



# Inclined lorentzian force effect on tangent hyperbolic radiative slip flow imbedded carbon nanotubes: lie group analysis

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## Nomenclature

### Dimensional Variables

$B_0$	Uniform magnetic field strength
$g$	Acceleration due to gravity(m/s <sup>2</sup> )
$k$	Thermal conductivity (W/(m.K))
$k^*$	Mean absorption coefficient
$T$	Temperature (K)
$q_r$	Radiative heat flux
$Q_0$	Heat generation orabsorption coefficient
$\bar{u}, \bar{v}$	Velocity components
$\bar{x}, \bar{y}$	Cartesian co-ordinates

### Dimensionless symbols

$C_f$	Local Skin friction
$N_r$	Radiation parameter
$Nu$	Local Nusselt number
$Pr$	Prandtl number
$Re_x$	Local Reynolds number
$S$	Suction/injection parameter
$We$	Weissenberg number

### Greek symbols

$\alpha$	Thermal diffusivity(m <sup>2</sup> /s)
$\beta$	Velocity slip parameter
$\delta$	Temperature slip parameter
$\lambda$	Heat source or sink parameter
$\mu$	Dynamic viscosity (Pa.s or (kg/ms)
$\mu_0$	Zero shear rate viscosity
$\mu_\infty$	Infinite shear rate viscosity
$\nu$	Kinematic viscosity of the base fluid (m <sup>2</sup> /s)
$\rho$	Density(Kg/m <sup>3</sup> )
$\rho C_p$	Heat capacitance (J/(kg.K))
$\sigma$	Electric conductivity (S/m)
$\sigma^*$	Stefan-Boltzmann constant(W/m <sup>2</sup> K <sup>4</sup> )
$\bar{\tau}$	Extra stress tensor
$\Gamma$	Time dependent material constant
$\phi$	Solid volume fraction of the Nanoparticles

### Subscripts

$nf$	Nanofluid
$f$	Base fluid
$s$	Nanoparticle
$w$	Wall
$x$	Local
$\infty$	Quiescent or far away from flow

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## Abstract

The present paper focuses on numerical study for an inclined magneto-hydrodynamic effect on free convection flow of a tangent hyperbolic nanofluid embedded with Carbon nanotubes (CNTs) over a stretching surface taking velocity and thermal slip into account. Two types of nanoparticles are considered for the study; they are single and multi-walled nanotubes. The presentation of single-parameter group (Lie group) transformations reduces the independent variable number by one, and hence the partial differential governing equations with the supplementary atmosphere into an ordinary differential equation with the appropriate suitable conditions. The obtained ordinary differential equations are then numerically solved by employing fourth order Runge-Kutta technique along with shooting method. The effects of the various parameters governing the flow field are presented with the help of graphs. The investigation reveals that the non-Newtonian MWCNTs Tangent hyperbolic nano-liquid reduces the friction near the stretching sheet contrasting SWCNTs. This combination can be used as a friction lessening agent/factor. Usage of CNTs shows an excellent performance in enhancing the thermal conductivity of the nanofluid and single wall carbon nanotubes (SWCNTs) has higher thermal conductivity than multi wall carbon nanotubes (MWCNTs) even in the presence of radiative heat transfer and heat source. Comparison with existing results available in literature is made and had an excellent coincidence with our numerical method.

## Keywords:

Aligned Lorentzian Force, Navier Slip, Thermal Slip, Radiation, Heat Generation/Absorption, Carbon Nanotubes, Lie Group Analysis.

## 1. Introduction

Warmth exchange in the recent era has become an inevitable one as increased productivity of compact objects. This warmth exchange liquid assumes a key part in the advancement of vitality proficient warmth exchange hardware including electronics, transportation and HVAC & R. The role of nanomaterials has turned top in handling such type of products. The non-Newtonian nanofluids groups' better long haul solidness

and rheological properties when contrasted and millimeter or micrometer measured molecule suspensions. Nanotubes are individuals from the fullerene auxiliary family and it is received from beneficiary long, empty structure with the divider composed by one particle thick sheets of carbon, called graphene. Carbon nanotubes are characterized into single or multi-walled carbon nanotubes. The investigation of carbon nanotubes (CNTs) is started in 1991, and it is opened up another period in materials science. These are the mind boggling structures that have a variety of intriguing electronic, attractive and mechanical properties. CNT is no less than 100 times more grounded than steel, however just a single 6th as substantial, so nanotube fibers could reinforce any material. Nanotubes warm conductivity is superior to copper. CNTs are prior utilized as a part of polymers to control or upgrade conductivity and are added to hostile to static bundling. CNTs have different structures, contrasting in thickness, length and number of layers. The physical attributes of nanotubes are distinctive relying upon how the graphene sheet is moved up from the tube starting it to act either metallic or a semiconductor. CNTs have various applications, for example, controlling the warm conductivity, field outflow, conductive properties, vitality stockpiling and conductive glue, atomic gadgets in light of CNTs, warm materials and basic outlining. Choi [1], investigated that the nanofluid is now one of the fastest growth rates in scientific papers in nanoscale science and technology. Chandrasekhar and Kasi [2] analyzed the heat and mass transfer effects on MHD nanofluid. Hayat *et al.* [3] discussed MHD nanofluid with power law velocity and chemical reaction. Gorla and Vasu [4] studied unsteady convective heat transfer to a stretching surface in a non-Newtonian nanofluid. Kameswaran *et al.* [5] studied numerically the convective heat transfer in the influence of non-linear Boussinesq approximation, thermal stratification and convective boundary conditions on non-Darcy nanofluid flow over a vertical wavy surface. Rizwan *et al.* [6] presented the thermos-physical effects of carbon nanotubes on MHD flow past a stretching sheet. Waqar *et al.* [7] studied the analysis of heat transfer on MHD water

functionalized carbon nanotubes past a static/moving wedge. Khan *et al.* [8] investigated the effect of heat transfer with carbon nanotubes past a flat surface in the presence of Navier slip. The slip flow and Convective heat transfer in MHD carbon a nanotube over a stretching surface was studied by Rizwan *et al.* [9].

