



## Research paper

# Robust Passivity-Based Voltage Control of Robot Manipulators

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## Abstract

**Background and Objectives:** This paper presents a robust passivity-based voltage controller (PBVC) for robot manipulators with  $n$  degree of freedom in the presence of model uncertainties and external disturbance.

**Methods:** The controller design procedure is divided into two steps. First, a model-based controller is designed based on the PBC scheme. An output feedback law is suggested to ensure the asymptotic stability of the closed-loop error dynamics. Second, a regressor-free adaptation law is obtained to estimate the variations of the model uncertainties and external disturbance. The proposed control law is provided in two different orders.

**Results:** The suggested controller inherits both advantages of the passivity-based control (PBC) scheme and voltage control strategy (VCS). Since the proposed control approach only uses the electrical model of the actuators, the obtained control law is simple and also has an independent-joint structure. Moreover, the proposed PBVC overcomes the drawbacks of torque control strategy such as the complexity of manipulator dynamics, practical problems and ignoring the role of actuators. Moreover, computer simulations are carried out for both tracking and regulation purposes. In addition, the proposed controller is compared with a passivity-based torque controller where the simulation results show the appropriate efficiency of the proposed approach.

**Conclusion:** The robust PBVC is proposed for EDRM in presence of external disturbance. To the best of our knowledge, it is the first time that a regressor-free adaptation law is obtained to approximate the lumped uncertainties according to the passivity-based VCS. Moreover, the electrical model of the actuators is utilized in a decentralized form to control each joint separately.

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## Introduction

Passivity-based control (PBC) scheme is widely extended to control dynamical systems due to its exclusive features. It introduces a useful concept of system's energy [1]. In other words, the stored energy of a passive system is less than the supplied energy. Hence, passivity property can be regarded as a suitable criterion for system's stability. Therefore, the asymptotic stability of the closed-loop system can be guaranteed under some circumstances. In addition, this property can be utilized to design a proper output feedback control law [2]-[7]. On the other hand, control of robot

manipulators is still an active topic due to its applications in industry, modern surgical operations, etc [8]. The passivity concept has been used to achieve privileged success in robot manipulators control field since late 1980s [9]-[12]. Since PBC approach makes good usage of physical property of the system, it avoids unnecessary large control effort which is usually seen in high gain controllers. The PBC scheme makes the controller design procedure easier and simplifies the structure of control law as well [13]. Moreover, it can be used with other control solutions to satisfy the control objectives. For instance, passivity-based adaptive method is used to

control robot manipulators in [13]. Also, the controller that is introduced in [14] is simplified with the help of the passivity property. In [15], a passivity-based controller is designed to guarantee a good tracking performance of the closed-loop system. A disturbance observer is utilized to estimate the effect of plant uncertainties and external disturbance. Another issue that is solved by passivity concept is defined in [16]. The effect of friction which is a harmful physical phenomenon for robotic systems is considered and an observer-based friction compensator is proposed. The analyses are done in an input-output analytical framework which is exactly the meaning of passivity concept. Another similar issue is introduced in [17]. The effect of temperature variations on friction forces is studied. They proposed a passivity-based adaptive scheme to make the closed-loop system strictly passive and then an outer-loop adaptive control law removes the temperature-dependent term. In [18], the problem of tracking control for robot manipulators with torque constraints is solved. Passivity is used to ensure the asymptotic stability of the closed-loop system as a strong tool. There is much research that has paid attention to the robot manipulators control field using PBC scheme. For instance, see [19]-[21][22].

The aforementioned papers are used the PBC scheme to design a torque control law for joint position tracking problem of the robot manipulators. The torque-based control strategies suffer from the complexity of manipulator dynamics, practical problems and ignoring the role of actuators. Furthermore, the torque command cannot be applied directly to the inputs of the actuators [23]. Hence, an efficient voltage control strategy (VCS) is proposed to solve the tracking problems of electrically-driven robot manipulators (EDRM) [24]. Accordingly, only the electrical model of the actuator is used to design a proper controller. The VCS provides accuracy, speed, simplicity of calculation, and robustness for the manipulator control system. According to these contents, proposing a voltage controller for EDRM based on the passivity concept inherits both advantages of the PBC scheme and VCS. In this paper, we aim to enhance the performance of the tracking controller for EDRM in presence of external disturbances by proposing passivity-based voltage control (PBVC) approach. In addition, the proposed controller is provided in two different orders. The summation of rotor current and its derivative is regarded as constant lumped uncertain term in the electrical equation of the actuators [25]. The design procedure is divided into two steps. First, a model-based controller is suggested using PBC scheme. Then a regressor-free adaptation law is obtained to estimate the lumped uncertain term and the external disturbance. Many pieces of researches consider a

limiter assumption, in which the manipulator equation must be written in the regression form (for example see [26]-[28]). However, this condition is not regarded in this paper. The computer simulations are carried out for tracking and regulation purposes. Moreover, the performance of the proposed controller is compared with a passivity-based torque controller.

In this paper the problem of EDRM control is studied based on PBC scheme for two different orders of control law. The main contributions of this paper are as follows:

- The robust PBVC is proposed for EDRM in presence of external disturbance.
- To the best of our knowledge, it is the first time that a regressor-free adaptation law is obtained to approximate the lumped uncertainties according to the passivity-based VCS.
- The electrical model of the actuators is utilized in a decentralized form to control each joint separately.

