



## Research paper

## Parameter Identification Method for Opinion Dynamics Models: Tested via Real Experiments

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### Abstract

**Background and Objectives:** One of the interesting topics in the field of social networks engineering is opinion change dynamics in a discussion group and how to use real experimental data in order to identify an interaction pattern among individuals. In this paper, we propose a method that utilizes experimental data in order to identify the influence network between individuals in social networks.

**Methods:** The employed method is based on convex optimization and can identify interaction patterns precisely. This technique considers individuals' opinions in multiple dimensions. Moreover, the opinion dynamics models that have been introduced in the literature are investigated. Then, the three models which are the most comprehensive and vastly accepted in the literature, are considered. These three models are then proven to satisfy the convexity condition, which means they can be used for the introduced method of identification.

**Results:** Four real experiments have been conducted in this research that their results verify the application of our method. The outcomes of these experiments are presented in this paper.

#### Conclusion:

Results show that the provide method is suited for parameter identification for opinion dynamic models.

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### Introduction

Complex networks have drawn the attention of scientists in the field of technology from all around the world. These networks have two types of dynamics; the dynamics of and the dynamical process on complex networks [1]. The first type, dynamics of complex networks, characterizes the unexplored aspects of growing networks by means of graph theory, which defines and evaluates an entropy rate that quantifies the information encoded in the network and measures its complexity. For the second, dynamical processes on

complex networks, we describe it as the behavior of quantum critical phenomena on complex topologies. Modeling the dynamical processes on complex networks is being used in many research fields (mathematics, physics, engineering, biology, social sciences, etc.) [2]-[5]. In the field of social networks, graphs are used to describe the links representing relationships or flows among entities. The study of opinion dynamics as a special process in social networks has recently been witnessed in an increasing number of works that rely on computational and agent-based models [6], [7]. The

social influence network theory has been around for a while and, *French and Harary's* formal theory of social power [8], [9] and *DeGroot's* consensus formation model [10] are two special cases of it. *Abelson* also introduced a continuous time equivalent model in 1964 [11] and that is the *Laplacian* flow dynamics. Another counterpart is the *Voter* model, in which each individual only has an opinion characterized by ( $\pm 1$ ) and it is referred to as the *Ising* model in the field of ferromagnetisms [12]. *Axelrod* proposed a simple but effective model and studied the convergence issue in a multidimensional space [13]. One of the well-known cases to use this type of model in analyzing the consensus agreement was presented in [14]-[18]. It was called the *Friedkin-Johnsen* (FJ) model. However, the FJ model itself was an extension of another mathematical model about the collective agreement in a working team that was proposed by *DeGroot* in 1974 [10]. Another interesting model that has been introduced by (*Altafini*, 2013) has received increasing attention lately [19]-[21]. In this model, individuals can have a negative impact on each other. Opposing the previous models (*French* 1956 and *DeGroot*, 1974), the opinions of individuals in this model can change with inverse proportions. This means that some people in the network have an adverse effect on each other. In contrast, there is another category of models which represents the dynamical behavior of agents regardless of the network graph, only in terms of the distance between agents' opinion [15], [16], [22]. There is a particular interest in modeling the opinion dynamics based on the notion of "Bounded Confidence", which was first suggested by *Deffuant* et al. in 2000 [23], and followed by *Hegselmann* and *Krause* in 2002 [24], [25]. Since then, this subject has been continuously studied and developed by other researchers [26], [27]. These models put forward a special kind of clustering, in which a delicate yet important point has been neglected, i.e., the relationship between individuals. Ignoring the communications between individuals in the second category may be considered a weakness, but both types are effective and yield reasonable performances. While the first category is widely used in microscopic applications, the second one has found its application in macroscopic approaches. In fact, when we are doing accurate calculations in small networks, the first group is useful, while the second group models find their applications at higher levels, for instance, in presidential elections. A rich survey on this research stream can be found in [28] and [29]. This paper extends the work that has been done in one of our recent papers [30]. In this paper, we used the identification technique proposed by [31] and complemented by [32]. This technique identifies the influence matrix of *Gene* regulatory networks (GRN). As

expressed in these prior papers, this method was applied to the static data from a genetic network that acted around its equilibrium point. However in this paper, we use the method on social network subjects and identify the influence matrix of dynamic behavior. Another prominent work done as a part of this research is the subjective experiments conducted. Only a few researchers have also done experiments on real-word networks [33]-[35]. We show the validity of this procedure by conducting four real experiments. Although all the experiments were successfully carried out using the method, due to page limitations, only the fourth one is reported on in this paper. Simulation results also show that the proposed method can identify the dynamics of a network (influence matrix) rather precisely. The rest of the paper is organized as follows. Mathematical preliminaries and formulation of the used models as well as the concept of multidimensional opinions and convex optimization are presented in Section I. Section II is the essential part of this study and describes the method we have used to identify the network influence matrix. Three theorems have been proved that show the convexity of the minimization program. Section III is dedicated to reporting the real experiment results. Three well-known models have been considered in the research. The identification method has been applied and their results have been shown in 2-dimensional diagrams. We introduced an error criterion which can show the differences between the three models. The dataset that was created as a part of the real-world experiments we conducted has been made available to other researchers. Finally, the conclusions wraps up the paper.