The stream and heat exchange along with magnetohydrodynamic has pulled in the consideration of numerous specialists due to the significance of the attractive impact on the limit layer control the liquid stream and its applications are essential in numerous building issues, for example, atomic reactors, MHD generators, plasma contemplates and geothermal vitality extractions and so on. Haq *et al.* [10] studied thermophysical effects of carbon nanotubes on MHD flow past a stretching sheet. Magnetohydrodynamic convective boundary layer flow of dissipating nanofluid past an exponential stretching sheet was presented by Sreenivasulu and Poornima [11]. Poornima *et al.* [12] analysed the effect of temperature dependent viscosity on mixed convection MHD dissipating cylindrical shaped Cu-water nanofluid flow over a vertical moving surface. Hayat *et al.* [13] analysed the Powell-Eyring MHD nanofluid flow over a non-linear stretching surface with variable thickness. Beg *et al.* [14] presented computational solutions for steady-state boundary layer flow of a nanofluid from an impermeable vertical flat wall in a porous medium saturated with a water-based dilute nanofluid containing oxytactic microorganisms. Reddy *et al.* [15] studied the flow and heat transfer characteristics of Williamson MHD Nanofluid over a stretching surface in the presence of variable thermal conductivity and variable thickness. Electrically magnetic field applied transversely normal to the fluid flow aligned is studied under Maxwell's law. This part has wide applications in the power generators, magnetic shielding, bearings, electrically leading fluid. An aligned magnetic effect on plasma structures was explained by Ergun *et al.* [16].

The study of thermal radiation and heat transfer is of great importance whenever the environment is at higher temperature and its applications plays an important role in the

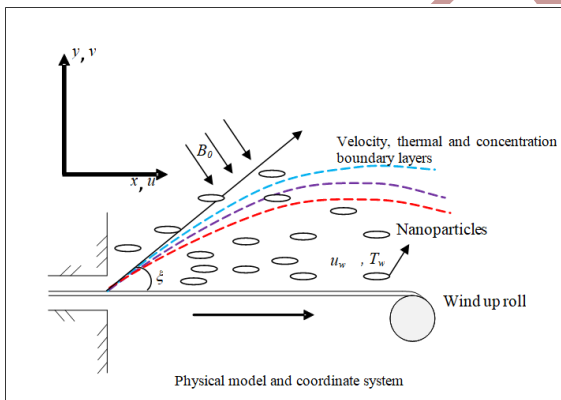
design of various advanced thermal conversion system having at high temperature. Rashidi *et al.* [17] explained the radiative effect on MHD nanofluid. Wahed and Akl [18] studied the hall current effect on MHD nanofluid with non-linear thermal radiation. Lu *et al.* [19] elucidated the non-linear thermal radiation effect on MHD Williamson nanofluid with constant wall temperature. Poornima *et al.* [20] proposed the slip flow of Casson rheological fluid under variable thermal conductivity and thermal radiation. Bhaskar Reddy *et al.* [21] investigated the effect of radiation on MHD slip flow of nanofluid over an exponential porous stretching sheet with viscous dissipation. Radiation influence on MHD Williamson nanofluid was studied by Kho *et al.* [22]. Boundary layer MHD convective flow of EG based Cu nanotubes past an extensible moving surface with thermal radiation was presented by Poornima and Sreenivasulu [23]. A radiation effect in stagnation point flow on carbon water nanofluid was presented by Hayat *et al.* [24]. Rashid *et al.* [25] explained the effect of radiation on water based metallic nanoparticles with prescribed surface temperature.

The tangent hyperbolic fluid model is enough competent to describe shear thinning phenomenon. It measures the fluid sustaining less flow with more rate of shear stress. Examples of the fluids having this property are ketchup, paint, blood etc. In literature many studied were reported on tangent hyperbolic fluid by considering different physical aspects. An important branch of the non-Newtonian fluid models is the hyperbolic tangent fluid model. The hyperbolic tangent fluid is used extensively for different laboratory experiments. Hussain *et al.* [26] discussed the dissipative impact on tangent hyperbolic flow under convective boundary condition. Zakir and Zaman [27] studied the MHD tangent hyperbolic flow past a stretching sheet with slip employing Lie group analysis. Akbar *et al.* [28] numerically analyzed the MHD tangent hyperbolic flow towards stretching sheet. Thammanna *et al.* [29] investigated combined effects of partial slip and Ohmic heating on MHD nanoflow. Naseer *et al.* [30] analyzed hyperbolic tangent flow over an exponential expanding cylinder. Sandeep [31]

studied the aligned Lorentzian force on thin nano-film suspended graphene particles. Sailaja *et al.* [32] explained the influence of aligned magnetic field over Casson film flow. Arifin *et al.* [33] described the aligned MHD effect past a two-phase dusty Casson flow with Newtonian heating. Rashidi *et al.* [34] discussed the squeezing flow between parallel plates under inclined magnetic field effect. Waini *et al.*[35] studied inclined Lorentz force effect on UCM fluid with stretching /shrinking of sheet. Ashwin kumar *et al.* [36] presented the effect of the aligned magnetic field on MHD nanoflow over a semi-infinite vertical plate with ferrous nanoparticles.

Motivating the above, the objective of present study is to study the Lie group analysis for the radiation and slip effects on boundary layer flow of carbon nanofluids with aligned magnetic field and heat generation /absorption.

**2. Mathematical Model**



**Fig.1.**Physical model and flow configuration

A steady two-dimensional boundary layer flow of an incompressible tangent hyperbolic fluid flow of a nanofluid past a stretching sheet with aligned magnetic field and partial slip is considered. The flow is considered along the  $\bar{x}$ -axis, which is taken along the stretching surface, and the  $\bar{y}$ -axis is taken perpendicular to sheet (Fig. 1). The flow is conformed to  $\bar{y} > 0$ . The surface of the plane is maintained at a constant temperature  $T_w$  higher than the constant

temperature  $T_\infty$  of the ambient nanofluid. The fluid is a water based nanofluid containing two kinds of nanotubes such as single wall carbon nanotubes (SWCNTs) and multiwall carbon nanotubes (MWCNTs). The nanofluid is assumed incompressible and the flow is assumed to be laminar. The thermophysical properties of the nanofluid are given in Table 1.