The rest of this paper is organized as follows. Section 2 introduces the fundamental definitions and lemmas of passive systems. Section 3 presents the problem formulation, including the mechanical equations of n-link robot manipulators and also the mechanical and electrical equations of the actuators. Section 4 discusses the controller design procedures and stability analysis. Section 5 presents simulation results for a two-link manipulator.

## Passive Systems

The basic view of passive systems is defined in this section. For this reason, consider the following nonlinear system.

$$\begin{aligned}\dot{\xi} &= F(\xi, \varpi) \\ \nu &= H(\xi)\end{aligned}\tag{1}$$

where  $\xi \in R^n$  is the state vector and  $\varpi \in R^m$ ,  $\nu \in R^m$  are input and output vectors of the system, respectively.

In addition,  $F: R^n \times R^m \rightarrow R^n$  and  $H: R^n \rightarrow R^m$  are continuous nonlinear functions which  $F$  is Locally Lipschitz in  $(\xi, \varpi)$ . Moreover, it is assumed that the origin is an open-loop equilibrium point for this system.

**Definition 1 [1]:** Suppose that there exists a positive semi-definite (PSD) storage function  $S: R^n \rightarrow R$  with  $S(0) = 0$  such that the inequality (2) is satisfied. Therefore, system (1) is passive between its input ( $\varpi$ ) and its output ( $\nu$ ).

$$\dot{S}(\xi) \leq \langle \nu, \varpi \rangle\tag{2}$$

where  $\langle \nu, \varpi \rangle = \sum_{i=1}^m (\nu_i \varpi_i)$  indicates the inner multiplication of the input and output vectors.

**Definition 2 [1]:** Zero-state observability defines a property in which the only solution of unforced system

$\dot{\xi} = F(\xi)$  that satisfies  $\{H(\xi) \equiv 0\}$  is  $\xi \equiv 0$ . Such systems are called zero-state observable (ZSO).

**Lemma 1 [1]:** Consider the following particular affine form of system (1):

$$\dot{\xi} = f(\xi) + g(\xi)\varpi \quad (3)$$

where  $f: R^n \rightarrow R^n$  and  $g: R^n \rightarrow R^{n \times m}$  are locally Lipschitz functions. System (3) is passive between its input and defined output  $v = (\Delta g(\xi))^T$ , if there exists a PSD storage function  $S$  (as defined before) such that:

$$\Delta f(\xi) \leq 0 \quad (4)$$

where  $\Delta = (\partial S(\xi) / \partial \xi)^T = [\partial S(\xi) / \partial \xi_1 \dots \partial S(\xi) / \partial \xi_n]$  shows a row vector.

**Lemma 2 [1]:** The output feedback control law  $\varpi = -K\Omega(v)$ , (where  $\Omega(0) = 0$ ,  $v^T \Omega(v) > 0$  for all  $v \neq 0$  and  $K$  is a diagonal matrix and positive) guarantees the asymptotic stability of the equilibrium points for system (1), if it is:

- a) passive with a Positive Definite (PD) storage function, and
- b) ZSO.

In this paper,  $\Omega(\cdot)$  defines a function with introduced features in this lemma.

**Remark 1 [1]:** If the storage function of lemma 2 is a radially unbounded PD function, the equilibrium points of the closed-loop system will be globally asymptotically stable.

### Problem Formulation

The mechanical model of  $n$ -link robot manipulators can be described as follows [8]:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (5)$$

where  $q, \dot{q}, \ddot{q} \in R^n$  are the vector of position, velocity, and acceleration of the robot joint, respectively.

$M(q) \in R^{n \times n}$  is the inertia matrix,  $C(q, \dot{q}) \in R^n$  and  $G(q) \in R^n$  show the vector of Coriolis and Centrifugal,

and the gravitational torques, respectively.  $\tau \in R^n$  is the torque vector of the robot joints as well. Here, the desired position, velocity and acceleration vectors are considered as  $q_d, \dot{q}_d, \ddot{q}_d \in R^n$ , respectively.

The mechanical and electrical models of a permanent magnetic dc motor are introduced as [8]:

$$J_m \ddot{\theta}_m + B_m \dot{\theta}_m + r\tau = \tau_m \quad (6)$$

$$V = RI_a + L\dot{I}_a + K_b r^{-1} \dot{\theta}_m \quad (7)$$

where  $J_m \in R^{n \times n}$  and  $B_m \in R^{n \times n}$  are the inertia and damping diagonal matrices of the motors, respectively.  $\theta_m \in R^n$  shows the rotor position and  $r \in R^{n \times n}$  is the

gear coefficient matrix. In addition,  $K_b, R$  and  $L$  are  $n \times n$  diagonal matrices that represent the back-emf constant, resistance and inductance of motors, respectively. Moreover,  $\tau_m \in R^n$ ,  $V \in R^n$  and  $I_a \in R^n$  are the vector of motor torque, motor voltage and rotor current. The motor torque is proportional to the rotor current with a coefficient  $K_m = K_b$  (see 8)). Furthermore, the rotor position is proportional to the robot joint position with the gear coefficient matrix as shown in (9).

$$\tau_m = K_m I_a \quad (8)$$

$$q = r\theta_m \quad (9)$$

In this paper, the electrical model of the motor is considered in two different orders as below.

$$V = \dot{q} + \delta_1, \quad \delta_1 = RI_a + L\dot{I}_a + (K_b r^{-1} - I)\dot{q} \quad (10)$$

$$V = \ddot{q} + \delta_2, \quad \delta_2 = RI_a + L\dot{I}_a + K_b r^{-1} \dot{q} - \ddot{q} \quad (11)$$

**Assumption 1:** Precise measurement of  $\dot{I}_a$  is not possible in practice. Thus,  $\delta_{1,2} \in R^n$  is introduced as an uncertain term here. On the other hand, since the variation of  $\delta_{1,2}$  is smaller than the sampling frequency of the system, it can be regarded as a constant unknown parameter [25].