## Technical Work Preparation

### • Preliminaries and Terminology

#### A. Mathematical Notations

In this paper, we used the following matrix notation. If  $\mathbf{X}$  is a real matrix with  $N$  rows and  $M$  columns, we write:  $\mathbf{X} = [x_{ij}] \in \mathbb{R}^{N \times M}$ . The symbol  $x_{ij}$  refers to the entry of  $\mathbf{X}$  at the  $i$ th row and  $j$ th column. A single index such as  $\mathbf{X}_j$  refers to the column vector corresponding to the  $j^{\text{th}}$  column of  $\mathbf{X}$  and the operator  $\text{Var}[\mathbf{X}_j] \in \mathbb{R}_+$  is the variance of  $\mathbf{X}_j$ . It is possible to have a matrix in more dimensions. In this case, we write:  $\mathbf{X} = [x_{ijk}] \in \mathbb{R}^{N \times M \times P}$ , the symbol  $x_{ijk}$  refers to the entry of  $x_{ij}$  at  $k$ th page. Furthermore,  $|\mathbf{X}|$  stands for determinant of matrix  $\mathbf{X}$  and  $\|\mathbf{X}\|$  is the norm of the matrix. We denote the transpose of a vector  $\mathbf{X}$  by  $\mathbf{X}^T$  or  $\mathbf{X}'$ .

We use  $\mathbf{G}(\mathbf{V}, \mathbf{H}, \mathbf{A})$  notation, (sometimes called:  $\mathbf{G}(\mathbf{A})$ ) for the directed graph, where  $\mathbf{V}$  stands for a finite set of vertices,  $N = |\mathbf{V}|$  is the number of agents and  $\mathbf{H}$  is the set of edges in the network. Matrix  $\mathbf{A}$  is the adjacency matrix of graph:  $\mathbf{G}$ . In this paper, the

assumptions are:

1. Time is discrete:  $t = 1, 2, \dots, M$ , where  $M$  is the number of iterations in the experiments.
2. There is a set of  $N$  agents each with its own opinion at time( $t$ ):  $x_1(t), x_2(t), x_3(t), \dots, x_N(t)$ .
3. Each individual has its own opinion, represented by a real number:  $x_i(t) \in [-1, 1]$ ,  $i = 1, 2, \dots, N$ ;  $t = 1, 2, \dots, M$ .
4. The profile of all opinions at time  $t$  is a vector:  $\mathbf{X}_t = [x_1(t), x_2(t), \dots, x_N(t)]'$ .
5. Gathering all vectors results in a matrix:  $\mathbf{X} = [x_{it}] \in \mathbb{R}^{N \times M}$ .

### B. Classical Model (French-DeGroot)

DeGroot presented a model that describes how the group might reach a consensus and form a common subjective probability distribution, simply by revealing their individual distributions to each other and pooling out their opinions [10]. Formally, suppose there are  $N$  agents located in a directed graph:  $\mathbf{G}(\mathbf{V}, \mathbf{H}, \mathbf{A})$ . The Link  $(i, j) \in \mathbf{H}$  has the weight:  $0 \leq a_{ij} \leq 1$ , which captures the homophily between agents  $i$  and  $j$ . We have:

$$\sum_{j=1}^N a_{ij} = 1, i = 1, 2, \dots, (N = |V|) \quad (1)$$

In this situation  $\mathbf{A} \in \mathbb{R}^{N \times N}$  is a stochastic matrix and we have a homogeneous Markov chain with a finite number of states [36]. Opinions of agents lie on a real line. At time  $t$ , let  $x_i(t)$  denote the amount of opinions of agent  $i$ . The update rule in this model is as follows [37], [38]:

$$x_i(t+1) = \sum_{j=1}^N a_{ij}(t)x_j(t), \forall i = 1, 2, \dots, N \quad (2)$$

If we consider the vector  $\mathbf{X}$  as the stated variable in this network:  $\mathbf{X}' = [x_1 \ x_2 \ \dots \ x_N]$  we would have a dynamical equation for vector  $\mathbf{X}$  as follows:

$$\mathbf{X}(t+1) = \mathbf{A}(t) \mathbf{X}(t) + \mathbf{B}(t) d(t) \quad (3)$$

where matrix  $\mathbf{A}(t) = [a_{ij}(t)] \in \mathbb{R}^{N \times N}$  is the dynamic interaction of agents in time  $t$ , and  $d(t)$  can be the control signal, disturbance or measuring noise which is injected by matrix  $\mathbf{B}(t)$  into the system. In this research, we consider the network to be without any input and the system initiates from an initial condition and goes to the final equilibrium point by using this dynamic matrix  $\mathbf{A}(t)$ . In (3), matrix  $\mathbf{X}$  is known since it is being measured (possibly with noise). The goal of the method that is used in this paper is to find the unknown matrix  $\mathbf{A}(t) = \mathbf{A} \in \mathbb{R}^{N \times N}$ , which models the influence of network interactions, and explains the opinion distribution experiments [28]. Considering (3),  $\mathbf{X} \in \mathbb{R}^{N \times M}$  is the matrix of all opinions in the experiment, where  $N$  is the number of individuals and  $M$  is the number of iterations in the experiments. We introduce

Matrix  $\mathbf{X}_{next}$  to emphasize that it is one step ahead in time compared to matrix  $\mathbf{X}$ , and we can rewrite (3), via a matrix equation as follows:

$$\mathbf{X}_{next} = \mathbf{A} \mathbf{X} + \mathbf{B} \mathbf{U} \quad (4)$$

If we assume that the measurements are noise-free and we have a sufficient number of independent experiments ( $M = N$ ),  $\mathbf{X}$  would be an invertible matrix, and we could obtain  $\mathbf{A}$  using [30]:

$$\mathbf{A} = \mathbf{X}_{next} \mathbf{X}^{-1} \quad (5)$$

However, the absence of noise is not realistic, except for when we are dealing with simulated data. Furthermore, any mathematical model for opinion dynamics has special constraints that should be considered for extracting matrix  $\mathbf{A}$ . Consequently, we define identification error  $\mathbf{E}(t)$  as:

$$\mathbf{E}(t) = \mathbf{X}(t+1) - \mathbf{A} \mathbf{X}(t) = [e_i(t)], \quad i = 1, 2, \dots, N \quad (6)$$

and we try to minimize  $\mathbf{E}$  as a function of  $\mathbf{A}$  with respect to the selected model constraints. In the paper, the total squared error as presented in [31] has been used.