**Table 1.** Thermal properties of base fluid and nanoparticles [8]

Physical properties	Nanoparticles		
	Base fluid Water	Single wall carbon nanotubes	Multi wall carbon nanotubes
$\rho$ (kg/m <sup>3</sup> )	997	2600	1600
$C_p$ (J/kg K)	4179	425	796
$k$ (W/m K)	0.613	6600	3000
$\beta \times 10^{-5}$ (K <sup>-1</sup> )	21	0.33	0.72

The constitutive equation for hyperbolic tangent fluid model is given by [28]

$$\bar{\tau} = \left[ \mu_\infty + (\mu_0 + \mu_\infty) \tanh(\Gamma \bar{\gamma})^n \right] \bar{\gamma}$$

Where  $\bar{\tau}$  is the extra stress tensor,  $\mu_0$  is the zero shear rate viscosity,  $\mu_\infty$  is the infinite shear rate viscosity,  $\Gamma$  is the time dependent material constant and  $n$  is the flow behaviour index. The term  $\bar{\gamma}$  can be expressed as

$$\bar{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\gamma}_{ij} \bar{\gamma}_{ij}} = \sqrt{\frac{1}{2} \Pi}$$

where  $\Pi = \frac{1}{2} \text{tr}(\text{grad}V + (\text{grad}V^T))^2$ . As we concerned with infinite shear rate viscosity problem, so in the following we restrict ourselves to the case  $\mu_\infty = 0$ . The fluid under consideration that described shear thinning, so we must take  $\Gamma \bar{\gamma} < 1$ . Then Eq. (1) reduces to

$$\begin{aligned} \bar{\tau} &= \mu_0 \left[ (\Gamma \bar{\gamma})^n \right] \bar{\gamma} = \mu_0 \left[ (1 + \Gamma \bar{\gamma} - 1)^n \right] \bar{\gamma} \\ &= \mu_0 \left[ (1 + n(\Gamma \bar{\gamma} - 1))^n \right] \bar{\gamma} \end{aligned}$$

The boundary layer and Boussinesq approximations are assumed to be valid. Under

these assumptions, the governing boundary layer equations can be written in dimensional form as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{1}{\rho_{nf}} [\mu_{nf} (1-n) \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \sqrt{2n} \Gamma \left( \frac{\partial u}{\partial y} \right) \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 \text{Sin}^2(\xi) \bar{u}] \quad (2)$$

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha_{nf} \frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial \bar{y}} + \frac{Q_0}{(\rho C_p)_{nf}} (T - T_\infty) \quad (3)$$

where  $\bar{u}$  and  $\bar{v}$  are velocity components in the  $\bar{x}$  and  $\bar{y}$  directions,  $T$  is the local Temperature of the nano fluid,  $g$  is the acceleration due to gravity,  $\sigma$  is the electric conductivity,  $B_0$  is the uniform magnetic field strength,  $\rho_{nf}$  is the effective density of the nanofluid,  $\mu_{nf}$  is the effective dynamic viscosity of the nanofluid and  $\alpha_{nf}$  is the thermal diffusivity of the nanofluid,  $q_r$  is the radiative heat flux and  $Q_0$  is the volumetric rate of heat generation.

boundary conditions of these equations are

$$\bar{u} = \bar{u}_w(\bar{x}) + N \frac{\partial \bar{u}}{\partial \bar{y}}, \bar{v} = \bar{v}_w, T = T_w + L \frac{\partial T}{\partial \bar{y}} \text{ at } \bar{y} = 0$$

$$\bar{u} \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } \bar{y} \rightarrow \infty \quad (4)$$

where  $\bar{u}_w(\bar{x}) = a \bar{x}$  is the stretching velocity.

The Rosseland approximation for radiative heat flux is

$$q_r = -\frac{4 \sigma^* \partial T^4}{3 k^* \partial \bar{y}} \quad (5)$$

Where  $\sigma^*$  is the Stefan-Boltzmann constant and  $k^*$  is the mean absorption coefficient. If the temperature differences within the flow are adequately small, then Eq. (5) can be linearized by expanding  $T^4$  into the Taylor series about  $T_\infty$ , which after ignoring the higher-order terms

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

In sight of Eqs. (5) and (6), Eq. (3) becomes

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha_{nf} \left( 1 + \frac{4}{3} R \right) \frac{\partial^2 T}{\partial \bar{y}^2} + \frac{Q_0}{(\rho C_p)_{nf}} (T - T_\infty) \quad (7)$$

where  $R = 16\sigma^* T_\infty^3 / 3k_f k^*$  is the radiation constraint.

Several researchers projected nanofluid models are effective only for spherical or rotational elliptical particles with small axial ratio and these models do not describe the properties of space distribution of the carbon nanotubes on thermal conductivity. Nevertheless the carbon nanotubes can be regarded as rational elliptical model with large axial ratio. A theoretical model based on Maxwell theory including rotational elliptical nanotubes with very large axial ratio and reimbursing the effects of space distributions on carbon nanotubes (CNTs) was proposed by Xue [37].

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{CNT}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$$

$$(\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_{CNT},$$

$$\frac{k_{nf}}{k_f} = \frac{1 - \phi + 2\phi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \phi + 2\phi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}},$$

$$(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_{CNT},$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (8)$$

Where  $\phi$  is the solid volume fraction of nanotubes. Here the subscripts  $nf$ ,  $f$  and  $CNT$  represent the thermo physical properties of the nanofluids, base fluid and nano-solid particles respectively.

we introduce the following non-dimensional variables:

$$x = \sqrt{\frac{a}{\nu_f}} \bar{x}, y = \sqrt{\frac{a}{\nu_f}} \bar{y}, u = \frac{\bar{u}}{\sqrt{a\nu_f}},$$

$$v = \frac{\bar{v}}{\sqrt{a\nu_f}}, \theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{9}$$

Eqs. (1), (2) and (7) take the subsequent non-dimensional form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{(1 - \phi + \phi \rho_{CNT} / \rho_f)}$$

$$\left\{ \frac{1}{(1 - \phi)^{2.5}} (1 - n) \frac{\partial^2 u}{\partial y^2} + \sqrt{2n} \Gamma a \left( \frac{\partial u}{\partial y} \right) \frac{\partial^2 u}{\partial y^2} - \text{Sin}^2 \xi M u \right\}$$

$$\tag{11}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr} [1 - \phi + \phi(\rho Cp)_{CNT} / (\rho Cp)_f]} \left( 1 + \frac{4}{3} R \right)$$

$$\left( \frac{k_{nf}}{k_f} \right) \frac{\partial^2 \theta}{\partial y^2} + \frac{\lambda}{[1 - \phi + \phi(\rho Cp)_{CNT} / (\rho Cp)_f]} \theta$$

$$\tag{12}$$

with the corresponding boundary conditions

$$u = x + \sqrt{\frac{a}{\nu_f}} N \frac{\partial u}{\partial y}, v = \sqrt{a\nu_f} v_w, \left. \begin{array}{l} \\ \theta = 1 + \sqrt{\frac{a}{\nu_f}} L \frac{\partial \theta}{\partial y} \end{array} \right\} \text{at } y = 0 \tag{13}$$

$$u \rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty$$

where  $\text{Pr} = \nu_f / \alpha_f$  is the Prandtl number  $\lambda = Q_0 / a(\rho Cp)_f$  is the coefficient of heat generation/absorption and  $M = \sigma B_0^2 / \rho_f a$  is the magnetic parameter.