### Controller Design and Stability Analysis

In order to design a proper control law, a model-based procedure is carried out. For this reason, it is considered that  $\delta_{1,2}$  is a known constant parameter and a proper output feedback control law is proposed based on PBC scheme. This process is discussed in subsection A. Then a regressor-free adaptation law is obtained to approximate  $\delta_{1,2}$  which is described in subsection B.

#### A. PBVC

In this subsection, two lemmas are introduced to show the model-based voltage control laws according to the PBC theorem.

**Lemma 3:** The control law (12) ensures the joint position tracking of system (10) in which  $e = q - q_d$ .

$$V = \dot{q}_d + \delta_1 + \varpi \quad (12)$$

where

$$\varpi = -Ke \quad (13)$$

and  $K$  is a diagonal matrix and positive.

**Proof:**  $\varpi$  is defined as an auxiliary input for passivation in (12). In fact, it has the function of damping injection to the closed-loop system. The following closed-loop system will be obtained by substituting (12) into (10).

$$\dot{e} = \varpi \quad (14)$$

where  $e = [e_1 \dots e_n]^T$ ,  $\varpi = [\varpi_1 \dots \varpi_n]^T$ .

Consider the following PD storage function:

$$\dot{S}(e) = 0.5 \|e\|^2 \quad (15)$$

where  $\|e\| = \sqrt{\sum_{i=1}^n e_i^2}$  shows 2-norm of the  $e$ . The time derivative of (15) yields:

$$\dot{S} = \sum_{i=1}^n (e_i \dot{e}_i) = \sum_{i=1}^n (e_i \varpi_i) = \langle e, \varpi \rangle \quad (16)$$

Using (16) and lemma 1, the closed-loop system (14) is passive between the auxiliary input  $\varpi$  and the defined output  $v = e$ . In addition, it is easy to demonstrate the ZSO property of (14), according to definition 2. Thus, the output feedback control law (13) guarantees the asymptotic stability of the equilibrium point of the closed-loop system (14), for  $\Omega(v) = v$ , according to lemma 2. Therefore,  $e \rightarrow 0$  and  $q \rightarrow q_d$ . Furthermore, since the storage function (15) is a radially unbounded PD function, the global asymptotic stability of system (14) is ensured as explained in remark 1. ■

**Lemma 4:** Consider system (11). The following voltage controller ensures the global asymptotic stability of the tracking error vector for joint positions.

$$V = q_d - K_1 e - K_2 \dot{e} - K_3 \int_0^t e(\sigma) d\sigma + \delta_2 + \varpi \quad (17)$$

where

$$\varpi = -K(B^T P X) \quad (18)$$

and  $K_1, K_2, K_3$  &  $K$  are positive diagonal matrices. In addition,  $P$  is the solution of Lyapunov equation  $A^T P + P A = -I$ . Also, the parameters  $A$ ,  $X$  and  $B$  are defined as:

$$A = \begin{bmatrix} 0_{n \times n} & I_{n \times n} & 0_{n \times n} \\ 0_{n \times n} & 0_{n \times n} & I_{n \times n} \\ -(K_3)_{n \times n} & -(K_1)_{n \times n} & -(K_2)_{n \times n} \end{bmatrix},$$

$$X^T = \begin{bmatrix} (\int e)^T & e^T & \dot{e}^T \\ x_1^T & x_2^T & x_3^T \end{bmatrix}, B^T = [0_{n \times 2n}, I_{n \times n}].$$

**Proof:** Similar to the proof of lemma 3, the auxiliary input  $\varpi$  is used for passivation. Substituting (17) into (11) obtains the following closed-loop system.

$$\ddot{e} + K_1 \dot{e} + K_2 \dot{e} + K_3 \int e = \varpi \quad (19)$$

Now, consider the state-space representation of (19).

$$\dot{X} = A X + B \varpi \quad (20)$$

In which  $X$ ,  $A$  and  $B$  are introduced before. Also, we have  $(\int e)^T = X_1^T = [x_{11} \dots x_{1n}]$ ,  $e^T = X_2^T = [x_{21} \dots x_{2n}]$  and  $\dot{e}^T = X_3^T = [x_{31} \dots x_{3n}]$ . Consider the following radially unbounded PD storage function candidate for (19) which  $P$  is a symmetric PD matrix.

$$S(X) = 0.5 X^T P X \quad (21)$$

The time derivative of (21) is written as:

$$\dot{S} = 0.5 X^T (A^T P + P A) X + X^T P B \varpi \quad (22)$$

$A$  is Hurwitz, since  $K_1, K_2$  and  $K_3$  are diagonal positive matrices. If  $Q$  is a symmetric and PD matrix, then there exists a unique symmetric and PD matrix  $P$  which

satisfies the Lyapunov equation  $A^T P + P A = -Q$  [25].

Choosing  $Q = I$  and  $v^T = (X^T P B)_{1 \times n}$ , (22) will be changed as:

$$\begin{aligned} \dot{S} &= -0.5 \sum_{i=1}^n (x_{1i}^2 + x_{2i}^2 + x_{3i}^2) + \sum_{i=1}^n (v_i \varpi_i) \\ &\leq \sum_{i=1}^n (v_i \varpi_i) = \langle v, \varpi \rangle \end{aligned} \quad (23)$$

Therefore, according to lemma 1, the closed-loop system (19) is passive between the auxiliary input  $\varpi$  and defined output  $v$ .

