of circumferential waves of the roof from the number of circumferential waves corresponding to the fundamental frequencies, the steel tank had the lowest natural frequencies of the roof in comparison with other tanks, and with decreasing the number of circumferential waves, this result was correct for the composite2 tank. For both cylinder and roof modes, in all circumferential waves, the composite3 tank had the highest natural frequencies Similar to the result obtained for the fundamental frequencies, the internal support had approximately no effect on the natural frequencies of the first vibration mode of the cylindrical shell corresponding to different numbers of circumferential waves.

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