



Research Paper

A PFC-based Hybrid Approach for Control of Industrial Heating Furnace

H. Nasiri Soloklo, N. Bigdeli*

Department of Electrical Engineering, Imam Khomeini International University, Qazvin, Iran.

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*Corresponding Author's Email
Address:

n.bigdeli@eng.ikiu.ac.ir

Abstract

Background and Objectives: In this paper, a predictive functional control based on Laguerre functions is designed for control of an industrial heating furnace. The fractional order model (FOM) of the heating furnace is assumed as the plant model.

Methods: For designing the predictive functional controller (PFC), a reduced integer order approximation of the fractional order heating furnace model is derived. The order of the reduced integer model is determined based on Hankel singular values of the original system. Coefficients of the reduced integer model are assumed to be unknown. Unknown parameters are then obtained by minimizing a many-objective fitness function including weighted summation of differences of step responses, steady state errors, maximum overshoots as well as magnitude of frequency responses of the original and reduced systems. Routh-Hurwitz criteria are used as stability criteria and added to optimization problem as its constraints. The optimization tool is Genetic algorithm.

Results: Advantages of the proposed method are preserving stability and focusing on various important features of both time and frequency responses of system. In addition, it uses a direct order reduction method without the need to intermediated approximations such as Oustaloup approximation.

Conclusion: Laguerre-based PFC controller has been evaluated via two scenarios and the obtained results represent the satisfactory performance of the proposed controller.

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Introduction

Pressure and Temperature controls have a major impact on industrial processes efficiency and quality, and therefore accurate and precise control of these quantities is very important, in such systems [1]. Heating furnace is a mechanical appliance that is used for industry as well as household purposes. Heating furnace is used at the appropriate temperature to warm different substances [2]. In the literature, many strategies have been proposed for control of the industrial processes such as heating furnace [3]-[4].

Heating furnace model can be an integer or fractional

order and therefore, the control strategies are divided to integer or fractional approaches, as well. Also, heating furnace control strategies can be classified to classic and advance control methods. The proportional-integral-derivative (PID) controller is one of the most usual classic controllers, applied in control of heating furnace [5]. The PID controller is widely used and has extended drastically which has improved the characteristics of system output. Many methods have been proposed for PID controller design, such as Ziegler-Nichols approaches, evolutionary algorithms, Fuzzy methods and so on [6]. First time, Igor Podlubny has proposed

fractional order PID (FOPID) by combination of fractional calculus and PID controller [7] and then, many researchers have used the FOPID for many industrial applications [2], [8]-[9]. The FOPID controller does not have any steady state error, while has satisfactory gain and phase cross over frequency. In addition, the FOPID controller is robust to disturbances and noise [10]. Furthermore, when the system is multivariate, nonlinear, or the set point is changing, the PID and FOPID controllers do not work well. Thus, modern strategies have been developed to control the industrial systems such as heating furnace [11]-[12]. One of the powerful methods provided by researchers to control industrial processes is model predictive control (MPC). MPC controllers are highly popular due to their ability to deal with multivariable, nonlinear, discrete time, time invariant and non-minimum phase systems [13]-[16]. In recent years, model predictive controllers are used for control of industrial process such as heating furnace [17]-[21]. However, computational burden of MPC have often been pointed to as the most serious constraint facing its practical implementation. Predictive functional control (PFC) has been pioneered for fast processing systems in the last decade as the most common industrial MPC tool [22]-[23]. Compared to the various MPC configurations, the benefit of PFC is its ability in turning a QP problem into a square equation system that allows simpler implementation in practice [24]. Two essential features of PFC which make it distinguishable from other MPC variants are making the control signal by using the linear combination of certain predetermined basis functions and evaluating the cost function at coincidence points, along the horizon. A constant future input assumption has been used by the nominal PFC for decreasing the complexity of formulation and computational burden [25]. In some situations, these assumptions can be useful, where short prediction of problem is necessary to capture the core dynamics of the system [26]. Nevertheless, actual closed-loop performance can be degenerated when prediction horizon is long or severe disturbances exist [27]. The consistency of the prediction will be improved by using the Laguerre functions [27]. In this matter, instead of constant future inputs in nominal PFC, the future input is predicted by considering the desired pole which exponentially convergences to steady state. The constrained solution will be more effective and accurate when the appropriate decisions have been made. In Laguerre-based PFC (LPFC), Laguerre functions are generally applied as a black box model to describe the plant [27]. In this structure, the system order reduces which significantly decreases the computational volume of the controller.

