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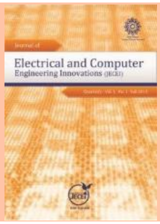

Design of a Novel Framework to Control Nonlinear Affine Systems Based on Fast Terminal Sliding-Mode Controller

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ABSTRACT

In this paper, a novel approach for finite-time stabilization of uncertain affine systems is proposed. In the proposed approach, a fast terminal sliding mode (FTSM) controller is designed, based on the input-output feedback linearization of the nonlinear system with considering its internal dynamics. One of the main advantages of the proposed approach is that only the outputs and external states of the system should be measured. Moreover, in order to realize finite-time convergence of the output variables, a set of switching manifolds with a recursive procedure is utilized. Finally, robust stability and efficacy of the proposed control law are shown through computer simulations.

1. Introduction

The sliding mode control (SMC), is a famous method in nonlinear control field. The main advantages of SMC are its robustness against parameter variations and external disturbances and its simple implementation [1, 2]. However, a sliding mode controller only guarantees asymptotic convergence of the state variables on sliding surface [3, 4]. Therefore, the states of the system cannot reach an equilibrium point in a finite time. While, finite-time convergence of the state variables is necessary for many practical systems such as guidance problem and robot manipulators. In recent decades, a terminal sliding mode (TSM) method has been developed to realize finite-time stabilizing of the state variables on a sliding surface [5, 6]. The literature on TSM shows a variety of approaches. In [7, 8], a finite-time disturbance observer based on NTSMC has been developed for flexible air-breathing hypersonic vehicle and active permanent magnet linear motors, respectively. A research branch has proposed the TSM controller for applicant systems, for instance in [9], robust TSM for the lateral motion of under-actuated autonomous underwater vehicles has been developed and in [10, 11], the TSM method has been proposed for the control of hovering and landing of asteroids and induction motors. Some other research works have focused on transforming the dynamic model of a system into its corresponding canonical form.

The TSM controller is applicable in the case of nonlinear systems with special structures. For instance, in [12], first a nonholonomic system is converted into a chain form by a particular transformation; then, the FTSM controller is applied to it. Also, in [13], a pure feedback form has been considered for the nonlinear system and then, it is transformed to the canonical form and finally, the FTSM controller is designed. Indeed, design of a FTSM controller is straightforward for a canonical dynamical form [14, 15]. However, one of the challenges in design procedure of a FTSM controller is transformation of the system dynamics to a canonical form.

In the last decade, an approach has been proposed to transform a nonlinear system to the canonical form using the input-output feedback linearization method [16]. However, this method is applicable only for systems with a relative degree of n without internal dynamics and it is necessary to measure all the state variables. This paper considers finite-time stabilizing of nonlinear systems in the presence of model uncertainties and external disturbances.

The main result of this paper is an extension of the FTSM technique to design a robust controller based on the input-output feedback linearization with considering the internal dynamic of nonlinear systems. First, based on the normal form representation of the system, the system is divided into two parts, which are called the internal and the external parts. Then, the external part of the system is transformed to a canonical form using the input-output feedback linearization method.

It is worth noting that, this approach is different from previous versions of FTSM. In previous versions, it is required to measure all the state variables of the system; whereas, in the proposed method only the outputs and the external states should be measured. Also, in order to ensure finite-time convergence of the output variables, a set of sliding surfaces with recursive procedure are utilized.

This paper is organized as follows: First, in Section 2, the dynamical model of the system and some preliminaries are introduced. In Section 3, the proposed FTSM controller is designed based on the input-output feedback linearization. Section 4 represents the efficacy of the proposed method using computer simulations. Finally, the conclusion of the paper are presented in Section 5.