Any identification procedure has two phases. In the first phase, we have:

$$Error\ Criterion = \sum_{t=1}^M \sum_{i=1}^N e_i(t)^2 \quad (7)$$

And then, we use the following criterion for the second phase:

$$Error\ Criterion = \sum_{t=1}^M r_t \sum_{i=1}^N e_i(t)^2 \quad (8)$$

In the last equilibrium,  $\mathbf{E}_t = \mathbf{E}(t) = [e_i(t)]$  is the identification error in the  $t$ th step of the experiment, and  $r_t = (\text{Var}(\mathbf{E}_t))^{-1}$  is the inverse of error variance in the same step of previous phase. We use this part for the improvement of identification at any time that the result of our algorithm in the previous phase is not very close to the dynamic matrix [31]. To determine this fact, we check the error between our estimation and the real experiment at all times ( $t = 1, 2, \dots, M$ ). The intuition behind this weight is that identification errors in experiments with more reliable data (smaller variance), weigh more than those coming from less reliable data [30].

### C. Altafini Model

The original Altafini model coincides with the Abelson model; however the matrix  $\mathbf{A}(t)$  need not be non-negative. The continuous time model of Altafini is as in the following equation:

$$\dot{x}_i(t) = \sum_{j=1}^N |a_{ij}| (x_j(t) \text{sgn}(a_{ij}) - x_i(t)), \quad \forall i = 1, 2, \dots, N \quad (9)$$

and it is equal to the linear system:

$$\dot{\mathbf{X}} = -\mathbf{L} \mathbf{X} \quad (10)$$

where  $\mathbf{L} = [l_{ij}]$  is the standard Laplacian matrix

associated with matrix  $A$  whose elements are:

$$l_{ij} = \begin{cases} \sum_{j \neq i} a_{ij}, & i = j \\ -a_{ij}, & i \neq j \end{cases} \quad (11)$$

The discrete time *Altafini* model can be considered as follows [29]:

$$\mathbf{X}(t+1) = \mathbf{A} \mathbf{X}(t) \quad (12)$$

where:  $\sum_{j=1}^N |a_{ij}| = 1, \forall i = 1, 2, \dots, N$

$$a_{ii} \geq 0, -1 \leq a_{ij} \leq +1$$

In this equations,  $|\mathbf{A}| = [a_{ij}]$  is a stochastic matrix.

#### D. Friedkin-Johnsen (FJ) Model

This model was introduced in 1999 [15] and is one of the connection-based models. The opinions in this model change according to the following equation [39]:

$$\mathbf{X}(t+1) = \Lambda \mathbf{A} \mathbf{X}(t) + (\mathbf{I} - \Lambda) \mathbf{X}(0), \quad (13)$$

$t = 1, 2, \dots, M$

where matrices  $\Lambda$  &  $A$  are positive and  $I$  is the unit matrix.  $A = [a_{ij}] \in \mathbb{R}_+^{N \times N}$  is the matrix of interpersonal influence, and  $\Lambda = \text{diag}(\lambda_i) \in \mathbb{R}_+^{N \times N}$  is the matrix of actors' susceptibilities to interpersonal influence and  $\mathbf{X}(0)$  is the vector of initial opinion of individuals. In this equation, we have the following constraints:

$$\begin{aligned} \sum_{j=1}^N a_{ij} &= 1, & i = 1, 2, \dots, N \\ 0 \leq a_{ij} &, & i, j = 1, 2, \dots, N \\ 0 \leq \lambda_i \leq 1 &, & i = 1, 2, \dots, N \end{aligned} \quad (14)$$

#### E. Two or Multidimensional Opinions

As seen in most papers about opinion dynamics, researchers assume the opinion of the individuals as real numbers and shape them using a one-dimensional array. Considering individuals' opinions as real numbers is an underestimation of them. The issue which is neglected here is that a person's opinion should be considered in a multidimensional vector due to the number of subjects which have been noticed by individuals. The matter that is pointed to in this important subject is that the opinion of a person about two topics (for example two election candidates) can't be a scalar value. In order to give a picture of individuals' opinions through a better method, they could be presented as vectors.

In this way, any element of vector  $\mathbf{X}$  in (3), (i.e.,  $\mathbf{x}_i$ ) is a vector and has its own elements:  $\mathbf{x}_i = [x_i^1 \dots x_i^D]^T$  where  $D$  is the number of topics in the subject. For example, consider an election with two separate candidates. The opinions of individuals in the network have two dimensions ( $D = 2$ ) and can be represented using the following notation:

$$\mathbf{x}_i = \begin{bmatrix} x_i^1 \\ x_i^2 \end{bmatrix}, \quad \forall i = 1, 2, \dots, N \quad (15)$$

where  $N$  is the number of individuals in the network.

The figure below (Fig. 1) portrays the opinion of 5

nodes in the network at the beginning of an election. As displayed in the figure, opinions of people are placed on a 2-dimensional plane. Any individual can have separate opinions about two candidates. This causes their opinions to be placed anywhere on the plane, not just on a straight line. Although this method makes the equations more complicated than before, it can be an effective method to use. The method for considering the opinions multi-dimensionally has been considered in previous papers; however our approach is simple and different from those [39], [40].

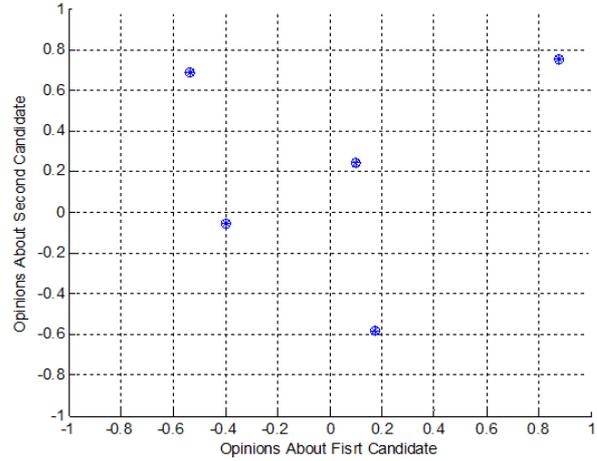


Fig. 1: Multidimensional individuals' initial opinions, (A case study).