By presenting the stream function  $\psi$ , which is defined by  $u = \partial \psi / \partial y$  and  $v = -\partial \psi / \partial x$  then the system (11) and (12) becomes

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \frac{1}{(1 - \phi + \phi \rho_{CNT} / \rho_f)}$$

$$\left\{ \frac{1}{(1 - \phi)^{2.5}} (1 - n) \frac{\partial^3 \psi}{\partial y^3} + \sqrt{2n} \Gamma a \left( \frac{\partial \psi}{\partial y} \right) \frac{\partial^3 \psi}{\partial y^3} - M \text{Sin}^2 \xi \frac{\partial \psi}{\partial y} \right\} \tag{14}$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr} [1 - \phi + \phi(\rho Cp)_{CNT} / (\rho Cp)_f]} \left( 1 + \frac{4}{3} R \right) \left( \frac{k_{nf}}{k_f} \right)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{\lambda}{[1 - \phi + \phi(\rho Cp)_{CNT} / (\rho Cp)_f]} \theta$$

$$\tag{15}$$

with the boundary conditions

$$\left. \begin{array}{l} \frac{\partial \psi}{\partial y} = x + \sqrt{\frac{a}{\nu_f}} N \frac{\partial^2 \psi}{\partial y^2}, \\ \frac{\partial \psi}{\partial x} = -\sqrt{a\nu_f} v_w, \theta = 1 + \sqrt{\frac{a}{\nu_f}} L \frac{\partial \theta}{\partial y} \end{array} \right\} \text{at } y = 0$$

$$u \rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty \tag{16}$$

### 3. Lie Group Analysis

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations (Mukhopadhyay *et al.* [38])

$$\Gamma_1 : x^* = x e^{\epsilon \alpha_1}, y^* = y e^{\epsilon \alpha_2},$$

$$\psi^* = \psi e^{\epsilon \alpha_3}, \theta^* = \theta e^{\epsilon \alpha_4} \tag{17}$$

Substituting (17) in (14) and (15) we get,

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)} \left( \frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial y^{*2}} \right) = \frac{1}{(1-\phi + \phi \rho_{CNT} / \rho_f)} \left[ \begin{aligned} & \frac{(1-n)}{(1-\phi)^{2.5}} e^{\varepsilon(3\alpha_2-\alpha_3)} \frac{\partial^3 \psi^*}{\partial y^{*3}} + \\ & e^{\varepsilon(5\alpha_2-2\alpha_3-\alpha_5)} \sqrt{2na} \Gamma^* \frac{\partial^2 \psi^*}{\partial y^{*2}} \frac{\partial^3 \psi^*}{\partial y^{*3}} \\ & - M e^{\varepsilon(\alpha_2-\alpha_3)} \frac{\partial \psi^*}{\partial y^*} \end{aligned} \right] \quad (18)$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_4)} \left( \frac{\partial \psi^*}{\partial y^*} \frac{\partial \theta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \theta^*}{\partial y^*} \right) = \frac{1}{Pr} \frac{(1+R)}{[1-\phi + \phi(\rho_{CNT} / \rho_f)]} \left( \frac{k_{nf}}{k_f} \right) e^{\varepsilon(2\alpha_2-\alpha_4)} \frac{\partial^2 \theta^*}{\partial y^{*2}} + \frac{1}{Pr} \frac{\lambda}{[1-\phi + \phi(\rho_{CNT} / \rho_f)]} e^{-\varepsilon\alpha_4} \theta^* \quad (19)$$

The system of equations will remain unchanged under the group of transformations  $\Gamma_1$ , and we would have the following families among the parameters, namely

$$\begin{aligned} \alpha_1 + 2\alpha_2 - 2\alpha_3 &= 3\alpha_2 - \alpha_3 \\ &= 5\alpha_2 - 2\alpha_3 - \alpha_5 = \alpha_2 - \alpha_3 \\ \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 &= 2\alpha_2 - \alpha_4 = -\alpha_4 \end{aligned}$$

these relations give

$$\alpha_3 = \alpha_1, \alpha_2 = \alpha_4 = 0 \text{ and } \alpha_5 = -\alpha_1$$

The set of transformations of the  $\Gamma_1$  group reduces to

$$\Gamma_1 : x^* = x e^{\varepsilon\alpha_1}, y^* = y, \psi^* = \psi e^{\varepsilon\alpha_1}, \theta^* = \theta, \Gamma^* = \Gamma_p e^{-\varepsilon\alpha_1} \quad (20)$$

Expanding (20) in power of  $\varepsilon$  by Taylor's technique and keeping terms up to the order of  $\varepsilon$ , we get

$$\begin{aligned} x^* - x &= x \varepsilon \alpha_1, y^* - y = 0, \psi^* - \psi = \psi \varepsilon \alpha_1, \\ \theta^* - \theta &= \theta \varepsilon \alpha_1, \Gamma^* - \Gamma_p = -\Gamma_p \varepsilon \alpha_1 \end{aligned} \quad (21)$$

The characteristic functions are

$$\frac{dx}{x\alpha_1} = \frac{dy}{0} = \frac{d\psi}{\psi\alpha_1} = \frac{d\theta}{0} = \frac{d\Gamma_p}{-\Gamma_p\alpha_1} \quad (22)$$

from which, we obtain the following similarity variables

$$\eta = y^*, \psi^* = x^* f(\eta), \theta^* = \theta(\eta), \Gamma_p = \frac{1}{x^*} \Gamma_0 \quad (23)$$

With the help of these relations, the Eqs.(18) and (19) become

$$(1-n)f'''' + (1-\phi)^{2.5} \left\{ \begin{aligned} & [1-\phi + \phi(\rho_{CNT} / \rho_f)] (ff'' - (f')^2) \\ & + nW_e f'' f''' - M \sin^2 \xi f' \end{aligned} \right\} = 0 \quad (24)$$

$$\left( 1 + \frac{4}{3} R \right) \left( \frac{k_{nf}}{k_f} \right) \theta'' + Pr \quad (25)$$

$$[(1-\phi + \phi(\rho_{CNT} / \rho_f)] f \theta' + \lambda \theta = 0$$

the agreeing boundary conditions (16) become

$$\begin{aligned} f'(0) &= 1 + \beta f''(0), f(0) = S, \\ \theta(0) &= 1 + \delta \theta'(0) \\ f'(\infty) &\rightarrow 0, \theta(\infty) \rightarrow 0 \end{aligned} \quad (26)$$

where  $\beta = \sqrt{a/v_f} N$  is the velocity slip parameter,  $S = -\sqrt{a/v_f} v_w$  is the suction/injection parameter,  $\delta = \sqrt{a/v_f} L$  is the temperature slip parameter and  $W_e = \sqrt{2} \Gamma a$  be the Weissenberg number.