As another fact, PFC is dependent on the discrete time

state space model of system., while, representing fractional order systems as discrete time state space models is not a straight forward task [28]. Therefore, it is preferable to convert the heating furnace FOM to the integer order model. Several methods have been proposed for integer order approximation of fractional order systems and controllers such as Oustaloup approximation algorithm [29], Matsuda's approximation algorithm [30], Carlson method [31] and et al. Unfortunately, obtained integer order approximations by these methods are of high order, although are accurate enough to approximate the fractional order models. Design and implementation of model predictive control of these high order models are very difficult, time consuming, and sometimes impossible. Therefore, via employing model order reduction (MOR) techniques (to reduce the order of integer approximations), the volume of calculations is drastically decreased. Davison [32] first presented model order reduction in 1966. Since 1966, many researchers have extended MOR techniques. Balanced truncation method was proposed by Moore [33], which is based on analysis of controllability and observability Gramians via solving Lyapunov equations. This method is more popular because of its ease of use, preserving stability, good approximation capability and having computable explicit error bounds. However, the balanced truncation method suffers from steady state error. The other difficulty of balanced truncation method is the need for solving the Lyapunov equations. In 1984, Glover introduced an optimal Hankel norm approximation method, and its advantages and disadvantages were almost similar to balanced truncation method [34]. MOR based on Krylov subspace methods is another important method in order reduction. The main advantage of using a Krylov model order reduction is significantly shortening of the computational time while an induced error is almost negligible or very low [35]. However, these methods do not necessarily preserve stability of the model. Within the field of computational fluid dynamics (CFD), proper orthogonal decomposition, also known as Karhunen-Loeve decomposition is created [35]. The POD method could be applied to nonlinear partial differential equations. Evolutionary algorithms such as Genetic algorithm and Harmony search algorithm have been used for improving the optimality of model order reduction methods [36]-[37]. Nowadays, model order reduction of nonlinear system has been considered by researchers [38]. Nonlinear model order reduction is divided to some main method such as trajectory piecewise linear model (TPWL) [39], model reduction of bilinear [40], quadratic bilinear [41] systems and proper orthogonal decomposition. In summary, up to the knowledge of the authors, in current literature, in

model-based control of industrial processes such as heating furnace, high integer order approximation of its FOM is obtained by conventional approximation methods such as Oustaloup method [42]-[46]. This high order model is next reduced by different model order reduction approaches, and then the controller is designed. Thus, designing of MPC controller for heating furnace system has three general steps as integer approximation, order reduction and controller design [47]. To increase the efficiency and quickness of MPC design, in this paper, reduced integer order approximation of heating furnace model is determined directly, without applying conventional integer approximation methods such as Oustaloup method. At first, a desired fixed structure for reduced integer approximation of heating furnace model is considered, which their numerator and denominator are unknown and should be determined. The order of the fixed structure model is calculated based on concept of Hankel singular values. In the next stage, step response and magnitude of frequency response of heating furnace is calculated. By minimizing a fitness function using Genetic algorithm (GA), the unknown parameters of reduced integer order model of heating furnace are determined. The proposed many-objective fitness function is sum of integral square error (ISE) of step responses, difference of magnitude of frequency responses and difference of maximum overshoots and steady state errors of heating furnace FOM and its reduced order model (ROM). Because heating furnace model is stable, thus reduced integer order approximation of these fractional order systems must be stable, too. In order for satisfying the stability of integer order approximation, Routh-Hurwitz criterion is used. These stability conditions are added to optimization problem as constraints and therefore the problem of optimization is converted to a constrained optimization problem. Consequently, reduced integer approximation of heating furnace model is obtained by minimizing the proposed many-objective fitness function via Genetic algorithm. Finally, obtained reduced integer approximation of the heating furnace by the proposed method, is used as a model for predictive controller. Predictive functional controller based on Laguerre function is used to control a heating furnace which leads to reduction in volume of calculations and consequently fast tracking capability. To demonstrate the ability and accuracy of proposed method, PFC is designed in presence of disturbance. Based on the above discussions, the main contributions of the paper can be summarized as follows

- Unlike what is common in the related literature such as [43]-[45], a new direct optimal integer order approximation method is proposed for fractional order

systems without applying the conventional integer approximation methods such as Oustaloup method.

- In spite of some related researches such as [47], in which the order of the reduced integer approximation is selected via trial and error, in this paper, a systematic approach based on Hankel singular values is followed for order selection of the reduced model.

- Unlike some related articles [42],[46] that use classical model order reduction methods such as BT to reduce the order of the approximated model of FO systems; in this paper, the ROM is obtained via minimizing a many-objective fitness function including both time and frequency domain properties of the system via Genetic algorithm.

- Stability criteria of the ROM have been embedded to optimization procedure via some optimizing constraints coming from Routh-Hurwitz stability analysis method.

- Unlike the most related work in [47], which uses PFC method for controller design, in this paper, Laguerre-based PFC method is applied for controller design which increases the speed of convergence and reduces the computational burden.

The paper is organized as follows: preliminary mathematical formulations include fractional order calculus and genetic algorithm are explained in Section II. Design of PFC controller is illustrated in Section III. The heating furnace and its modeling as a case study is investigated in Section IV. The effectiveness of the proposed method is shown through results and discussion in Section V. In Section VI, conclusion of article is presented.

Preliminary Mathematical Formulations

In this section, a brief overview of the mathematical tools used in this article are shortly overviewed. At first, fractional calculus is introduced. Then, the Genetic algorithm is described as a powerful tool for solving the optimization problems.

A. Fractional Order Calculus

Today, the fractional calculus is used in many fields of engineering. Among various definition of fractional order derivative provided by mathematicians, three definitions are most commonly used in engineering as Grünwald–Letnikov, Riemann–Liouville and Caputo definitions [48]-[49]. Caputo fractional derivatives are more popular among above-mentioned definitions than the others. The Caputo definition is explained by:

$$D_t^q = \begin{cases} \frac{1}{\Gamma(n-q)} \int_0^t (t-\tau)^{n-q-1} \frac{d^n f(\tau)}{d\tau^n} d\tau & n-1 < q < n \\ \frac{d^n u(x,t)}{dt^n} & q = n \in N \end{cases} \quad (1)$$

The Caputo fractional derivative's Laplace transformation is then defined by

$$L\{D_t^q f(t)\} = s^q F(s) - \sum_{k=0}^{n-1} s^{q-1-k} f^{(k)}(0) \quad (2)$$

A standard FOM can be provided by the generalized fractional differential equation as:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\sum_{i=1}^p b_i s^{\beta_i}}{1 + \sum_{i=1}^n a_i s^{\alpha_i}} \quad (3)$$

where $y(t)$ and $u(t)$ are output and input functions, respectively. Also a_i and b_i are real coefficients.