#### F. Convex Optimization

The method used in this paper is based on an optimization technique called convex programming [41]. Basically, convex programming is a mathematical theory for minimization of a convex cost function in relation to a convex set of feasible solutions. Formulating the identification problem as a convex programming one, could attract attention because there are many techniques for solving convex programming problems efficiently. The convex optimization problems that we state in this paper have been solved using MATLAB with toolbox CVX [42].

##### • Identification Procedure

Identification of opinion dynamics in a consensus agreement network is intended to estimate the influence matrix among individuals and possibly the diagonal matrix of actors' susceptibilities to the social influence in some models. We walk the reader through the network matrix identification process (via convex optimization) by taking one of the models mentioned before: i.e. the classical model of French-DeGroot. In this case, we define the identification error matrix ( $E \in \mathbb{R}^{N \times M}$ ) as the following for one-dimensional opinions:

$$\mathbf{E} = \mathbf{X}_{next} - \mathbf{A} \mathbf{X} = [e_{it}], \quad (16)$$

$i = 1, 2, \dots, N \ \& \ t = 1, 2, \dots, M$

and as the following equation for multidimensional opinions:

$$\mathbf{E}^k = (\mathbf{X}_{next}^k - \mathbf{A} \mathbf{X}^k) = [e_{it}]^k, \quad (17)$$

$$k = 1, 2, \dots, D$$

where  $D$  is dimension of opinions and  $\mathbf{E}^k$  is the  $k^{th}$  page of total identification error matrix.

As is clear from (17), the error is calculated over all pages ( $k = 1, 2, \dots, D$ ), and then, their norms are added together. We try to minimize  $\sum_{k=1}^D \|\mathbf{E}^k\|$  (as a function of  $\mathbf{A}$ ) while obtaining a minimal model for matrix  $\mathbf{A}$  and satisfying the prior constraints.

By defining  $\mathbf{E} = [e_{it}]$  as in (16), (one dimensional opinions), it can be formulated mathematically as the following optimization problem:

$$\underset{\mathbf{A}}{\text{minimize}} \sum_{t=1}^M r_t \left( \sum_{i=1}^N e_{it}^2 \right) \quad (18)$$

$$\text{subject to: } 0 \leq [a_{ij}], \sum_{j=1}^N a_{ij} = 1, \forall i$$

Note that the optimization variable is  $\mathbf{A} \in \mathbb{R}^{N \times N}$  and the computational error is:  $\sum_{t=1}^M r_t (\sum_{i=1}^N e_{it}^2) \in \mathbb{R}$ . Since in these equations, the *French-DeGroot* model has been utilized on the basis of this model, the constraints:  $\sum_{j=1}^N a_{ij} = 1$  &  $0 \leq [a_{ij}]$ ,  $\forall i, j$  are added to the equation.

#### A. Theorem I (French-DeGroot Model):

The minimization problem in (18), can be changed to a convex problem as follows.

##### Proof:

A convex optimization problem usually has the following form [41]:

$$\underset{x}{\text{minimize}} f_0(x) \quad (19)$$

$$\text{subject to: } f_i(x) \leq 0, \quad i = 1, 2, \dots$$

where the functions  $f_i$ ,  $\forall i = 0, 1, \dots$  are convex. This means that *domain*  $f_i(x)$  must be a convex set and we must have:

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y) \quad (20)$$

For any  $x, y \in \mathbb{R}^n$  and  $\forall \alpha, \beta \in \mathbb{R}_+$  with:  $\alpha + \beta = 1$ .

Now consider that  $f_0(\mathbf{A})$  in (18), is as the following:

$$f_0(\mathbf{A}) = \sum_{i=1}^N \sum_{t=1}^M e_{it}^2 = \|\mathbf{A} \mathbf{X} - \mathbf{X}_{next}\|_2^2 \quad (21)$$

Here,  $\mathbf{A} \in \mathbb{R}^{N \times N}$  is a variable,  $\mathbf{X}$  and  $\mathbf{X}_{next} \in \mathbb{R}_+^{N \times M}$  are known and  $f_0: \mathbb{R}^{N \times N} \rightarrow \mathbb{R}_+$ . Parameter  $r_t \in \mathbb{R}_+$  is inverse of the error variance in the  $t^{th}$  step. This formulation is not convex [31] in this current format. So, to make it be convex, we solve the problem iteratively in several iterations and in each iteration the variance ( $r_t$ ) is considered to be a constant, as the following:

- In the first iteration  $r_t$  is considered as 1.

- In the next phases, it is computed as the error variance in the previous phase.

With those preliminaries, the function  $f_0(\mathbf{A})$  is a simple sum-square function, and it is a special case of

convex function (refer to part 1.3 of [41]). Moreover, we have:  $0 \leq a_{ij} \leq 1$ ,  $i, j = 1, 2, \dots, N$ . This means that the domain of  $f_0(\mathbf{A})$  is a finite real number set and it is a convex set. We may represent the constraints of (18), as  $N$  separated relations:

$$f_i(\mathbf{A}) = \sum_{j=1}^N a_{ij} = 1, \quad i = 1, 2, \dots, N \quad (22)$$

For convexity of these functions we should have:

$$f_i(\alpha \mathbf{A} + \beta \mathbf{B}) \leq \alpha f_i(\mathbf{A}) + \beta f_i(\mathbf{B}), \quad \forall i \quad (23)$$

for any  $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{N \times N}$  and  $\alpha, \beta \in \mathbb{R}_+$  with  $\alpha + \beta = 1$ .