2. PRELIMINARIES

Consider a nonlinear dynamical system as follows:

$$\begin{aligned} \dot{x} &= f(x) + g(x)\big(u + d(t,x)\big) \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $f(x)$, $g(x)$ are the nonlinear smooth functions in a domain $D \subset R^n$ and $u \in R$ is the scalar control input. Also, $h(.)$ is a scalar smooth field on R^n and $d(t,x)$ represents the bounded disturbances of the system. In order to apply TSMC technique to system (1), the following conversion is used to transform dynamical system (1) into the normal form:

$$\boldsymbol{z} = \begin{bmatrix} f_1(x) \\ M \\ f_{n-r}(x) \\ ---- \\ h(x) \\ L_f h(x) \\ M \\ L_f^{r-1} h(x) \end{bmatrix} = \begin{bmatrix} \eta \\ --- \\ \zeta \end{bmatrix} \tag{2}$$

where r is the relative degree of system (1)($1 \le r \le n$), and $L_f h$ depicts the Lie derivative of h with respect to f. Also, the function ϕ_i may be chosen such that the following conditions are ensured [17]:

1. Transformation (2) is a diffeomorphism function on a domain $D_0 \subset D$.
2. $\frac{\partial \phi_i}{\partial x} g(x) = 0$ for $1 \le i \le n-r$, $\forall x \in D_0$

where $\gamma(x)$, $\alpha(x)$ and $f_{in}(\eta,\zeta)$ are nonlinear functions evaluated using the system equations [13]. Also, $\eta \in R^{n-r}$, $\zeta \in R^r$ are the internal and external state vectors, respectively and the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are considered as follows:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & & \ddots & & \vdots \\ \vdots & & & 0 & 1 \\ 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \tag{3}$$

Let us define $g_e(\eta,\zeta) = \gamma(\eta,x)$ and $f_e(\eta,\zeta) = -\gamma(x)\alpha(x)$. Also, an external disturbance appears in the derivative of the last state, therefore, equations (3) may be rewritten as,

$$\begin{cases} \dot{\eta} = f_{in}(\eta,\zeta) \\ \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = \zeta_3 \\ \vdots \\ \dot{\zeta}_{r-1} = \zeta_r \\ \dot{\zeta}_r = f_e(\eta,\zeta) + g_e(\eta,\zeta)\big(u + d(t,x)\big) \\ y = \zeta_1 \end{cases} \tag{4}$$

In the differential equations (4), it is assumed that $f_e(\eta,\zeta) = f_n(\zeta) + \Delta f(\eta,\zeta)$ and

$g_e(\eta,\zeta)=g_n(\zeta)+\Delta g(\eta,\zeta)$ where $\Delta f(\eta,\zeta)$ is an uncertain term corresponding to unmodeled dynamics of the system; Also $\Delta g(\eta,\zeta)$ is the uncertainty of the input coefficient. Moreover, $f_n(\zeta)$ and the reference equation, $g_n(\zeta)$, are the nominal parts of $f_e(.)$ and $g_e(.)$ functions, respectively. Let us define $\delta(x,t)$ as the summation of uncertainties and disturbances, *i.e.*,

$$\delta(t,x,u)=\Delta f(\eta,\zeta)+\Delta g(\eta,\zeta)(u+d(t,x)) \tag{5}$$

After applying the transformation (2), system (1) may be rewritten in the normal form, as follows:

$$\begin{aligned}&\dot{\eta}=f_{in}(\eta,\zeta)\\&\dot{\zeta}=A\zeta+B\gamma(x)\left[(u+d(t,x))-\alpha(x)\right]\end{aligned} \tag{6}$$

Theorem 1 [18]: Suppose that, for a nonlinear dynamical system $\dot{x}=F(x)$, with the initial condition $x(t_0)=x_0$, there exists a continuous function $V(x)$, defined on an open neighborhood of the origin, $U\subset R^n$, such that:

1. $V(x)$ is positive definite on U
2. There exist real numbers $c>0$ and $0<\beta<1$, such

that,

$$\dot{V}(x)\leq -cV^{\beta}(x),\quad \forall x\in U \tag{7}$$

Then, there exists an area $U_0\subseteq U$ such that any $V(x)$, originated from U_0 will reach $V(x)=0$ in a finite time.

For the Proof, See [18].