B. Genetic Algorithm

One of the most popular and powerful global search algorithms is Genetic algorithm which is based on the genetic and natural selection mechanism. The Genetic algorithm includes concepts such as population, selection, crossover and mutation. At first, GA start with initial population as an initial answer. Then, for each individual of population, fitness is evaluated. In the next step, GA uses the selection, crossover and mutation to generate next generation and new population is generated again. This methodology is repeated until the criteria for the termination have been met. A typical ordinary GA is explained in detail in [50]-[51].

Design of Proposed Fractional Order Laguerre-Based PFC

In this section, a detailed overview of the proposed method would be presented. In order to better explanation, the general procedure of the proposed method is shown in the flowchart of Fig. 1.

According to the flowchart of Fig.1, step and frequency responses of fractional order system are calculated. Then, a ROM as a transfer function with unknown parameters is considered. Order of this reduced model is obtained by Hankel singular values of Oustaloup approximation of fractional order system. Stability criteria are added to optimization problem as constraints.

By minimizing the many-objective fitness function, integer approximation of fractional order system is determined. This minimization is done by GA.

At last, Laguerre-based PFC is applied to control of the proposed ROM. In subsection A, a new approach for integer order approximation of fractional order systems is suggested.

In subsection B, Laguerre-based predictive functional controller is formulated for reduced integer order approximation model in order to good reference tracking and disturbance rejection goals.

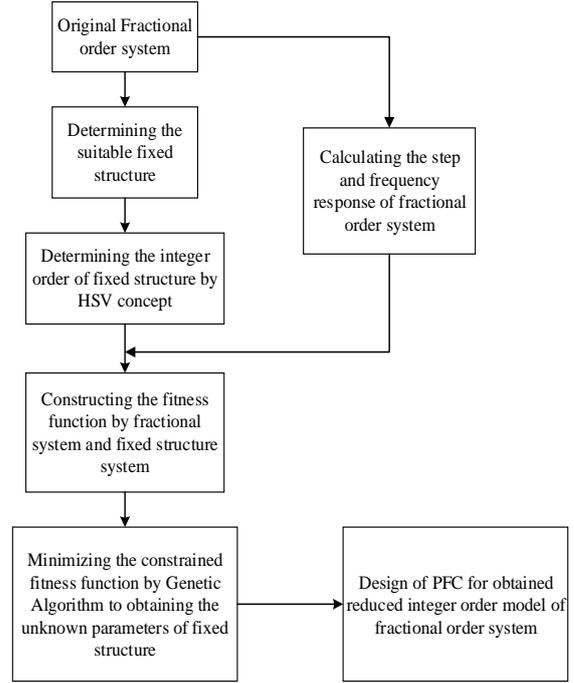


Fig. 1: The flowchart of the proposed method.

A. Reduced Integer Order Approximation

A general form of a stable linear time invariant fractional order system is presented by transfer function model as follows

$$G_f(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_0 s^{\alpha_0}} \quad (4)$$

where, a_i and b_i are respectively coefficients of numerator and denominator, which are known parameters. Also, α_i and β_i are arbitrary positive real numbers.

Let G_r denote the reduced integer order approximation with a fixed structure as follows

$$G_r(s) = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \dots + c_r}{s^r + d_1 s^{r-1} + \dots + d_r} \quad (5)$$

where, coefficients of numerator and denominator and the order are unknown. Also, r is an integer and positive variable. These unknown parameters of the reduced integer order approximation should be determined such that the important characteristic of the fractional order system and the reduced integer order approximation are almost identical.

To determine the order of the fixed structure (5), the concept of Hankel singular value (HSV) is used [52]. Hence, Hankel singular values of Oustaloup approximation are calculated. The order of the fixed structure model is equals to number of larger Hankel singular values. After determining the order of the

reduced approximation, unknown coefficients of (5) are determined by minimizing the suitable fitness function. Genetic algorithm is used as a minimizer of many-objective fitness function. The desired many-objective fitness function is defined as follows

$$J' = w_1 \int_0^{t_f} (y_f - y_r)^2 dt + w_2 |mag_f - mag_r| + w_3 |OS_f - OS_r| + w_4 |e_{ss_f} - e_{ss_r}| \quad (6)$$

where, y denotes the step response, mag is the magnitude of the frequency response, OS is overshoot and e_{ss} is the steady state of error. Also, f and r indices represent the fractional order system and reduced integer order approximation, respectively. In addition, t_f is final time, and w_1 , w_2 , w_3 and w_4 are weighted coefficients.

Reduced integer order approximation has to be stable, because fractional order system is assumed stable. To achieving this goal, Routh-Hurwitz criterion is used as follows.