In the left hand of (23), for any  $i = 1, 2, \dots, N$ , we have:

$$f_i(\alpha \mathbf{A} + \beta \mathbf{B}) = \sum_{j=1}^N (\alpha a_{ij} + \beta b_{ij}) \quad (24)$$

It is equal to:

$$\sum_{j=1}^N \alpha a_{ij} + \sum_{j=1}^N \beta b_{ij} = \alpha \sum_{j=1}^N a_{ij} + \beta \sum_{j=1}^N b_{ij} \quad (25)$$

then, we have:

$$f_i(\alpha \mathbf{A} + \beta \mathbf{B}) = \alpha f_i(\mathbf{A}) + \beta f_i(\mathbf{B}) \quad (26)$$

■ If we use the *Altafini* model in (12), the minimization problem changes to the following equations:

$$\underset{\mathbf{A}}{\text{minimize}} \sum_{t=1}^M r_t \left( \sum_{i=1}^N e_{it}^2 \right) \quad (27)$$

$$\text{subject to: } -1 \leq \mathbf{A} \leq 1,$$

$$\sum_{j=1}^N |a_{ij}| = 1, \quad i = 1, 2, \dots, N$$

#### B. Theorem II (Altafini Model):

The minimization problem in (27), is a convex problem with the same trick as said in *theorem I*.

##### Proof:

The first part of proof is the same as *theorem I* and for the second part we have:

$$f_i(\mathbf{A}) = \sum_{j=1}^N |a_{ij}| = 1, \quad i = 1, 2, \dots, N \quad (28)$$

For convexity, we should have:

$$f_i(\alpha \mathbf{A} + \beta \mathbf{B}) \leq \alpha f_i(\mathbf{A}) + \beta f_i(\mathbf{B}), \quad \forall i \quad (29)$$

We have:

$$f_i(\alpha \mathbf{A} + \beta \mathbf{B}) = \sum_{j=1}^N |\alpha a_{ij} + \beta b_{ij}| \quad (30)$$

$$\leq \sum_{j=1}^N |\alpha a_{ij}| + \sum_{j=1}^N |\beta b_{ij}|$$

Since  $0 \leq \alpha, \beta$  is equal to:

$$= \sum_{j=1}^N \alpha |a_{ij}| + \sum_{j=1}^N \beta |b_{ij}| \quad (31)$$

$$= \alpha \sum_{j=1}^N |a_{ij}| + \beta \sum_{j=1}^N |b_{ij}|$$

Then, we have:

$$f_i(\alpha \mathbf{A} + \beta \mathbf{B}) \leq \alpha f_i(\mathbf{A}) + \beta f_i(\mathbf{B}) \quad (32)$$

It means that the theorem is proved. ■

In multi-dimensions, the optimization problem for the *French-DeGroot* model can be formulated as the following:

$$\begin{aligned} & \underset{\mathbf{A}}{\text{minimize}} \sum_{k=1}^D \|\mathbf{X}_{next}^{(k)} - \mathbf{A} \cdot \mathbf{X}^{(k)}\|_2^2 \\ & \text{subject to : } 0 \leq \mathbf{A} = [a_{ij}] \leq 1, \\ & \sum_{j=1}^N a_{ij} = 1, \quad i, j = 1, 2, \dots, N \end{aligned} \quad (33)$$

where  $\mathbf{A} \in \mathbb{R}^{N \times N}$ ,  $\mathbf{X} \in \mathbb{R}^{N \times M \times D}$ , and  $D$  is the number of dimensions of opinions.

**C. Theorem III (Multidimensional F-DeGroot Model):**

The minimization problem in (33), is a convex problem.

**Proof:**

In this situation,  $Dom f_0 = \mathbf{A}, -1 \leq \mathbf{A} \leq 1$  is a convex set and equations  $f_i(\mathbf{A}) = \sum_{j=1}^N a_{ij}$ ,  $\forall i = 1, 2, \dots, N$  are  $N$  convex functions; the same as theorem I&II. For  $f_0(\mathbf{A})$ , we have  $D$  separated sum-square functions which are added together as:

$$f_0(\mathbf{A}) = \sum_{k=1}^D f_0^{(k)}(\mathbf{A}), \quad k = 1, 2, \dots, D \quad (34)$$

where:

$$f_0^{(k)}(\mathbf{A}) = \|\mathbf{A} \mathbf{X}^{(k)} - \mathbf{X}_{next}^{(k)}\|_2^2 \quad (35)$$

and  $\mathbf{X}^{(k)}$  is the  $k^{th}$  page of matrix  $\mathbf{X}$  and  $f_0^{(k)}(\mathbf{A})$  is a sum-square function. As we know, affine functions do not change convexity and since "sum" is an affine function,  $f_0(\mathbf{A})$  is also a convex function with a domain of convex set ([41], Section 2.3). ■

The theorem for the multidimensional *Altafini* model is similar and is hence omitted.