3. Controller Design

In order to implement the FTSM controller, the conversion (2) is applied to system (1). So that, the external dynamics of the system is transformed to the canonical form, as follows:

$$\begin{cases}\dot{\zeta}_1=\zeta_2\\ \dot{\zeta}_2=\zeta_3\\ \quad\vdots\\ \dot{\zeta}_{r-1}=\zeta_r\\ \dot{\zeta}_r=y^{(r)}=f_e(\eta,\zeta)+g_e(\eta,\zeta)(u+d(t,x))\\ \quad=f_n(\zeta)+g_n(\zeta)u+\delta(t,\eta,\zeta)\end{cases} \tag{8}$$

As it is seen, the state variables η as well as the state variables ζ and the input variable u are appeared in equations (8). If the state variables η are not measureable, then implementation of the FTSM controller is not possible. Now, the state variables η are considered as disturbances in the external dynamics (8). Then, using an upper bound for these state variables, an FTSM controller may be designed.

Remark 1: Since, in the design procedure the internal state variables η are considered as disturbances in the external dynamics, the internal dynamics should be bounded. Therefore, the internal dynamics should be stable.

In this paper, in order to design the control law for tracking, the following recursive sliding surfaces are considered in which the number of sliding surfaces is equal to the relative degree of the system:

$$\begin{cases}S_0=\zeta_1-y_d=e\\ S_1=\dot{S}_0+\alpha_1 S_0+\beta_1 S_0^{\frac{q_0}{p_0}}\\ S_2=\dot{S}_1+\alpha_2 S_1+\beta_2 S_1^{\frac{q_1}{p_1}}\\ \vdots \qquad \vdots\\ S_{r-1}=\dot{S}_{r-2}+\alpha_{r-1}S_{r-2}+\beta_{r-1}S_{r-2}^{\frac{q_{r-2}}{p_{r-2}}}\end{cases} \tag{9}$$

where y_d is the desired output, $\alpha_{i+1},\beta_{i+1}>0$, and the parameters p_i and q_i are odd positive integers for $i=0,\ldots,r-2$, where $(q_i<p_i)$.

Remark 2: The sliding surface is equal to

$$S=(\zeta_1-y_d)+\int_0^{\infty}\left(-\varphi(\zeta_1-y_d)-\gamma(\zeta_1-y_d)^{\frac{q}{p}}\right)dt \tag{10}$$

when the relative degree is equal to one.

The following theorem proposes a stabilizing control law for system (8):

Theorem 2: Consider the dynamical system (8). Then, the following control law will stabilize the system in a finite time:

$$u=-\frac{1}{g_n(\zeta)}\left[F_n(\zeta)-y_d^{(r)}+\sum_{k=0}^{r-2}\alpha_{k+1}S_k^{(r-k-1)}+\sum_{k=0}^{r-2}\beta_{k+1}\frac{d^{r-k-1}}{dt^{r-k-1}}S_k^{\frac{q_k}{p_k}}+\varphi S_{r-1}+\gamma S_{r-1}^{\frac{q}{p}}\right] \tag{11}$$

where $\alpha_k,\beta_k,\varphi>0$ and p, q and γ are degrees of freedom. Also $S_k^{(r-k-1)}$ means $\frac{d^{r-k-1}}{dt^{r-k-1}}S_k$.

Proof: Consider the Lyapunov function $V=\frac{1}{2}S_{r-1}^2$. Using equations (9), the time derivative of V is as,

$$\begin{aligned}\dot{V}&=S_{r-1}\dot{S}_{r-1}\\&=S_{r-1}\left(\ddot{S}_{r-2}+\alpha_{r-1}\dot{S}_{r-2}+\beta_{r-1}\frac{d}{dt}S_{r-2}^{\frac{q_{r-2}}{p_{r-2}}}\right)\end{aligned} \tag{12}$$

Then, by substituting the derivatives of S_i's, step by step, the following equation is achieved:

$$\dot{S}_{r-1}=S_0^{(r)}+\sum_{k=0}^{r-2}\alpha_{k+1}S_k^{(r-k-1)}+\sum_{k=0}^{r-2}\beta_{k+1}\frac{d^{r-k-1}}{dt^{r-k-1}}S_k^{\frac{q_k}{p_k}} \tag{13}$$