The denominator of ROM approximation which is presented by (5) can be shown as below [37]:

$$s^r + h_1 s^{r-1} + (h_1 + h_2 + \dots + h_r) s^{r-2} + h_2 (h_3 + h_4 + \dots + h_r) s^{r-3} + [h_2 (h_4 + h_5 + \dots + h_r) + h_3 (h_5 + h_6 + \dots + h_r) + \dots + h_{r-2} h_r] s^{r-4} + \dots + h_{1+k} h_{3+k} \dots h_{r-2} h_r$$

which is constructed by taking the coefficients of the first two rows of the Routh-Hurwitz array with the elements of its first column given by

$$1, h_1, h_2, h_1 h_3, h_2 h_4, h_1 h_3 h_5, \dots, h_{1+k} h_{3+k} \dots h_{r-2} h_r \quad (8)$$

where, k is equal to 1 for even r and k is equal to 0 for odd r .

The relationships described in (9) are obtained by comparing the entries of the first row with $1, d_2, d_4, \dots$ and those of the second row with d_1, d_3, d_5, \dots :

$$\begin{cases} d_1 = h_1 \\ d_2 = (h_2 + h_3 + \dots + h_r) \\ d_3 = h_1 (h_2 + h_3 + \dots + h_r) \\ \vdots \\ d_r = (h_{1+k} h_{3+k} \dots h_{r-2} h_r) \end{cases} \quad (9)$$

The equation (7) is obtained by substituting the above equations in the denominator of the ROM.

Thus, the necessary and sufficient condition to be strictly on the left-half plane for all the poles of the reduced system is:

$$h_i > 0, \quad i = 1, \dots, r \quad (10)$$

and subsequently

$$d_i > 0, \quad i = 1, \dots, r \quad (11)$$

Therefore, integer ROM approximation is stable if and only if unknown parameters of (5) is determined by minimizing (6) subject to (11). Hence, optimization problem (6) is converted to constrained optimization problem and consequently fitness function as follow:

$$J : \begin{cases} J' \\ s.t. d_j > 0, j = 1, 2, \dots, r \end{cases} \quad (12)$$

The reduced integer order approximation which obtained by minimizing the J have an almost important characteristics of the fractional order system.

B. Design of Predictive Functional Controller Based on Laguerre Functions

In this subsection, predictive functional controller based on Laguerre function is designed for reduced integer order approximation which achieved in previous subsection.

The control input is calculated in the PFC algorithm as a linear combination of orthogonal basis functions such as Laguerre functions, Legendre functions, etc., and the output is a weighted combination of these basis functions. A series of orthonormal functions such as the Laguerre functions define the trajectory of the difference of the future control signal within a moving horizon frame [53]. That is, the control input can be presented as follows:

$$\Delta u(k) \approx \sum_{m=1}^N c_m l_m(k) \quad (13)$$

where, l_m and c_m are Laguerre functions terms and corresponding coefficients of Laguerre functions, respectively. Also, N is the number of Laguerre functions terms which used in series. Obviously, by increasing N , the rate of convergence of expansion is increased but volume of calculation is growth.

1) Laguerre functions

The discrete time transfer function of Laguerre functions in Z-domain is defined as follows [54]

$$\phi_m(z) = \frac{\sqrt{(1-\lambda)^2}}{z-\lambda} \left(\frac{1-\lambda z}{z-\lambda} \right)^{m-1} \quad (14)$$

where, λ is called scaling factor which $0 \leq \lambda < 1$. The Laguerre functions are orthogonal function and therefore following relationship is satisfied:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_m(\exp(j\omega)) \phi_n(\exp(j\omega))^* d\omega = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \quad (15)$$

where, (*) denotes complex conjugate operator.

According to (14), it is simply shown that the set of discrete Laguerre function established the following equation:

$$L(k+1) = \Omega L(k) \quad (16)$$

where

$$\beta = 1 - \lambda^2, \quad L(k) = [l_1(k) \quad l_2(k) \quad \dots \quad l_N(k)]$$

and

$$\Omega = \begin{bmatrix} \lambda & 0 & 0 & \dots & 0 \\ \beta & \lambda & 0 & \dots & 0 \\ -\lambda\beta & \beta & \lambda & \dots & 0 \\ \lambda^2\beta & -\lambda\beta & \beta & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{N-2} \lambda^{N-2} \beta & (-1)^{N-3} \lambda^{N-3} \beta & \dots & \beta & \lambda \end{bmatrix}$$

with initial condition of:

$$L(0)^T = \sqrt{(1-\lambda^2)} [1 \quad -\lambda \quad \lambda^2 \quad -\lambda^3 \quad \dots \quad (-1)^{N-1} \lambda^{N-1}]$$

2) Predictive functional controller

Consider a system with p input - q output which described by a discrete state space model as below

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) + \xi(k) \\ y(k) &= C_m x_m(k) + \eta(k) \end{aligned} \quad (17)$$

where, x_m is state variable and is assumed to have n_1 dimension. Let $\Delta x_m(k) = x_m(k) - x_m(k-1)$, $\bar{y}(k) = C_m x_m(k)$. Also, $x(k) = [\Delta x_m(k) \quad \bar{y}(k)]^T$ is chosen as a new state variable. We have,

$$\begin{aligned} x(k+1) &= \begin{bmatrix} A_m & 0_{n_1 \times q} \\ C_m A_m & I_{q \times q} \end{bmatrix} x(k) + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) \\ &\quad + \begin{bmatrix} \xi(k) - \xi(k-1) \\ 0_{q \times 1} \end{bmatrix} \\ y(k) &= [0_{q \times n_1} \quad I_{q \times q}] x(k) + \eta(k) \end{aligned} \quad (18)$$