**D. Theorem IV (Friedkin-Johnsen Model):**

The minimization problem for *FJ* model in (13), and (14), can be rearranged to a convex problem as follows

$$\mathbf{X}_{next} = \mathbf{B} \mathbf{X} + (\mathbf{I} - \mathbf{A}) \mathbf{X}_0 \quad (36)$$

Where:  $\mathbf{B} = \mathbf{A} \mathbf{A}$  which  $\mathbf{B} \in \mathbb{R}_+^{N \times N}$  is as the new optimization variable and the new constraints are accordingly as follows:

$$\begin{aligned} & \sum_{j=1}^N b_{ij} = \lambda_i, \quad i = 1, 2, \dots, N \\ & 0 \leq \lambda_i \leq 1, \quad i = 1, 2, \dots, N \end{aligned} \quad (37)$$

**Proof:**

Equation (13) is a vector one, whose matrix form can be written as:

$$\mathbf{X}_{next} = \mathbf{A} \mathbf{A} \mathbf{X} + (\mathbf{I} - \mathbf{A}) \mathbf{X}_0 \quad (38)$$

where:  $\mathbf{X}_0 \in \mathbb{R}^{N \times M}$  is a matrix composed of the initial states of individuals' opinions:

$$\mathbf{X}_{0j} = [x_1(0), x_2(0), \dots, x_N(0)]^T, \quad (39)$$

$\forall j = 1, 2, \dots, M$

The matrix  $\mathbf{X}_{next} \in \mathbb{R}^{N \times M}$  is one step ahead of experiment  $\mathbf{X} \in \mathbb{R}^{N \times M}$  and other parameters were

explained in the previous section. Choosing  $\mathbf{B} = \mathbf{A} \mathbf{A}$  yields equation (36).

Now, for constraints, consider that  $\mathbf{A} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$  and,

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{pmatrix} \quad (40)$$

So:

$$\begin{aligned} \mathbf{B} = \mathbf{A} \mathbf{A} &= \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1N} \\ b_{21} & b_{22} & \dots & b_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1} & b_{N2} & \dots & b_{NN} \end{pmatrix} \\ &= \begin{pmatrix} \lambda_1 \cdot a_{11} & \lambda_1 \cdot a_{12} & \dots & \lambda_1 \cdot a_{1N} \\ \lambda_2 \cdot a_{21} & \lambda_2 \cdot a_{22} & \dots & \lambda_2 \cdot a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_N \cdot a_{N1} & \lambda_N \cdot a_{N2} & \dots & \lambda_N \cdot a_{NN} \end{pmatrix} \end{aligned} \quad (41)$$

Then for any  $i = 1, 2, \dots, N$  we have:

$$\begin{aligned} \sum_{j=1}^N b_{ij} &= b_{i1} + b_{i2} + \dots + b_{iN} \equiv \\ \sum_{j=1}^N (\lambda_i a_{ij}) &= \lambda_i a_{i1} + \lambda_i a_{i2} + \dots + \lambda_i a_{iN} = \\ \lambda_i (a_{i1} + a_{i2} + \dots + a_{iN}) &= \lambda_i \sum_{j=1}^N a_{ij} \end{aligned} \quad (42)$$

Since (from (14),):

$$\sum_{j=1}^N a_{ij} = 1, \quad \forall i = 1, 2, \dots, N \quad (43)$$

Then we have

$$\sum_{j=1}^N b_{ij} = \lambda_i, \quad \forall i = 1, 2, \dots, N \quad (44)$$

Which is the same as equation (37) and this ends the proof.

**Results and Discussion**

Four experiments were conducted in this research. A group of students along with their supervising professors were involved in these consensus agreement experiments. Two different topics in the same area were introduced and the group was asked to reach an agreement in a finite time. The objective of these experiments was to identify an opinion influence network among participants based on three different models and then predict the model parameters. Table shows an overview of these experiments. Although all experiments have been done during this research, only the fourth one is reported on in the paper due to page limitations. The procedure followed in the experiment is given below:

- The subject was discussed and explained.
- Individuals were asked to express their opinion on both topics in the subject of the experiment with an ordered number:  $x_i(t) \in \mathbb{R}^2, i = 1, 2, \dots, N$  between:  $[0, 100]$ , where  $t$  is the step time and  $N$  is the number of agents.
- Then in the laboratory, we changed the data from:  $[0, 100]$  to:  $[-1, 1]$ .
- The initial opinion of the participants on the issue

was asked for and recorded.

- In any iteration, the members of the group discussed their point of view about the issue.
- All of the participants were allowed to send one post in any step.
- A finite amount of time was given to the individuals to send their opinions and record them.
- After some pre-specified time or upon reaching a consensus, the group members recorded their final opinions on both issues.

Table 1: Overview of experiments

No. of Exp.	No. of Participants	No. of Topics	No. of Iterations	Time Duration
I	N = 5	D = 2	M = 22	2.5 hours
II	N = 5	D = 2	M = 20	2 hours
III	N = 7	D = 2	M = 15	15 days
IV	N = 39	D = 2	M = 45	45 days

These experiments provide us with the data on the theoretical constructs involved in the influence model. Given these data, the interpersonal influence network ( $\mathbf{A} \in \mathbb{R}^{N \times N}$ ) and possibly the diagonal susceptibilities matrix ( $\mathbf{\Lambda} = \text{diag}(\lambda_i)$ ) in some models were computed through convex optimization. Although it was possible to identify the influence matrix by using one-dimensional opinions, which would give higher identification errors for the second dimension, our identifications were done using multidimensional opinions and resulted in lower errors for both dimensions. The error criterion that we used was the rational norm of the difference between the experimental results and those of simulations done based on the identified network:

$$\text{Error} = \frac{\|\mathbf{X}_{exp} - \mathbf{X}_{simu}\|_{fro}}{\|\mathbf{X}_{exp}\|_{fro}} \quad (45)$$

where  $\mathbf{X}_{exp}$  is the real opinions acquired through the experiments and  $\mathbf{X}_{simu}$  is the opinions identified by the method. In this equation, *Frobenius* norm has been utilized because it has the best similarity to the sum of elements' absolute values in sparse matrices [43].

As mentioned before in this paper, only the fourth experiment was reported. In this experiment, the number of participants was  $N = 39$  and the duration was  $M = 45$  steps. The subject of the experiment falls within the territory of technical issues related to operating systems for intelligent mobile phones. More precisely, we discussed and reordered the opinion of participants about the subject by introducing two topics: 1) Android and 2) iOS (iPhone Operating System).