The skin friction coefficient  $C_f$  and the Nusselt number  $Nu_x$ , which are defined as

$$\begin{aligned} C_f &= \frac{\mu_{nf}}{\rho_f u_w^2} \left[ (1-n) \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0} + \frac{n\Gamma}{\sqrt{2}} \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}^2 \right], \\ Nu_x &= \frac{\bar{x} k_{nf}}{k_f (T_w - T_\infty)} \left( -\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0} \end{aligned} \quad (27)$$

Using the Eqs. (8), (24) and (27), the coefficient of skin friction and local Nusselt number are described as

$$\text{Re}_x^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} \left[ (1-n)f''(0) + \frac{n}{2} W_e (f''(0))^2 \right],$$

$$\text{Re}_x^{-1/2} Nu_x = -\frac{k_{nf}}{k_f} \theta'(0)$$

(28)

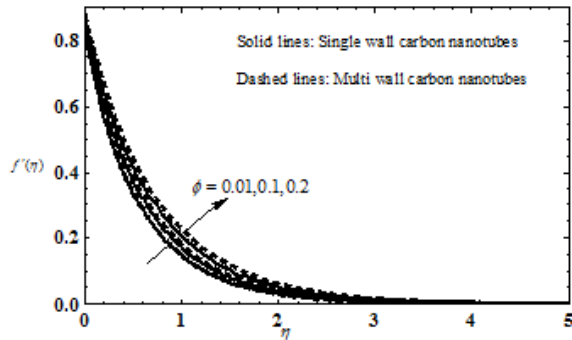
where  $\text{Re}_x = \bar{u}_w(\bar{x})\bar{x} / \nu_f$  is the local Reynolds number.

### 4. Results and Discussion

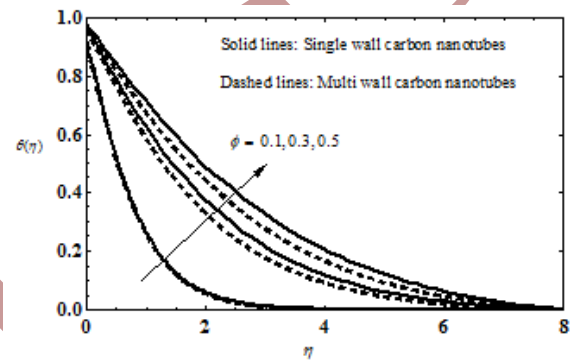
The coupled non linear representing boundary layer Eqs. (24) and (25) subjected to the boundary conditions (26) is explained numerically by utilizing fourth request Runge-Kutta strategy alongside shooting technique. First of all higher order non-linear equations are converted into simultaneous linear differential equations of first order and then transformed into an initial value problem by applying the shooting method. The resultant initial problem is solved by employing fourth order Runge-Kutta technique. A representative set of numerical results are presented graphically in Figs. 2–22. In order to assess the accuracy of numerical calculation, the present results for skin friction values are compared with the special case of Uma Maheswar Rao and KoteswaraRao [39] and Akbar *et al.* [28] and Ullah and Zaman [27] and found that the present results are in excellent agreement (Table. 2 & 3).

**Table 2.** Comparison results of surface skin friction for a particular case [39].

<i>N</i>	<i>We</i>	<i>M</i>	Uma Maheswara Rao and Koteswara rao[39]	Presentstudy
0.1			1.15892610	1.15893122
0.3			1.01304949	1.01303861
0.5	0.1		0.83768770	0.83768767
	0.5		1.14675023	1.14674032
	1.0	0.5	1.13077886	1.13077645
		1.0	1.33774399	1.33774398
		2.0	1.63740820	1.63772102

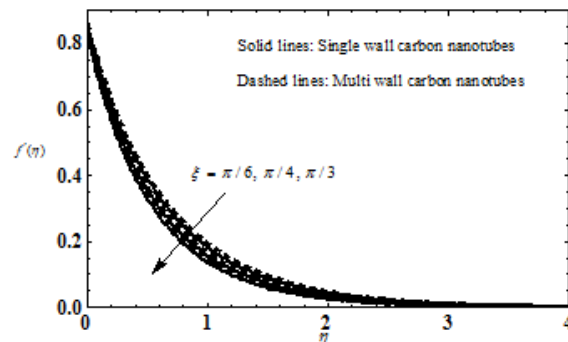


**Fig.2.** Velocity profiles for different values of  $\phi$



**Fig.3.** Temperature profiles for different values of  $\phi$

Iteration of velocity and temperature profile with nanoparticles volume fraction of CNTs is depicted in Figs. 2- 3, it is noticed that the warmth of the fluid to be an elevating function of  $\phi$  for both types of CNTs. Physically, the logic is increase in thermal conductivities of nanofluids due to supplement of more CNTs. The more the volume fraction of CNTs in the nanofluids, the more will be its thermal conductivity which in turn enhances the fluids temperature.



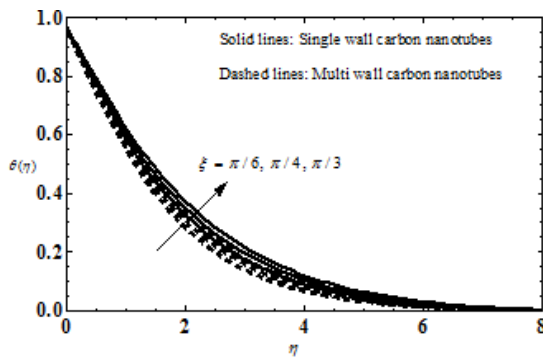
**Fig.4.** Velocity profiles for different values of  $\xi$



**Table 3.** Comparison results of surface skin friction for some particular cases.

n	M	Akbar <i>et al</i> [28]			ZakirUllah, GulZaman [27]			Present study		
		We=0.0	We=0.3	We=0.5	We=0.0	We=0.3	We=0.5	We=0.0	We=0.3	We=0.5
0.0	0.0	1	1	1	1	1	1	1	1	1
0.1	0.0	0.94868	0.94248	0.93826	0.94868	0.94248	0.93826	0.94860	0.94252	0.93321
0.2	0.0	0.89442	0.88023	0.87026	0.89442	0.88023	0.87026	0.89438	0.88019	0.87023
0.3	0.5	1.09544	0.98804	0.96001	1.02472	0.98804	0.96001	1.09551	0.98806	0.96032
0.3	1.0	1.26491	1.13454	1.09616	1.18322	1.13454	1.09616	1.26490	1.13448	1.09609
0.3	1.5	1.41421	1.26193	1.21235	1.32288	1.26193	1.21235	1.41423	1.26189	1.21229