On the other hand, $S_0^{(r)}=\zeta_1^{(r)}$, therefore,

$$\begin{cases}\dot{S}_{r-1}=\zeta_1^{(r)}+\sum\limits_{k=0}^{r-2}\left(\alpha_{k+1}S_k^{(r-k-1)}+\right.\\ \left.\beta_{k+1}\frac{d^{r-k-1}}{dt^{r-k-1}}S_k^{\frac{q_k}{p_k}}\right)=F_\varsigma(\zeta)+g_n(\zeta)u-y_d^{(r)}+\\ \delta(t,\eta,\zeta)+\sum\limits_{k=0}^{r-2}\alpha_{k+1}S_k^{(r-k-1)}\beta_{k+1}\frac{d^{r-k-1}}{dt^{r-k-1}}S_k^{\frac{q_k}{p_k}}\end{cases} \tag{14}$$

Now, by substituting the control signal (11) into (14) and doing some simplifications, the following equation is achieved:

$$\dot{S}_{r-1}=-\varphi S_{r-1}-\gamma S_{r-1}^{\frac{q}{p}}+\delta(t,\eta,\zeta) \tag{15}$$

Then, $\dot{S}_{r-1}$ is substituted in $\dot{V}$ as follow:

$$\dot{V}=S_{r-1}\dot{S}_{r-1}=-\varphi S_{r-1}^2-\gamma S_{r-1}^{\frac{p+q}{p}}+S_{r-1}\delta(t,\eta,\zeta) \tag{16}$$

where the perturbation term $\delta(t,\eta,\zeta)$ is unknown. However, some other information about it, like an upper bound on $\delta(t,\eta,\zeta)$, is known. Now, by inserting the control law (11) into (6), we obtain,

$$\delta(t,x,u)=\Delta f(\eta,\zeta)-\frac{\Delta g(\eta,\zeta)}{g(\eta,\zeta)}\left[F_n(\zeta)+\sum_{k=0}^{r-2}\alpha_k S_k^{(r-k-1)}+\sum_{k=0}^{r-2}\beta_k\frac{d^{r-k-1}}{dt^{r-k-1}}S_k^{\frac{q_k}{p_k}}+\varphi S_{r-1}+\gamma S_{r-1}^{\frac{q}{p}}\right]+\Delta g(\eta,\zeta)d(t,x) \tag{17}$$

According to [13], we may derive,

$$\left|-\frac{\Delta g(\eta,\zeta)}{g(\zeta)}\left(\sum_{k=0}^{r-2}\alpha_k S_k^{(r-k-1)}+\ldots\sum_{k=0}^{r-2}\beta_k\frac{d^{r-k-1}}{dt^{r-k-1}}S_k^{\frac{q_k}{p_k}}+\varphi S_{r-1}+\gamma S_{r-1}^{\frac{q}{p}}\right)\right|\le L \tag{18}$$

Also, suppose that the following term is bounded to ρ_{1_0}, *i.e.*,

$$\left|\Delta f(\eta,\zeta)-\frac{\Delta g(\eta,\zeta)}{g(\zeta)}F_n(\zeta)+\Delta g(\eta,\zeta)d(t,x)\right|\le\rho_{1_0}(t,\zeta) \tag{19}$$

Hence, $\delta(t,\eta,\zeta)$ is assumed to satisfy the following inequality

$$\left|\delta(t,\eta,\zeta)\right|<\rho_{1_0}(t,\zeta)+L,\forall(t,\eta,\zeta)\in[0,\infty)\times D\times R \tag{20}$$

where $\rho_{1_0}:[0,\infty)\times D\to R$ is a positive continuous function.