The experiment took over one month. Each step took about a day and began from 9 AM to 12 AM the next day. There were two phases every day: 1. Discussion (from 9 AM to 10 PM) and 2. Recording the opinions

(from 10 PM until 12 AM). The times are in GMT+4:30. We had participants from all around the world and we chose these timeframes for their convenience. In the experiment, the opinions were recorded invisibly by an observer who did not participate in the discussions.

Fig. 2 shows the opinions of the participants along each dimension. Real opinions of 39 individuals have been shown in different colors. As represented in the figure, the opinions of participants were converged in a bipartite situation. About 80 percent of the people believed the first choice, whereas about 13 percent believed the second one. There are 7 percent of people whose original beliefs did not change. In the following figures top charts show the first dimension and chart in the bottom of figures show the second dimension.

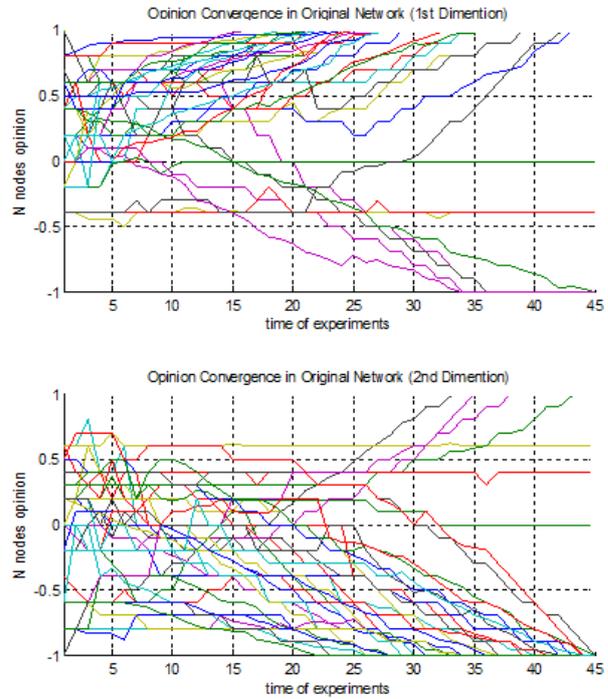


Fig. 2: Projection of the opinions in experiment IV. (Top first, down second dimension).

#### A. Fit Data to French-DeGroot Model

As described in (Section I), DeGroot introduced a mathematical model based on (2), where  $\mathbf{A}$  is a stochastic matrix. Through the influence matrix ( $\mathbf{A}$ ), various analyses can be done on the network of the experiment, however, we should first validate the model. The results we saw from this model showed the following error when compared to the experimental data:

$$\text{Error}_{DeG} = 0.3468 \quad (46)$$

which was defined as:

$$\text{Error} = \frac{\|\mathbf{X}_{exp} - \mathbf{X}_{simu}\|}{\|\mathbf{X}_{exp}\|} \quad (47)$$

in any dimension and is extended to multiple dimensions as:

$$Error = \frac{\sum_{k=1}^D \|X_{exp}^k - X_{simu}^k\|}{\sum_{k=1}^D \|X_{exp}^k\|} \quad (48)$$

where  $X_{exp}^k$  is the real experiment value in the  $k^{th}$  page,  $X_{simu}^k$  is the identified value,  $D$  is the dimension of opinions, and the subtitle of *Error* (e.g., *DeG* in this case) refers to the model that has been selected for this part of identification. Equation (46), shows the error between real opinions and the simulated ones in one-dimension and, it shows the value error for 2-dimensions. The one-dimension figures show that the classical model has not been able to follow the experimental data and there is an obvious difference between the experimental data samples and the simulated results.

One of the significant drawbacks of classical models such as *DeGroot* (which estimate the mean value of the opinions) is that they cannot follow the unusual behavior of opinions which naturally occurs in human society. As is clear in the figures, opinion values in the real experiments go beyond those which have been collected earlier. However, Fig. 3 shows that the *DeGroot* model cannot simulate this behavior and this results in a big error between the real experiments and the simulated ones.

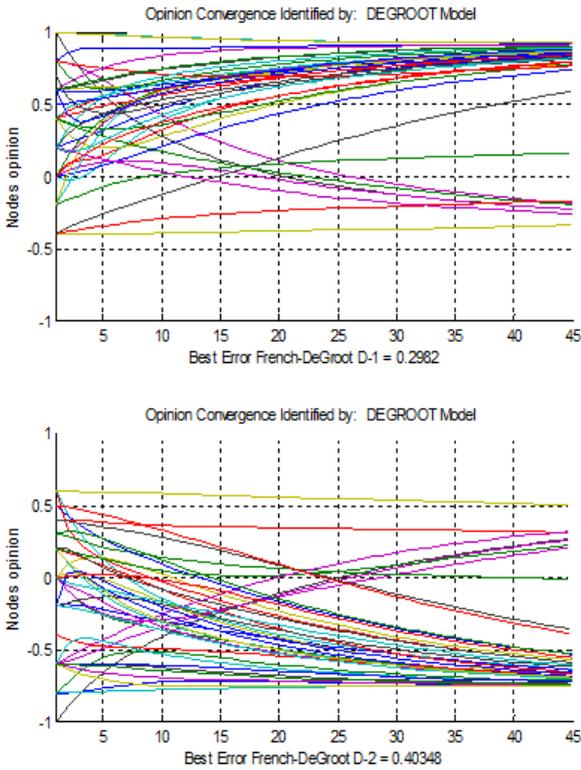


Fig. 3: Simulated opinions by *French-DeGroot* in experiment IV. (Top first, down second dimension).

### B. Fit Data to Friedkin-Johnsen Model

The next model which is also considered for validation of the method is the *Friedkin-Johnson* model explained by (13), and (14). Using this mathematical workaround, the FJ model leads to the following result:

$$Error_{FJ} = 0.3407 \quad (49)$$

which is lower than the error of the *DeGroot* model, though it is not better than *Altafini's*. Fig. 4 show the consensus process in this model.