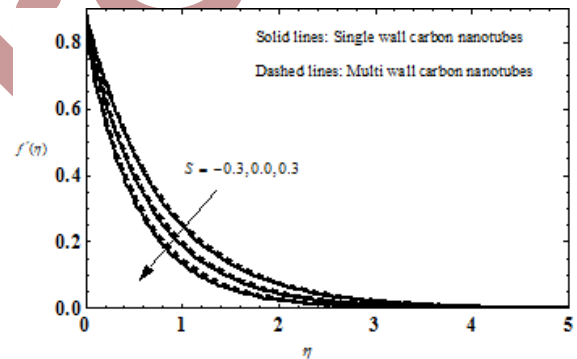
By maximizing the volume fraction  $\phi$  of CNTs, a gradual elevation is observed in thermal conductivity. Fig. 2 shows that there is an elevation in the velocity of the fluid for escalating  $\phi$  values. impact of  $\phi$  on the flow of Maxwell nanofluid. It is spotted that velocity profile maximizes with increasing  $\phi$  for both type of CNTs (SWCNT and MWCNT).



**Fig.5.** Temperature profiles for different values of  $\zeta$

Impact of aligned magnetic on  $f'(\eta)$  and  $\theta(\eta)$  (Figs. 4- 5), it is spotted that elevation in aligned magnetic parameter drops the velocity profiles and swifts the temperature profiles. This may happen due to the reason that an increase angle varying magnetic field strengthens the applied magnetic field. Quite commonly, increasing magnetic field parameter generates a resistive type of force to the tangent hyperbolic nano fluid flow, called drag force or Lorentzian force. This energy has the capacity to recede the movement of the nanoliquid for both types of CNTs. Although, it is seen that MWCNT velocity profiles elevates more than SWCNTs.

Interestingly, the thermal boundary layer thickens, rising the warmth in the nanoliquid. SWCNTs have higher rate of heat transfer than MWCNTs at the same volume fraction. It is due to the fact that SWCNT has greater thermal conductivity than MWCNT.



**Fig.6.** Velocity profiles for different values of  $S$

Influence of suction/injection parameter on  $f'(\eta)$  and  $\theta(\eta)$ (Fig. 6 - 7). It is detected that from these graphs velocity of CNTs based nanofluid declines continuously with increase in suction/injection parameter, for all values of  $\eta$  (Fig.6). MWCNTs velocity profiles have higher rate of values than SWCNTs distribution for same suction/injection variation. Similarly temperature profiles also decreases as rising suction/injection values(Fig.7). It is due to the fact that SWCNT has greater thermal conductivity than MWCNT The enhancement of heat in the nanofluid for SWCNT is higher than that of MWCNTs. It is due to the fact that

SWCNT has greater thermal conductivity than MWCNT.

Power law index ration effect on  $f'(\eta)$  and  $\theta(\eta)$  (Figs. 8 - 9). It is quite fascinating to observe that for accelerating power law index  $n$ , the momentum boundary layer thickness reduces showing a deceleration in the velocity profiles for both type of CNTs. But the thickness of the thermal boundary layer increases for increasing  $n$  values. Thus the heat in the fluid rises up.

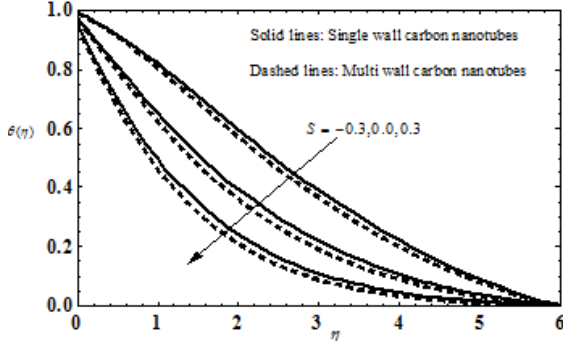


Fig.7. Temperature profiles for different values of  $S$

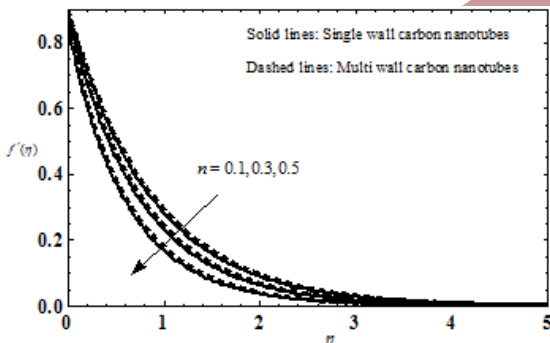


Fig.8. Velocity profiles for different values of  $n$

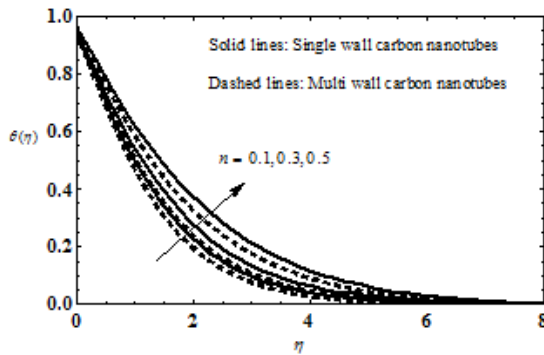


Fig.9. Temperature profiles for different values of  $n$

Consequence of strengthening fluid parameter  $We$  on  $f'(\eta)$  and  $\theta(\eta)$  (Figs. 10 -11). It is portrayed in Fig. 10, the influence of Weissenberg number on velocity profile. Escalating  $We$  values, recedes the momentum of the fluid since the fluid parameter is the ratio of relaxation time over process time. So, strengthening the fluid parameter implies enhancing the relaxation time, this in turn resists the flow and hence velocity reduces for both types of nanoparticles. Moreover, Multi walled nanotubes shows a better performance than single walled particles. While the thermal boundary layer thickens, enhancing the heat in the nanoliquid.