This system is ZSO with respect to definition 2 as well. Thus, according to Lemma 2 and remark 1, the output feedback control law (18) ensures the global asymptotic stability of equilibrium points of the closed-loop system (19) for  $\Omega(v) = B^T P X$ . Therefore,  $e \rightarrow 0$  and then  $q \rightarrow q_d$ . ■

### B. Robust PBVC

In the previous subsection, the model-based controller is designed for systems (10) and (11). Now, two regressor-free adaptation laws are obtained to estimate  $\delta_{1,2}$  for the model-based controllers (12) and (17). These are expressed in theorems 1 and 2.

**Assumption 2:** In this case, an external disturbance  $\phi(t) \in R^n$  is added to the lumped uncertain term  $\delta_{1,2}$ . Suppose that the upper bound of  $\|\phi(t)\| \leq \phi_{\max}$  is a constant and unknown parameter.

**Theorem 1:** Consider system (10). The tracking problem of joint position will be solved if the robust PBVC law is designed as:

$$V = \dot{q}_d + \hat{\delta}_1 - K e \quad (24)$$

where  $K$  and  $e$  are defined in (13). The adaptation law for  $\hat{\delta}_1$  is:

$$\frac{d\hat{\delta}_1}{dt} = -\Gamma^{-1} e \quad (25)$$

in which  $\Gamma$  is a positive diagonal matrix. This control scheme is shown in Fig. 1.

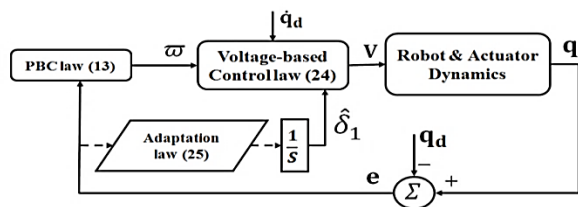


Fig. 1: The block diagram of the proposed controller for system (10).

**Proof:** Substituting (24) into (10) results in the following closed-loop system.

$$\dot{e} + K e + \tilde{\delta}_1 = 0 \quad (26)$$

where

$$\tilde{\delta}_1 = \delta_1 - \hat{\delta}_1. \quad (27)$$

In order to analyze the stability of the closed-loop system, the Lyapunov function is chosen as:

$$W(e, \tilde{\delta}_1) = 0.5 \|e\|^2 + 0.5 \tilde{\delta}_1^T \Gamma \tilde{\delta}_1 \quad (28)$$

Therefore, the time derivative of  $W$  is expressed as:

$$\begin{aligned}\dot{W} &= e^T \dot{e} + \tilde{\delta}_1^T \Gamma \frac{d\tilde{\delta}_1}{dt} = e^T (-Ke - \tilde{\delta}_1) - \tilde{\delta}_1^T \Gamma \frac{d\hat{\delta}_1}{dt} \\ &= -e^T Ke - \tilde{\delta}_1^T (e + \Gamma \frac{d\hat{\delta}_1}{dt}).\end{aligned}\quad (29)$$

The adaptation law will be obtained from the above equation.

$$e + \Gamma \frac{d\hat{\delta}_1}{dt} = 0 \rightarrow \frac{d\hat{\delta}_1}{dt} = -\Gamma^{-1}e \quad (30)$$

Thus:

$$\dot{W} \leq -e^T K e. \quad (31)$$

However, the last inequality confirms that  $\dot{W}$  is negative semi-definite. Therefore, Barbalat's Lemma [4] should be used to prove the asymptotic stability of the closed-loop system (26). The second derivative of  $W$  is:

$$\ddot{W} \leq -2\dot{e}^T K e \quad (32)$$

It is obvious that  $e$  and  $\tilde{\delta}_1$  are bounded. Therefore, from (26),  $\dot{e}$  is bounded. Thus,  $\ddot{W}$  is bounded and according to the Barbalat's Lemma,  $\dot{W} \rightarrow 0$  as  $t \rightarrow \infty$ . In other words, the asymptotic stability of the closed-loop system is guaranteed. Therefore,  $e \rightarrow 0$  and finally,  $q \rightarrow q_d$ . ■

**Theorem 2:** Consider system (11). The vector of joint position tracking error will converge to zero, if the voltage control law is designed as:

$$V = \ddot{q}_d - K_1 e - K_2 \dot{e} - K_3 \int_0^t e(\sigma) d\sigma + \hat{\delta}_2 - K(B^T P X) \quad (33)$$

where  $K_i (i=1,2,3)$  and  $K(B^T P X)$  are as defined in lemma 4 and the adaptation law for  $\hat{\delta}_2$  is as follows:

$$\frac{d\hat{\delta}_2}{dt} = -\Psi^{-1}B^TP^*X \quad (34)$$

where  $\Psi$  is a positive diagonal matrix.  $P^*$  is the solution of Lyapunov equation  $\bar{A}^T P^* + P^* \bar{A} = -I$ , in

which

$$\bar{A} = \begin{bmatrix} 0_{n \times n} & I_{n \times n} & 0_{n \times n} \\ 0_{n \times n} & 0_{n \times n} & I_{n \times n} \\ (-K'_3)_{n \times n} & (-K'_1)_{n \times n} & (-K'_2)_{n \times n} \end{bmatrix},$$

$$P = \begin{bmatrix} (P_1)_{n \times n} & (P_2)_{n \times n} & (P_3)_{n \times n} \\ (P_2)_{n \times n} & (P_4)_{n \times n} & (P_5)_{n \times n} \\ (P_3)_{n \times n} & (P_5)_{n \times n} & (P_6)_{n \times n} \end{bmatrix} \text{ and } \begin{cases} K'_1 = K_1 + K P_5 \\ K'_2 = K_2 + K P_6 \\ K'_3 = K_3 + K P_3. \end{cases}$$

This control scheme is shown in Fig. 2.