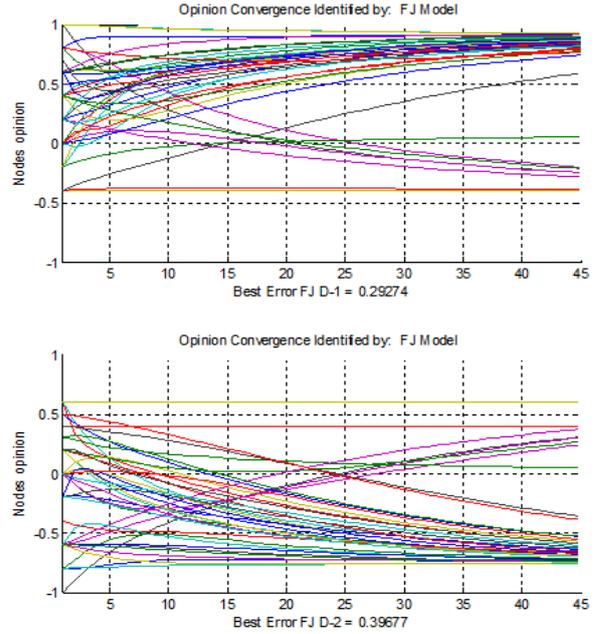


Fig. 4: Simulated opinions by FJ in Experiment IV. (Top first, down second dimension)

As seen in Fig. 4, the FJ model, similarly to the *DeGroot* model, fails to follow the experimental data that deviates from the mean value of the opinions. As mentioned earlier, this is a fundamental drawback of this family of models.

### C. Fit data to Altafini Model

Another model that was selected for identification is the *Altafini* Model, which was explained in (27). By choosing this model, the results got better and the prediction results were much closer to reality and the error was equal to:

$$Error_{Alta} = 0.2622 \quad (50)$$

Using the identification result, the opinions can be reconstructed and predicted along each dimension as shown in Fig. 5. It can be seen here, the *Altafini* model has been able to give a better approximation of the experimental data. Therefore intuitively, we can say that the *Altafini* model performs better than the *DeGroot* model. Table 2 shows the results from the experiment IV.

As seen in the table, the *Altafini* model provide the best results. The *French-DeGroot* model and the

*Friedkin-Johnsen* model have approximately same results.

Table 2: Identification errors in experiment IV

Models	Error in Dim_1	Error in Dim_2	Error in 2_Dims
<b>DeGroot</b>	0.2982	0.4035	0.3468
<b>FJ</b>	0.2927	0.3968	0.3407
<b>Altafini</b>	0.2018	0.3329	0.2622

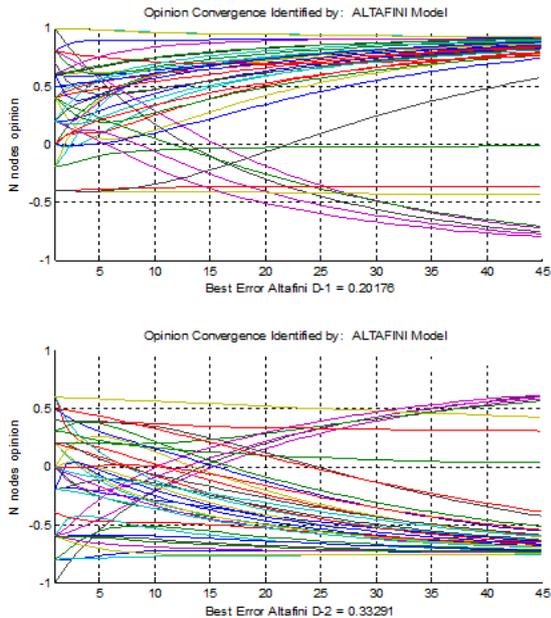


Fig. 5: Simulated opinions by *Altafini* in experiment IV. (Top first, down second dimension)

Table 3, shows 2-dimensional results in three other experiments. Although these experiments have not been conveyed in this paper, the results of all them emphasize the application of proposed method. These results show that the identification errors in DeGroot, FJ and Altafini models decrease respectively, as same as in the 4th experiment. It should be noted that all the experimental data in addition to supplementary materials accessible at the website of the *SyLiCon* lab [44].

Table 3: Identification errors in three other experiments

Model	Error in Exp. I	Error in Exp. II	Error in Exp. III
<b>DeGroot</b>	0.3129	0.3408	0.2786
<b>FJ</b>	0.3087	0.3845	0.2862
<b>Altafini</b>	0.2145	0.2088	0.1232

## Conclusion

In this paper, a method is proposed to identify the parameters of regular social networks models. The method is based on convex optimization. It should be

also noted that the provided method can be used also for models with two or multidimensional opinions. The three well-known models, i.e., the *French-DeGroot* model, the *Altafini* model, and the *Friedkin-Johnsen* model are considered to verify the method by real data. The analytical contribution of this paper is the analysis provided to show that the presented identification method can be used for these three models. Then, by using this method, the parameters (i.e., the influence matrix) for these models are calculated and simulations are conducted. By conducting four real experiments, we used mathematical criteria to evaluate the predictions of the three models and compared them against the real-world data. We showed that the proposed method can be used to identify the influence matrix within the real social networks and we extracted the error between the simulated results and the real experiments' ones. We provided a short yet comparative discussion on the results of the three experiments. However, there is still room for further evaluations and comparisons in future research. In a word, the main contribution of this paper is: 1) to propose a previously introduced method for identification of model parameters at the field of social networks, 2) to show analytically that the mentioned method is applicable for 3 well-known models of opinion dynamics, and 3) to conduct 4 real experiments to show that the method is applicable in practice.

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## Conflict of Interest

The S. M. Nematollahzadeh, S. Ozgoli, and M. Sayad Haghighi, declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

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