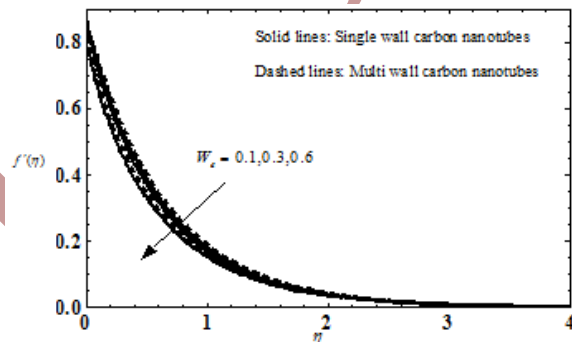


Fig.10 Velocity profiles for different values of  $We$

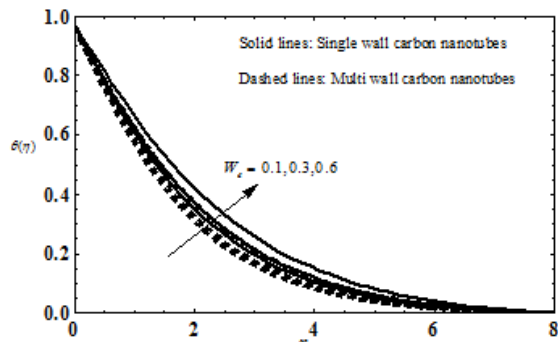


Fig.11. Temperature profiles for different values of  $We$

Radiation effect on  $\theta(\eta)$  is shown in Fig. 12. An increase in the radiative heat transfer rises the warmth in the nanofluid. Physically, enhancing radiation releases more heat energy inside the flow, the same helps in rising the temperature of the tangent hyperbolic nanofluid. Temperature

distribution for varying heat source/sink parameter is depicted in Fig. 13. An increase in  $\lambda$  values decreases the temperature of the fluid. It is due to the logic that heat generated in the nanofluid is absorbed by the stretching sheet, thus reducing the warmth in the nanoliquid.

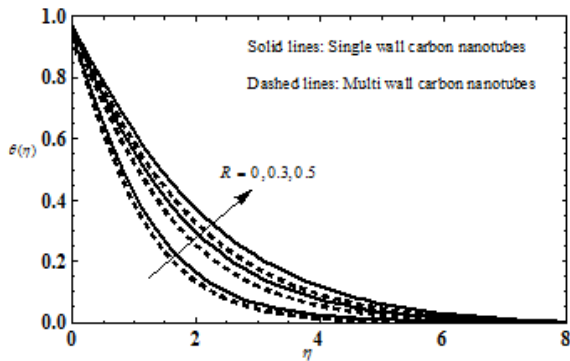


Fig.12. Temperature profiles for different values of  $R$

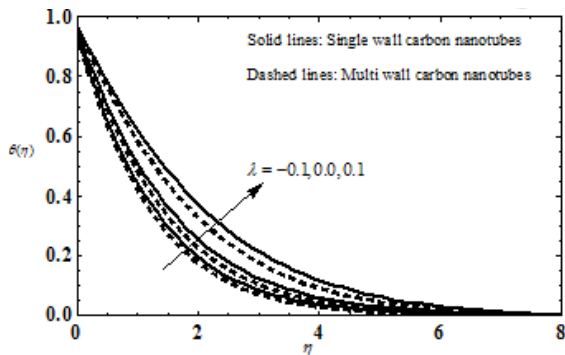


Fig.13. Temperature profiles for different values of  $\lambda$

Figs. 14 and 15 represents the Velocity and thermal slip effects on  $f'(\eta)$  and  $\theta(\eta)$ . Physically magnetic field together with slip effect acts as a retarding force. This retarding force can control the fluid's velocity which is useful in numerous applications such as magneto-hydrodynamic power generation and electromagnetic coating of wires and metal. As expected, escalating Navier slip parameter ( $\beta$ ) values depresses the momentum distribution at the free stream. Increase in the thermal slip parameter ( $\delta$ ) decreases the thermal boundary layer thickness, thereby decreasing the temperature of the nanofluid. It is because of the reason that a small

amount of heat is swapped away by fluid from the sheet.

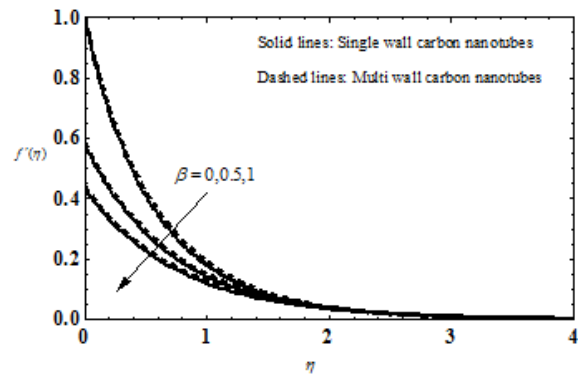


Fig.14. Velocity profiles for different values of  $\beta$

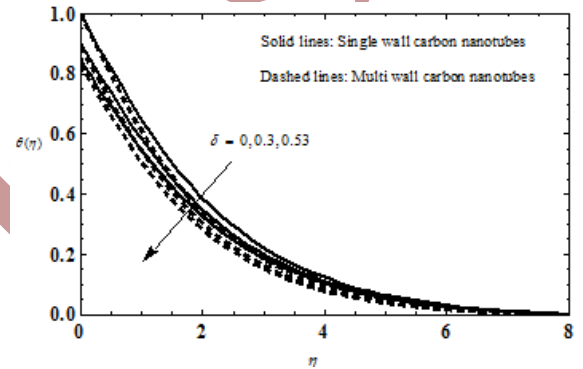


Fig.15. Temperature profiles for different values of  $\delta$

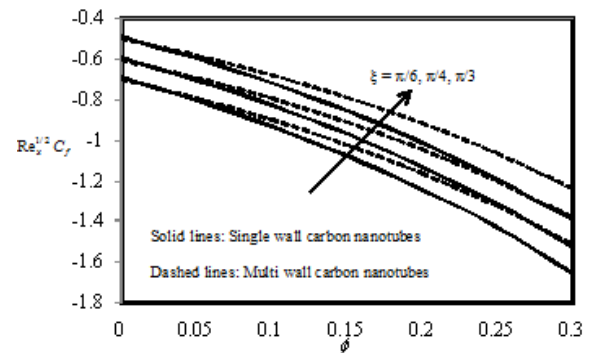


Fig.16. Surface skin friction versus volume fraction for different values of  $\xi$

The influence of various governing physical parameters on the surface skin friction and rate of heat transfer are shown in Figs. 16 -22. The variations of surface skin friction versus the CNT volume fraction for different values of aligned magnetic field is shown in Fig.16. It is

clear that the surface skin friction rate decreases with an increase in volume fraction of carbon nanotubes. Also it is evident that the surface skin friction increases for higher values of aligned magnetic field and the MWCNT have higher values than SWCNT.

From Fig. 17, it is seen that the behaviour of skin friction is decreases with an increase in nanoparticle volume fraction simultaneously it is evident that the skin friction increases with an increase in power law index  $n$ .