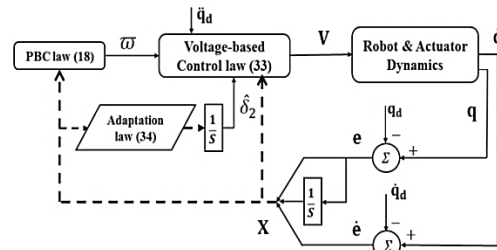


Fig. 2: The block diagram of the proposed controller for system (11).

**Remark 2:** As mentioned before,  $K_i (i = 1, 2, 3)$  and  $K$  are diagonal matrices and positive. According to the proof of lemma 4,  $P$  is the solution of Lyapunov equation of a linear system in companion form. Therefore,  $P_i (i = 1, \dots, 6)$  is a diagonal matrix which is positive for  $i = 1, 4, 6$  and is negative for  $i = 2, 3, 5$  [29]. Thus, the product of  $K$  and  $P_i$  is diagonal and thus,  $K'_i (i = 1, 2, 3)$  is a diagonal matrix that is defined in (34). Therefore,  $K'_2$  is positive since  $K_2$ ,  $K$  and  $P_6$  are positive. In addition,  $K'_1$  is positive if  $K_1 > KP_5$  and  $K'_3$  is positive if  $K_3 > KP_3$ .

**Proof of Theorem 2:** Substituting (33) into (11) results in the below closed-loop system.

$$\ddot{e} + K_1' e + K_2' \dot{e} + K_3' \int_0^t e + \tilde{\delta}_2 = 0 \quad (35)$$

where

$$\tilde{\delta}_2 = \delta_2 - \hat{\delta}_2 \quad (36)$$

The state-space equation of (35) can be shown as:

$$\dot{X} = \bar{A}X + B\alpha \quad (37)$$

in which,  $\alpha = -\tilde{\delta}_2$  shows the input of the system. In order to analyze the stability of the closed-loop system (37), the Lyapunov function is chosen as:

$$W = 0.5X^T P^* X + 0.5\tilde{\delta}_2^T \Psi \tilde{\delta}_2 \quad (38)$$

Now, the time derivative of  $W$  can be expressed as:

$$\begin{aligned} \dot{W} = & 0.5X^T(\bar{A}^TP^* + P^*\bar{A})X - \tilde{\delta}_2^TB^TP^*X \\ & + \tilde{\delta}_2^T\Psi\frac{d\hat{\delta}_2}{dt} \end{aligned} \quad (39)$$

$K'_1$ ,  $K'_2$ , and  $K'_3$  are positive diagonal matrices according to the conditions of remark 2. Moreover,  $\bar{A}$  is Hurwitz. If  $Q$  be a PD and symmetric matrix, there exists a unique PD and symmetric matrix  $P^*$  which satisfies the Lyapunov equation  $\bar{A}^T P^* + P^* \bar{A} = -Q$  [1]. Choosing  $Q=I$ , the time derivative of  $W$  can be rewritten as follows:

$$\dot{W} = -0.5X^T X - \tilde{\delta}_2^T (B^T P^* X + \Psi \frac{d\hat{\delta}_2}{dt}) \quad (40)$$

Therefore, the adaptation law is obtained from the



above equation as:

$$\frac{d\hat{\delta}_2}{dt} = -\Psi^{-1}B^T P^* X \quad (41)$$

Thus, the first-time derivative of  $W$  can be rewritten as below:

$$\dot{W} \leq -0.5X^T X \quad (42)$$

However, the last inequality confirms that  $\dot{W}$  is negative semi-definite. Hence, Barbalat's Lemma should be used to prove the asymptotic stability of the closed-loop system (37). The second derivative of  $W$  is as follows:

$$\ddot{W} \leq -\dot{X}^T X \quad (43)$$

It is obvious that  $X$  and  $\tilde{\delta}_2$  are bounded. Therefore, from (37),  $\dot{X}$  is bounded. Thus,  $\ddot{W}$  is bounded and according to the Barbalat Lemma,  $W \rightarrow 0$  as  $t \rightarrow \infty$ . In other words, the asymptotic stability of the closed-loop system is guaranteed. Therefore,  $X \rightarrow 0$  and  $\int e \rightarrow 0$ ,  $e \rightarrow 0$  and  $\dot{e} \rightarrow 0$ . ■

**Remark 3:** A voltage-based controller is designed for robot manipulators using passivity property in [30]. However, the PBC theorem is not used to design the controller and only the output strictly passivity property is employed. As described here (Lemma 2) the PBC theorem offers an output feedback law to guarantee the global asymptotic stability of the equilibrium points of the system. Furthermore, an identification method is used to compute the robot manipulator parameters in [30], and a limiter assumption is considered for the manipulator structure as well. It is supposed that the manipulator equation should be written in the regression form. While the proposed controllers of this paper are independent of robot manipulator dynamics and only a regressor-free adaptation law is used to approximate the behavior of the lumped uncertain term. In addition, a decentralized structure is used to control each joint separately.