Or equivalently:

$$\begin{aligned} x(k+1) &= Ax(k) + B \Delta u(k) + \bar{\xi}(k) \\ y(k) &= Cx(k) + \eta(k) \end{aligned} \quad (19)$$

where

$$A = \begin{bmatrix} A_m & 0_{n_1 \times q} \\ C_m A_m & I_{q \times q} \end{bmatrix}, \quad B = \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \quad \text{and}$$

$$C = [0_{q \times n_1} \quad I_{q \times q}]$$

Based on (13), the future control trajectory at future time m can be presented by

$$\Delta u(k_i + m) = \sum_{i=1}^N l_i(m) \epsilon_i = L(m)^T \eta \quad (20)$$

where, $\eta^T = [c_1 \quad c_2 \quad \dots \quad c_N]$. The prediction of the future state variable becomes:

$$x(k_i + m | k_i) = A^m x(k_i) + \sum_{i=0}^{m-1} A^{m-i-1} B L(i)^T \eta \quad (21)$$

By applying a set of Laguerre functions, the convolution sum in (21) can be solved. In [55], the following relation which is a closed-form solution for convolution sum have been proved.

$$A S_c(m) - S_c(m) \Omega^T = A^m B L(0)^T - B L(m)^T \quad (22)$$

where, $S_c(m) = \sum_{i=0}^{m-1} A^{m-i-1} B L(i)^T$.

The solution of (22) can be found by column by column iteratively, because Ω is the lower triangular matrix. Let $V(k)$ and $W(k)$ denotes the k th column of $A^m B L(0)^T - B L(m)^T$ and k th column of $S_c(m)$ respectively. Then, we let $W(1) = (A - \lambda I)^{-1} V(1)$ and $\gamma(1) = 0$ for $k = 1, 2, \dots, N-1$.

$$\gamma(k+1) = -\lambda \gamma(k) + (1 - \lambda^2) W(k) \quad (23)$$

$$W(k+1) = (A - \lambda I)^{-1} (V(k+1) + \gamma(k+1)) \quad (24)$$

It is obvious that if eigenvalues of A is equal to λ , the solution does not exist.

The prediction of output can be obtained by using the (21) as follows

$$y(k_i + m | k_i) = C A^m x(k_i) + \varphi(m)^T \eta \quad (25)$$

where, $\varphi(m)^T = \sum_{j=1}^{m-1} C A^{m-j-1} B L(j)^T$.

Based on MPC theorem, the control law that leads to the predicted output tracks the future trajectory of the reference input, $r(k_i + m)$, is specified by minimizing the following fitness function

$$J = \sum_{m=1}^{N_p} \left([r(k_i + m) - y(k_i + m | k_i)]^T \times Q \right. \\ \left. \times [r(k_i + m) - y(k_i + m | k_i)] \right) \quad (26)$$

$$+ \sum_{m=0}^{N_p-1} \Delta u(k_i + m)^T R \Delta u(k_i + m)$$

where, N_p is the prediction horizon that

$0 \leq m \leq N_p$. Also, R and Q are symmetric positive definite matrix. Based on orthogonality of Laguerre functions in sufficiently large prediction horizon and diagonal matrix of R , the cost function of (26) is converted to

$$J = \sum_{m=1}^{N_p} \left(\begin{array}{c} [r(k_i+m) - y(k_i+m|k_i)]^T \times Q \\ \times [r(k_i+m) - y(k_i+m|k_i)] \end{array} \right) + \eta^T R \eta \quad (27)$$

If this fitness function is considered without constraints, the optimization problem has a unique solution. The optimal solution of (27) is achieved by quadratic programming method as follows

$$\hat{\eta} = \left(\sum_{m=1}^{N_p} \varphi(m) Q \varphi(m)^T + R \right)^{-1} \times \left(\sum_{m=1}^{N_p} \varphi(m) Q (r(k_i+m) - CA^m x(k_i)) \right) \quad (28)$$

Based on the optimal parameter vector η which achieved by minimization of (27), the receding horizon control law is obtained as follows

$$\Delta u(k_i) = L(0)^T \hat{\eta} \quad (29)$$

If the MPC control problem has constraints, the cost function of (27) is converted to constrained optimization problem and therefore the problem does not have unique solution, but has a suboptimal solution.

In general, procedure of the proposed method can be summarized as follow as:

Step 1: Consider a desired fixed structure for reduced integer order model approximation of the fractional order system which order and their coefficients of numerator and denominator are unknown.

Step 2: Determine the order of the reduced integer model approximation by concept of Hankel singular values.

Step 3: Minimize the constrained fitness function (15) by Genetic algorithm to find the best parameters for the reduced integer order model approximation.

Step 4: Design of the predictive functional controller based on Laguerre functions for the reduced integer order model approximation which obtained in step 3 by minimizing the unconstrained or constrained (28) by quadratic programming method.

Case Study

The proposed method is tested on an industrial heating furnace. A heating furnace is simply a thermal enclosure used for multiple purposes to heat materials below or above their melting points. Heating furnaces

are used in many industries, including iron and steel manufacturing industries, non-ferrous metal extraction industries, ceramic processing industries, glass manufacturing industries, etc.

Heating furnace is categorized by different ways. One of these classifications is batch heating furnace and continuous heating furnace. Another classification is based on their application; for example: annealing furnace, quenching furnace, forging furnace, blast furnace and reheating furnace. Heating furnace has been included in many physical and chemical processes. Some of these processes are coking for converting the coal into coke, roasting for converting the supplied to oxide, matte smelting for separating the gangue from liquid metal supplied, reduction smelting for produce hot metal, Zinc and lead, refining for production of steel and electrolysis and reduction of metal oxide to remove oxygen [56].