$$\begin{aligned}\dot{V}&\le-\varphi S_{r-1}^2-\gamma S_{r-1}^{\frac{p+q}{p}}+\left|S_{r-1}\right|\left(\rho_{1_0}(t,\zeta)+L\right)\\&\le-\gamma S_{r-1}^{\frac{p+q}{p}}+\left|S_{r-1}\right|\left(\rho_{1_0}(t,\zeta)+L\right)\end{aligned} \tag{21}$$

If an appropriate γ is chosen such that the following condition is satisfied, the FTSM controller results a negative definite $\dot{V}$ which leads to asymptotic stability of system (8):

$$\gamma>\left|\frac{1}{S_{r-1}^{q/p}}\right|\left(\rho_{1_0}(t,\zeta)+L\right)\Rightarrow\gamma>\left|\frac{1}{S_{r-1}^{q/p}}\right|\left(\rho_{1_0}(t,\zeta)+L\right) \tag{22}$$

If γ has the above condition and $\varepsilon=\inf\left(\gamma-\left|\frac{1}{S_{r-1}^{q/p}}\right|\left(\rho_{1_0}(t,\zeta)+L\right)\right)$ hence, inequality (21) may be rewritten as follows:

$$\dot{V} \le -\varepsilon\left|S_{r-1}\right| \le \sqrt{2}\left(-\varepsilon\right)V^{0.5} \tag{23}$$

Furthermore, according to the theorem 1, we can reach finite-time stability, if ε has a positive value. Consequently, according to the terminal attractor $\dot{S}_{r\text{-}1} = -\varphi S_{r\text{-}1} - \gamma S_{r\text{-}1}^{q/p}$ in a finite time, the states will reach the sliding manifold $S_{r-1} = 0$ [18]. According to (9), it can be easily shown that when $S_{r-1} = 0$, the sliding surface S_{r-2} will converge to the origin in a finite time, and this continues for $S_{r-1},\ldots,S_1$ and S_0. Therefore, finite-time stability of system (8) with the proposed control law is proved. As a result, the outputs of the system will converge to the origin in a finite time.

Remark3: In order to solve singularity problem, the surfaces should converge to zero sequentially from $k = r-2$ to $k = 0$, if

$$\frac{q_k}{p_k} > \frac{r-k-1}{r-k}, \quad \forall k = r-2,\ldots,0 \tag{24}$$

Then, sliding surfaces converge to zero sequentially from $k = r-2$ to $k = 0$ with no occurring singularity.

4. Computer Simulations

In this section, computer simulations are performed to illustrate the effectiveness of the proposed control law (11) in finite-time stabilization of two examples. The external disturbance $d(t)$ is considered as: $d(t) = 2u(t-3) + 2\sin(0.8\pi t)$

4.1. Stabilization Example

Consider the following uncertain nonlinear system [13]:

$$\begin{cases} \dot{x}_1 = -x_1 + \dfrac{2+x_3^2}{1+x_3^2}u \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_1x_3 + u + d(t) \\ y = x_2 \end{cases} \tag{25}$$

where $x_i \in R$ (for $i = 1,..,3$), $u \in R$ and the relative degree of the system is $r = 2$. Thus, the normal form of the system is as follows [13]:

$$\begin{cases} \dot{\eta} = \left(-\eta + \zeta_2 + \tan^{-1}\zeta_2\right)\left(1 + \dfrac{2+\zeta_2^2}{1+\zeta_2^2}\zeta_2\right) \\ \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = \left(-\eta + \zeta_2 + \tan^{-1}\zeta_2\right)\zeta_2 + u \\ y = \zeta_1 \end{cases} \tag{26}$$

where $\eta = x_1$ and $(\zeta_1,\zeta_2) = (x_2,x_3)$. The terminal sliding manifolds S_0, S_1 are designed as,

$$\begin{aligned} S_0 &= \zeta_1 \\ S_1 &= \zeta_2 + \zeta_1 + \zeta_1^{\frac{3}{5}} \end{aligned} \tag{27}$$

where, the parameters are selected as $\alpha_0, \beta_0 = 1$ and $q_0 = 3, p_0 = 5$. According to the theorem 2, the terminal sliding mode controller will be as follows:

$$u = -\left(-a\zeta_2 + \left(\zeta_2 + \tan^{-1}\zeta_2\right)\zeta_2 + \frac{3}{5}\zeta_2\zeta_1^{-\frac{2}{5}} + \gamma S_1^{\frac{3}{9}} + \varphi S_1\right) \tag{28}$$

In the simulations, the control and surface parameters are $q = 1, p = 3$ and $a = -0.5, \gamma = 10, \varphi = 5$, respectively.