## Results and Discussions

MATLAB/Simulink software is utilized for computer simulations to show the efficiency of the theoretical discussions. For this reason, first, the proposed robust PBVC laws (24) and (33) are applied on a two-link manipulator which is driven by permanent magnetic dc motors (subsection A and B). Then, the proposed controller (33) is compared with the passivity-based torque controller which is given in [18] (subsection C).

Schematic view of this system is shown in Fig. 3. Moreover, the parameters of the manipulator and dc motors are given in Tables 1 and 2, respectively [8].

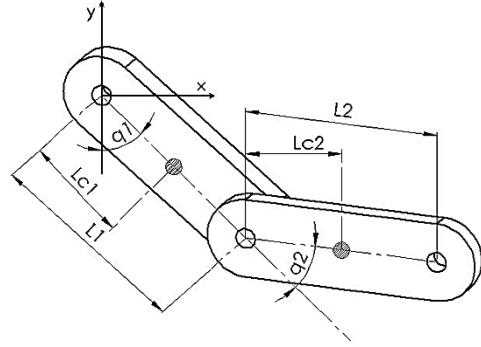


Fig. 3: Schematic view of a two-link manipulator [8].

Table 1: The Parameters of the Manipulator

Link	$l(m)$	$l_c(m)$	$m(kg)$	$I(kg.m^2)$
1	1	0.5	1	0.12
2	1	0.5	2	0.25

Table 2: The Parameters of the DC Motors 1, 2

$R$ (Ohm)	$L(H)$	$J_m$ (Nm.s <sup>2</sup> / rad)	$B_m$ (Nm.s / rad)	$K_b$ (V s / rad)	$r$
1.6	0.001	0.0002	0.001	0.26	0.01

The simulations are done for regulation and tracking objectives in subsections A and B. In addition, a sinusoidal signal with variable frequency (as seen in Fig. 4) is injected to the input of each motor as an external disturbance  $\phi(t) \in R^2$ .

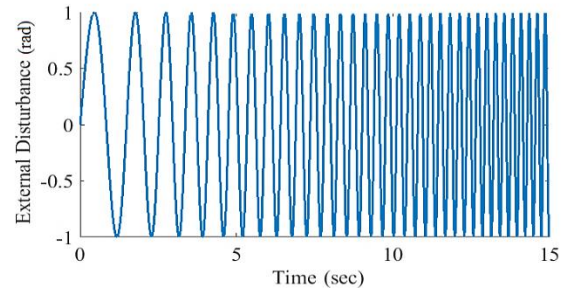


Fig. 4: The external disturbance.

The initial conditions of the manipulator joints are considered as  $q_1(0) = q_2(0) = 0.2$  that is equivalent to 10 deg. Furthermore, the RMS value  $e$  and  $V$  is calculated to evaluate the behavior of the proposed controllers numerically. This criterion is calculated as bellows for  $5 \leq t \leq 15$  to prevent the transient effects.

$$\begin{aligned} RMS\{e\} &= \sqrt{\frac{1}{10} \int_{t=5}^{t=15} \|e\|^2 dt} \\ RMS\{V\} &= \sqrt{\frac{1}{10} \int_{t=5}^{t=15} \|V\|^2 dt} \end{aligned} \quad (44)$$

### C. Regulation Purpose

In this case, a constant value of  $\text{rad}$  is regarded as a desired value. Therefore, the steady state value of the smooth function  $q_d(t) = 1 - e^{-2t}$  is the desired joint angle (as shown in Fig. 5). The tuning parameters of the proposed control laws are set as Table 3 for this case.

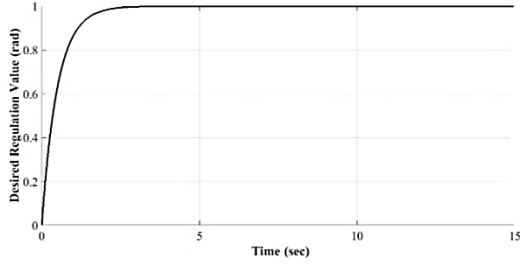


Fig. 5: The desired value (1 rad) for regulation purpose.

Table 3: Tuning Parameters for Regulation Purpose

Control Law (24)	Control Law (33)
$K = \text{diag}(100, 100)$	$K_1 = \text{diag}(250, 250)$ , $K_2 = \text{diag}(40, 40)$
$\Gamma = \text{diag}(0.005, 0.005)$	$K_3 = \text{diag}(300, 300)$ , $K = \text{diag}(250, 250)$
	$\Psi = \text{diag}(1000, 1000)$

The regulation error time history of the closed-loop systems are shown in Fig. 6. Moreover, as seen in Table 4, the order of regulation error is  $10^{-6}$  for both controllers. It can be inferred from Fig. 7, that the motor voltages are located in an acceptable range for practical considerations as well [24]. The RMS index is given in Table 4 for regulation error and motor voltages. They reveal good performance of the proposed controllers.

Table 4: The RMS Criteria for Regulation Performance

		Controller (24)	Controller (33)
$RMS\{e\}(\text{rad})$	Joint 1	$5.3233e^{-6}$	$8.4294e^{-6}$
	Joint 2	$5.0776e^{-6}$	$7.83e^{-6}$
$RMS\{V\}(\text{volt})$	Joint 1	6.9136	5.1406
	Joint 2	6.4347	4.7388

### D. Tracking Purpose

In this case, it is considered that the desired trajectory is smooth enough such that all its derivatives up to second order are bounded. For this reason, function  $q_d = 1 - \cos(\pi t/10)$  is defined as the desired trajectory which is shown in Fig. 8. The tuning parameters of the proposed control laws are set as Table 5.