Various models have been proposed for the heating furnace system. These models can be integer order or fractional order. The approximate heating furnace modeling includes the fuel mass gas flow rate and also the pressure inside the furnace as input and output values, respectively. The approximate dynamic heating furnace modeling includes the mass, energy and the momentum balances. It also includes the heat transfer from the hot flue gas to water, flue gas flow from the boiler model and steam model [10]. In [7], three model is proposed for heating furnace model. First model which is integer order is obtained by second-order differential equation. Second and third models are FOMs that obtained by three term fractional order differential equations. It was shown that FOM of heating furnace is more accurate than integer order model. Therefore, fractional order differential equation of heating furnace is considered as

$$a_2 y^{(\alpha_1)}(t) + a_1 y^{(\alpha_2)}(t) + a_0 y(t) = u(t) \quad (30)$$

where, $\alpha_1 = 1.31$, $\alpha_2 = 0.97$, $a_0 = 1.69$, $a_1 = 6009.52$ and $a_2 = -14994.3$. Also, y is pressure in Pascal unit as output of the system. Therefore, transfer function form of heating furnace model can be obtained as follows [4], [7], [47]:

$$G(s) = \frac{1}{14994.3s^{1.31} + 6009.52s^{0.97} + 1.69} \quad (31)$$

Igor Podlubny showed that this model is most accurate model that proposed for heating furnace model [7].

Results and Discussion

In this section, a heating furnace plant which illustrated in section 4 as an industrial process is considered. Control of heating furnace pressure is

important at industrial applications. Therefore, the proposed method has been used to control the pressure of heating furnace.

As mentioned in previous sections, the FOM that presented as a heating furnace model cannot be used directly to integer order model predictive controller. Also, conventional integer order approximation method such as Oustaloup approach leads to high integer order approximation. Therefore, the reduced integer order model approximation of heating furnace plant is achieved using the proposed method.

At first, the order of the reduced integer model approximation is determined. For determining the order of the reduced model approximation of heating furnace, Hankel singular values (HSV) of Oustaloup approximation of (31) are calculated. Fig. 2 shows the HSV chart of integer approximation model.

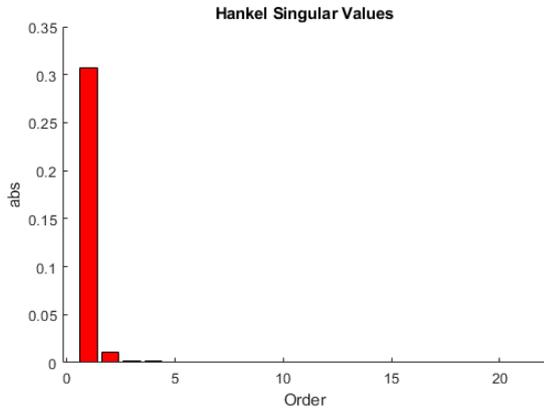


Fig. 2: HSV of Oustaloup integer approximation.

As shown in Fig. 2, the first two modes are relatively large and hence, have more energy and are more important. Therefore, the order of the reduced model approximation can be selected as 2. Then, a fixed structure for ROM approximation is considered as follows based on obtained order which determined by the concept of Hankel singular values:

$$G_r = \frac{c_1s + c_2}{s^2 + d_1s + d_2} \quad (32)$$

where c_i and d_i are unknown parameters.

The coefficients of numerator and denominator of ROM of (32) are obtained by minimizing the constrained fitness function of (12). The Genetic algorithm is applied as the optimization tools. The genetic algorithm parameters which have been used for tuning GA have been presented in Table 1. The GA parameters have been selected based on the requested approaches in [57]. Because number of unknown parameters which should be determined are 4, population size is selected 20. Mutation rate has been chosen 0.03 via trial and error to yield proper convergence rate. The mutation rate has been chosen 0.03 via trial and error to yield

proper convergence rate. Also, crossover ration is depended on the problem and has been selected as 0.9, in this paper.

Table 1: Genetic algorithm parameters for the test System 1

Parameter	Value
Population size	20
Selection function	Stochastic uniform
Crossover ration	0.9
Migration	0.2
Mutation rate	0.03

The ROM approximation of heating furnace plant is described as follows:

$$G_r(s) = \frac{0.0001117s + 2.691 \times 10^{-8}}{s^2 + 0.000421s + 4.566 \times 10^{-8}} \quad (33)$$

To verify the efficiency and ability of the proposed method, the step response of the heating furnace FOM, Oustaloup model and the proposed reduced integer order model approximation are compared in Fig. 3. Also, some of most important characteristics of the step response such as steady state, overshoot, integral square error (ISE) as an error measurement index, are considered. The compared results are shown in Table 2. Based on Fig. 3 and Table 2, it can be seen that the achieved ROM approximations are good low order approximations that preserve the specifications of the original fractional order system, properly. The proposed model is so similar to the fractional order system and have relatively the same characteristics such as overshoot, settling time. Also, the ISE index of the proposed method is smaller than the obtained model by Oustaloup method.