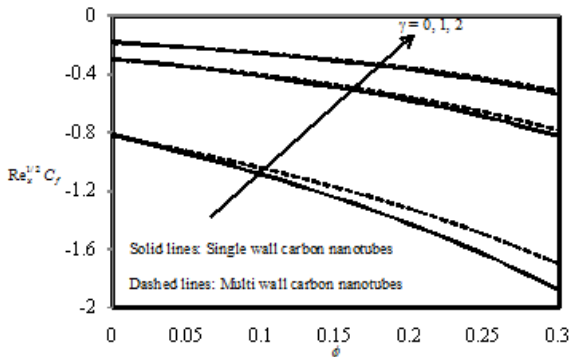


Fig.17. Surface skin friction versus volume fraction for different values of  $\gamma$

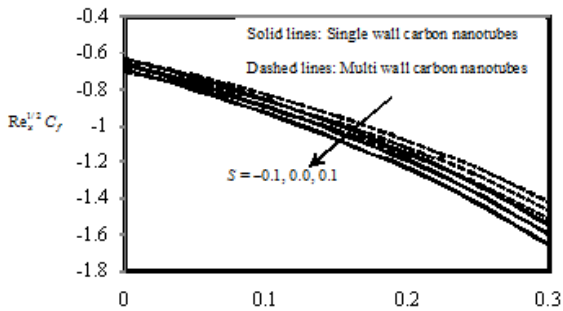


Fig.18. Surface skin friction versus volume fraction for different values of  $S$

Also it is clear that the values of surface skin friction have higher values in the case of MWCNT's than the SWCN's. The same behaviour is noticed in an increase in Weissenberg number  $We$ , this can be shown in Fig. 18.

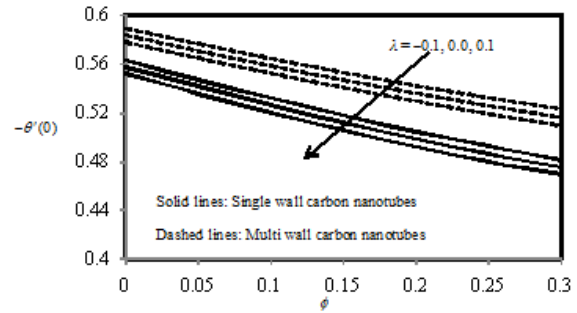


Fig.19. Heat transfer rate versus volume fraction for different values of  $\lambda$

Fig. 19 shows the surface skin friction against the volume fraction parameter  $\phi$  in the presence of velocity slip parameter  $\beta$ . It is clear from this that the surface skin friction decreases with an increase in volume fraction. Also it is noticed that the skin friction increases with the higher values of slip parameter  $\beta$  and initially single wall carbon nano tubes has higher values than multi wall carbon nano tubes, then it shows opposite for away from the surface. The heat transfer rate at the surface decreases with an increase in the nano fluid volume fraction in the presence of radiation parameter  $R$ .

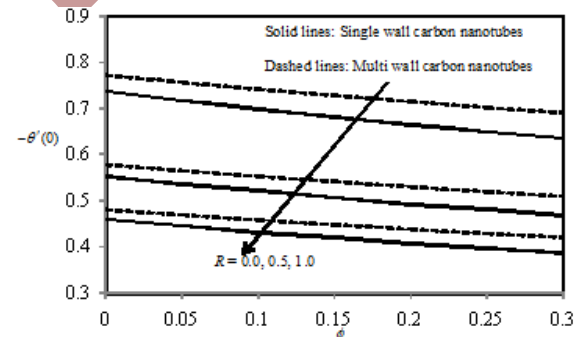


Fig.20. Heat transfer rate versus volume fraction for different values of  $R$

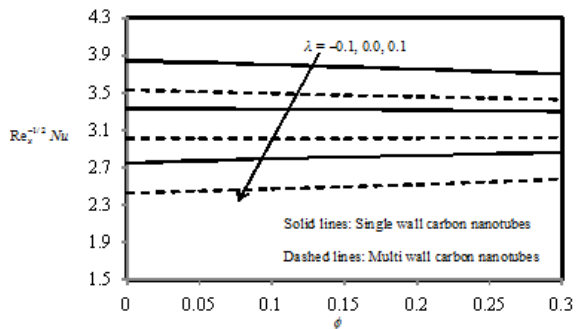


Fig.21. Heat transfer rate versus volume fraction for different values of  $\lambda$

Also it is evident that rate of heat transfer decreases with an increase in radiation parameter, which is shown in Fig.20. The variations in the rate of heat transfer against the volume fraction of carbon nanotubes for different values of heat generation or absorption is shown in Fig. 21. The rate of heat transfer decreases with an increase in volume fraction in case of absorption and it shows n opposite behaviour in case of generation. Also it is evident that the rate of heat transfer decreases for increasing values of heat generation or absorption and SWCNT dominates the MWCNT.

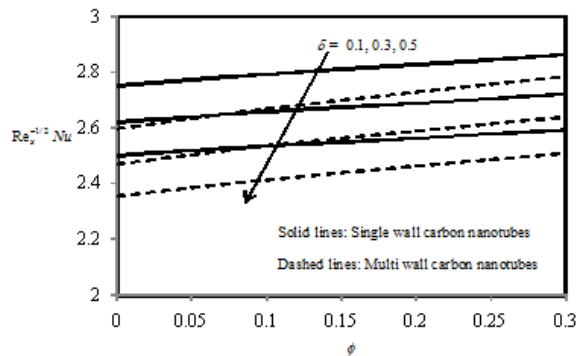


Fig.22. Heat transfer rate versus volume fraction for different values of  $\delta$

The rate of heat transfer at the surface against the volume fraction of carbon nano tubes for various values of temperature slip is shown in Fig. 22. From this, it is observed that the rate of heat transfer increases with an increase in volume fraction. Also it is notices that the rate of heat transfer decreases for higher values of slip parameter  $\delta$ .

### 5. Conclusions

A mathematical model has been framed to study the influence of aligned magnetic field and slip effects on boundary layer flow of carbon nanotubes past a vertical stretching sheet with thermal radiation and heat source or sink. From the investigation, the following summary can be drawn:

- The nanofluids with SWCNTs have higher effective thermal conductivities as compared to that having MWCNTs.
- The enhancement of heat transfer rate is seen with increasing volume fraction of nanoparticles which was our main target because the purpose of adding nanoparticles to our working fluid is to enhance its rate of heat transfer and to make it useful for the practical purpose.
- Shear stress drops for escalating volume fraction of CNTs while it accelerates for accelerating fluid parameter / Weissenberg number. MWCNTs profiles are elevating than SWCNTs.
- Heat transfer rate falls for rising radiative heat transfer parameter or heat source/sink parameter or thermal slip parameter.
- Skin friction drops for elevating aligned Lorentzian force or CNT volume fraction. MWCNTs dominate SWCNTs.

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