According to Fig. 9, the behavior of joint position tracking is satisfactory. The order of tracking error is  $10^{-3}$  for both controllers (see Table 6). In addition, the

control efforts are depicted in Fig. 10. The amplitude of the motor voltages has suitable range for practical considerations as well. The RMS index is calculated for tracking error and motor voltage vectors as shown in Table 6. These values indicate acceptable performance of the proposed controllers for tracking purpose.

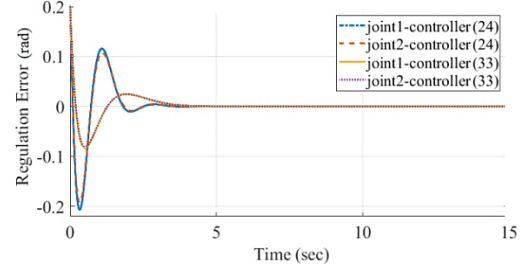


Fig. 6: The regulation error of the joints.

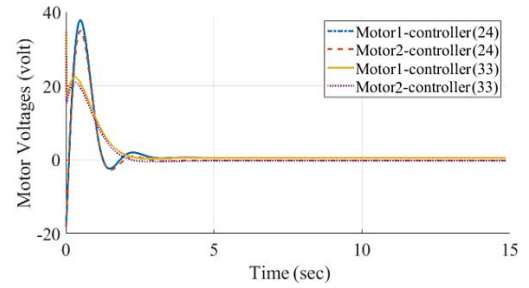


Fig. 7: The behavior of the motor voltages.

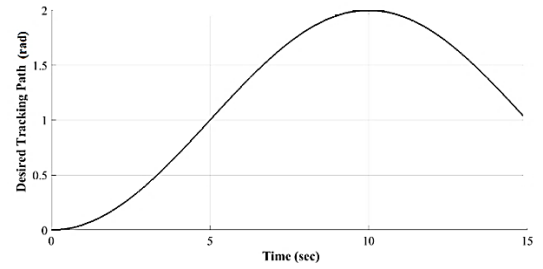


Fig. 8: The desired trajectory for tracking goal.

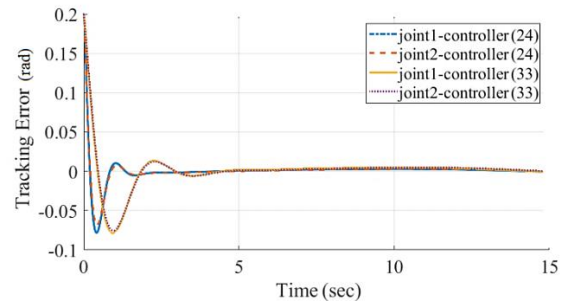


Fig. 9: The tracking error of the joints.

Table 5: Tuning Parameters for Tracking Purpose

Control Law (24)	Control Law (33)
$K = \text{diag}(150, 150)$	$K_1 = \text{diag}(180, 180)$ , $K_2 = \text{diag}(40, 40)$
$\Gamma = \text{diag}(0.0013, 0.0013)$	$K_3 = \text{diag}(550, 550)$ , $K = \text{diag}(0.5, 0.5)$
	$\Psi = \text{diag}(0.001, 0.001)$

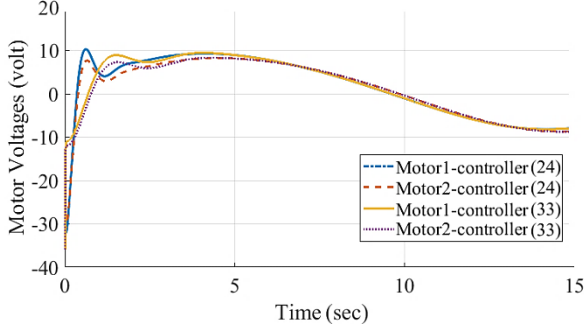


Fig. 10: The motor voltage time histories.

Table 6: The RMS Criteria for Tracking Performance

		Controller (24)	Controller (33)
$RMS\{e\} (rad)$	Joint 1	0.0023	0.0035
	Joint 2	0.0023	0.0034
$RMS\{V\} (volt)$	Joint 1	7.3705	6.8373
	Joint 2	6.9135	6.4466

### E. Comparison Study

Consider the following passivity-based torque control law that is given in [18]:

$$\tau = \underbrace{M(q) [\ddot{q}_d - k_d \dot{e} - k_p e]}_u + C(q, \dot{q}) \dot{q} + G(q) + \psi \quad (45)$$

$$\psi_i = (\bar{\tau}_i - |u_i|) f_i(u_i) \quad (46)$$

$$f_i(u_i) = \begin{cases} 0 & \text{if } |u_i| \leq \bar{\tau}_i \\ 1 & \text{if } u_i > \bar{\tau}_i \\ -1 & \text{if } u_i < -\bar{\tau}_i \end{cases} \quad (47)$$

in which,  $\psi$  is considered as a torque limiter term and  $\bar{\tau}_i$  is the desired upper bound for the  $i$ -th torque. The control law (45) is a nonlinear controller which depends on the mechanical equations of the manipulator and needs a function to limit the upper bound of the torque. Here, the performance of this control law is shown according to the defined conditions of this paper. In other words, we use the torque-based control law (45) to track the desired trajectory  $q_d = 1 - \cos(\pi t/10)$ . The tuning parameter of this controller is considered as  $k_d = \text{diag}(6, 10)$  and  $k_p = \text{diag}(13.2, 20)$ . The upper bound of the torques are given as  $T = [45 \ 12]^T$  (Nm) [18]. Moreover, the external disturbance signal that is shown in Fig. 4 is added to the input of the system. The performance of the control law (45) is weak as shown in Fig. 11. The tracking error is not converged to zero.