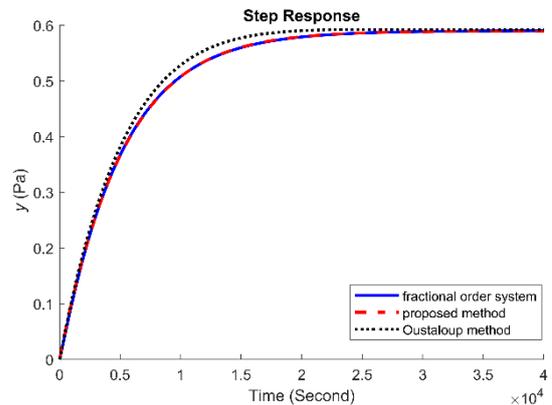


Fig. 3: The step responses of the fractional order system of the heating furnace model, the proposed reduced integer order approximation model, and the obtained integer order model by Oustaloup approximation.

The plot of the absolute error is presented to show the accuracy of the proposed method, in Fig. 4. It can be declared that the absolute error of the proposed method

is relatively less than Oustaloup approximation method.

Once, the fractional order system of heating furnace is approximated by the reduced integer order model, predictive functional controller based on Laguerre functions can be designed. Model predictive parameters such as sampling time, control horizon, prediction horizon, and weighted functions were selected as empirically 5, 10, 10, 15000 and 0.3, respectively. Also, scaling factor and number of Laguerre functions were selected 0.45 and 5, respectively. It is expected that using the proposed method, the control of heating furnace pressure is very well done. In the following, the performance of the proposed control system is investigated for two scenarios.

A. First Scenario: Design of PFC for Tracking Purpose

In this subsection, Laguerre-based PFC is designed for the reduced integer order model of heating furnace which obtained in previous section. The pressure of heating furnace is change as %100, %200, %-100 and %0, accordingly. The purpose of design is tracking the set-point variations.

Table 2: Comparison of methods for the test system 1

	FO system	Oustaloup method	Proposed method
Order	1.31	21	2
Steady state value	0.5903	0.5919	0.5894
Overshoot	0	0.1419	0
Rise time (sec)	1.107e3	0.97e3	1.103e3
Settling time (sec)	1.97e3	1.55e3	1.915e3
ISE	-	5.7843	0.0322

In Fig. 5, the obtained results by the proposed method have been shown. Also, to show the ability and efficiency of the proposed method, the results are compared with the PFC-based on Oustaloup model and Laguerre-based PFC (LPFC) based on the proposed model of [47]. In Table 3, the characteristics of the obtained results such as overshoot, rise time, settling time, root mean square (RMS) of tracking error and elapsed time in designing the PFC are compared with PFC on Oustaloup model and LPFC on model of [47].

The RMSE index has been defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_{c_i} - y_{r_i})^2}{N}} \quad (34)$$

in which, y_c , y_r and N are step responses of controller and reference and the number of vector elements of responses, respectively.

It can be observed that by using the proposed method, the better characteristics such as lower

overshoot, rise time and settling time can be reached compared with PFC-based on Oustaloup method. Besides, as seen, although the LPFC based on the proposed model of [47] performs faster than that of the proposed method, but overshoot appears in the closed loop response, when the set-point changes. Also, RMS and elapsing simulation time of the proposed method is smaller than the other methods. It is obvious that the volume of computations in Laguerre-based PFC design is decreased in the proposed method. Furthermore, it is observed from Fig. 5 that when the reference input changes, the proposed controller follows the set-point smoother, without noticeable overshoot.

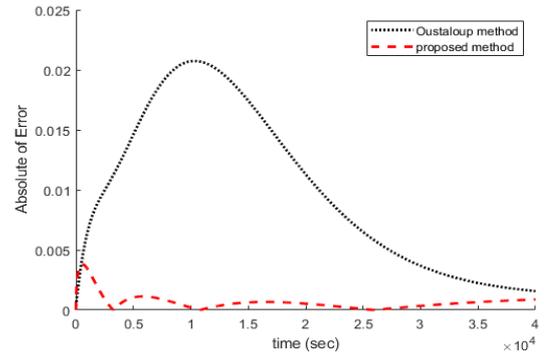


Fig. 4: The plot of absolute of error versus time for the test system (31).

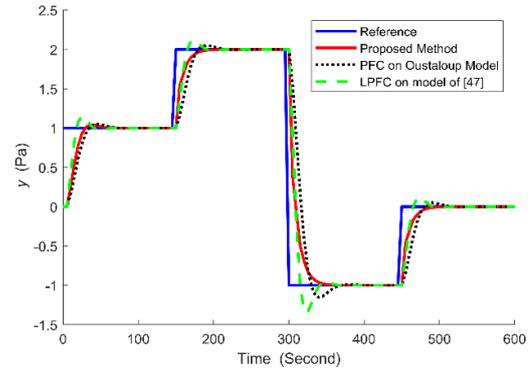


Fig. 5: Comparison of the proposed Laguerre-based PFC with PFC-based on Oustaloup model and that of [47].

Table 3: Performance comparison of the proposed Laguerre-based PFC with PFC-based on Oustaloup model and that of [47]

	Laguerre based PFC on proposed model	PFC on Oustaloup model	LPFC on model by [47]
Maximum Overshoot (%)	0.016	2.5851	5.7297
Rise time (sec)	3.41	4.1387	2.05
Settling time (sec)	11	13.1263	12
RMSE	0.3824	0.4877	0.3965
Elapsed time (sec)	0.034	0.052	0.0419

Finally, the control effort response which obtained for

PFC controller is plotted in Fig. 6, which seems more practical as it has lower maximum effort as well as smoother variations compared with PFC based on Oustaloup method as well as the method of [47].