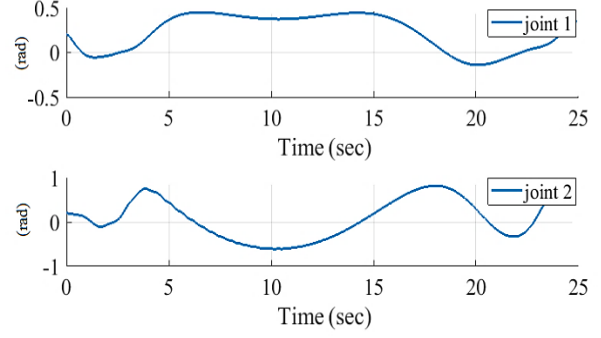


Fig. 11: Tracking error of the joints for controller (45).

Fig. 12 is depicted to compare the performance of the proposed controller (33) and the given control law (45). Moreover, the time history of the torques is shown in Fig. 13. As seen, the amplitude of the torques is located in the proper interval since using the limiter constraint (46). However, the proposed controllers (24) and (33) don't need any limiter condition to ensure achieving standard upper bound of the voltages.

Thus, the efficiency of the proposed approach is shown. The simple structure, proper accuracy and robustness against the external disturbance and model uncertainties of the actuators are the advantages of the proposed scheme.

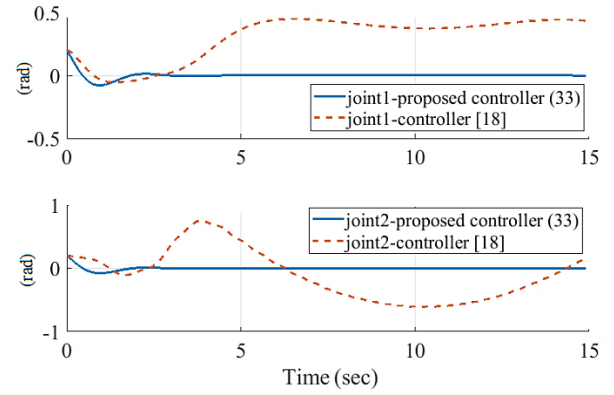


Fig. 12: Tracking error comparison for the proposed controller (33) and given controller (45).

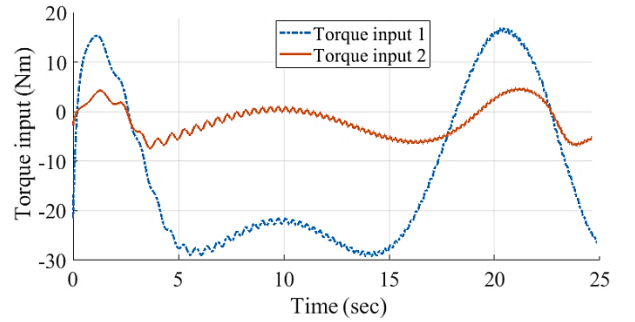


Fig. 13: The torque amplitude of the joints for controller (45).



## Conclusion

In this paper, the joint position tracking problem of the electrically driven robot manipulators has been considered. As a novelty, the voltage inputs of the actuators were proposed based on passivity theorem. The proposed controller does not have the drawbacks of the torque controllers such as complexity. Moreover, a regressor-free adaptation law was obtained to approximate the uncertainties and external disturbances of the system. The external disturbance was regarded as a chirp signal in the simulations to include more frequencies. The design procedure was independent of the robot manipulator dynamics and was done in a decentralized form as well. This feature is one of the most important advantages of the voltage control strategy. The computer simulation results show good performance for regulation and tracking purposes. Furthermore, the RMS criterion was used to show the efficiency of the proposed approach numerically. As shown in the context, the RMS value was about  $2 \times 10^{-3}$  and  $3 \times 10^{-3}$  for joint positions tracking error which is an acceptable range in practical considerations. Moreover, the proposed PBVC was compared with a passivity-based torque control law. This controller was nonlinear which depends on the mechanical equations of the manipulator and needs a function to limit the upper bound of the torque. Moreover, the simulation results shown the efficiency of the proposed controller in the presence of chirp signal as the external disturbance of the system.

## Author Contributions

Both of the authors have the same allotment of the contributions of the paper and controller design. H. Chenarani wrote the paper and M.M. Fateh was edited the context. The simulation part was provided by H. Chenarani.

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## Conflict of Interest

The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

## Abbreviations

<i>PBVC</i>	Passivity-based Voltage Controller
<i>PBC</i>	Passivity-based Control
<i>VCS</i>	Voltage Control Strategy
<i>EDRM</i>	Electrically Driven Robot Manipulator
<i>ZSO</i>	Zero-State Observable

<i>PD</i>	Positive Definite
<i>PSD</i>	Positive-semi Definite
$M(q)$	inertia matrix
$C(q, \dot{q})$	Coriolis and Centrifugal torque
$G(q)$	gravitational torque
$\tau$	Torque of the joints
$q$	Joint position
$\dot{q}$	Joint velocity
$J_m$	Inertia Matrix of the motors
$B_m$	Damping Matrix of the motors
$\theta_m$	Rotor position
$r$	Gear coefficient matrix
$K_b$	Back-emf constant
$R$	Resistance of the motors
$L$	Inductance of the motors
$\tau_m$	Motor torque
$V$	Motor voltage
$I_a$	Rotor current

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