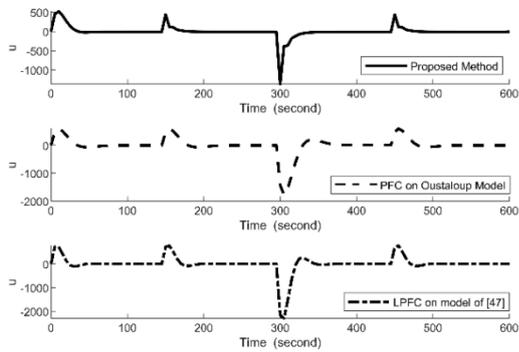


Fig. 6: Control effort signal.

B. Second Scenario: Design of PFC for Disturbance Rejection Purpose

In this section, effectiveness of proposed controller is investigated in presence of disturbance. A disturbance signal with 0.2 amplitude was added to output of system and as expected, the proposed controller rejected the effect of this disturbance signal on output. The obtained results have been shown in Fig. 7.

It can be seen that the proposed method is quite effective in rejecting the disturbance effects and has better performance and smoother than integer order which achieved by Oustaloup method.

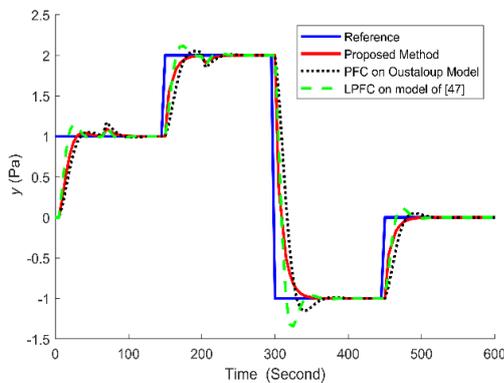


Fig. 7: Performance of the Proposed Laguerre-based PFC compared with PFC-based on Oustaloup model and that of [47] in presence of disturbance.

Conclusion

In this paper, the predictive functional controller based on the Laguerre functions was designed to control the pressure of the industrial heating furnace for tracking pressure variations. Because the model of heating furnace is fractional order, it is needed to be approximate by integer order model. Classical integer approximating method such as Oustaloup method leads to high integer order model. Also, model predictive controller has high computational complexity, in this

manner. Therefore, a new method for reduced integer order approximation of fractional-order system was presented. Well-known Hankel singular values concept were applied for determining the order of reduced integer order model of heating furnace. Genetic algorithm is used for minimizing the constrained fitness function including the weighted summation of ISE indices, absolute of difference of magnitude of frequency responses, difference of steady state errors and maximum overshoots, and the constraints are some stability conditions added to optimization problem to ensure stability. Then, predictive functional controller based on Laguerre functions was used to control pressure of achieved reduced integer order model of heating furnace. The designed controller has low computational complexity and fast simulation time and is quite effective in rejecting disturbance and tracking the pressure set-point variations.

Author Contributions

N. Bigdeli and H. Nasiri Solokloo designed the research idea and completed the proposed method. H. Nasiri Soloklo made the simulations. H. Nasiri Soloklo and N. Bigdeli interpreted the results and wrote the manuscript.

Conflict of Interest

The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

Abbreviations

a_i, b_j	Coefficients of fractional order Transfer Function (TF)
c_i, d_i	Unknown coefficients of reduced TF
CFD	Computational Fluid Dynamics
D_t^q	Fractional derivative operator
e_{ss}	Steady state error
f	Continuous function
FOM	Fractional Order model
GA	Genetic Algorithm
G_f	General fractional order TF
G_r	Reduced integer order TF
HSV	Hankel Singular Values
ISE	Integral Square Error
J	Constrained fitness function for reduced integer model
J'	Unconstrained fitness function
LPFC	Laguerre-based PFC
m	Number of terms in numerator of fractional TF
mag	Magnitude of frequency response
MOR	Model Order Reduction
MPC	Model Predictive Control
n	Number of terms in denominator of

	fractional TF
N_p	Prediction horizon
OS	Overshoot
PFC	Predictive Functional Controller
Q	Symmetric positive definite matrix
q	Order of fractional derivative
QP	Quadratic Programming
R	Symmetric positive definite matrix
r	Order of reduced model
RMSE	Residual Mean Square Error
t_f	Final time
TPWL	Trajectory Piecewise Linear Model
$w_1, w_2, w_3,$ w_4	Weighting coefficients
x	State vector
y	Output vector
y_r	Step response of reduced order model
α_i, β_j	Orders of fractional TF
Γ	Gamma function
λ	Scaling factor
ϕ	Discrete time transfer function of Laguerre functions

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Biographies



Hasan Nasiri Soloklo was born in Tehran in 1986. He received his M.Sc. degree in Control Engineering from Shahid Bahonar University of Kerman in 2012. Currently he is a Ph.D. candidate in Imam Khomeini International University of Qazvin. His research interests include model order reduction and evolutionary computation.



Nooshin Bigdeli was born in 1978 in Iran, and completed her Ph.D. degree in Electrical Engineering majoring in Control at Sharif University of Technology, Tehran, Iran in 2007. She is currently an associate professor of Electrical Engineering Department of Imam Khomeini International University, Qazvin, Iran. Her research interests include fractional order systems, intelligent systems and chaos control, model predictive control as well as signal, image and time series analysis.

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