The time responses of external states (x_2, x_3) and the internal state x_1, are illustrated in Fig. 1. It is seen that the external states of the system has been reached to zero in a finite time and the internal state has a stable behavior.

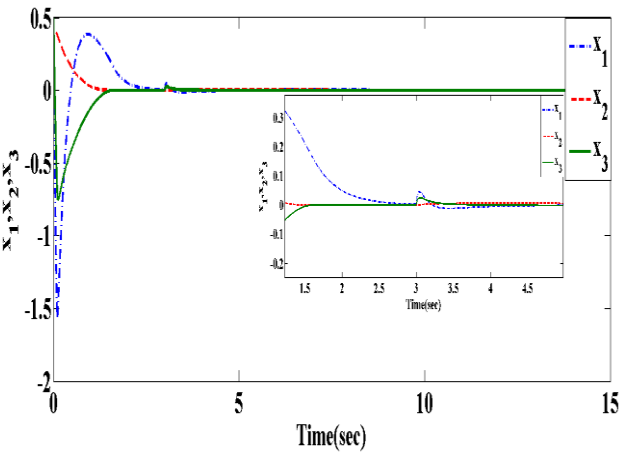

Figure 1: Time responses of the state variables.

Also, the time responses of the sliding surfaces and the control signal are shown in Figs. 2, 3. As it is seen, in this case, S_0 and S_1 converge to origin in a finite time and FTSM controller is robust to the bounded system uncertainties and disturbances.

4.2. Tracking Example

The second example, is used to evaluate the performance of the proposed method. Consider the following system:

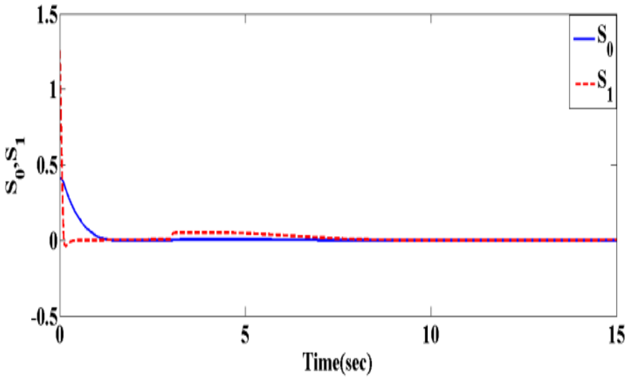

Figure 2: The time responses of the sliding surfaces.

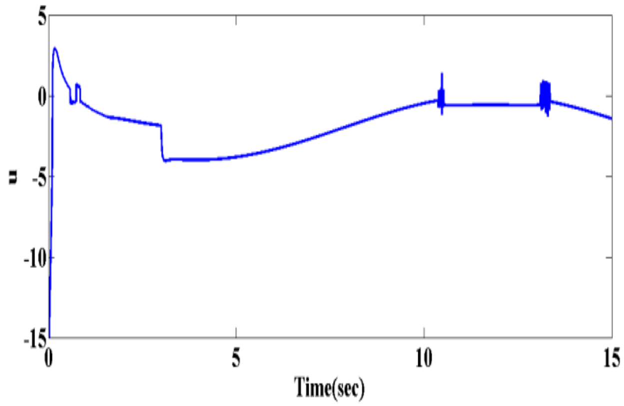

Figure 3: The time response of the control signal, in a closed-loop system.

$$\begin{cases} \dot{x}_1 = -x_1 + 0.5x_2^2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_1 x_3 + x_1 u + d(t,x) \\ y = x_2 \end{cases} \tag{29}$$

It can be easily checked that the relative degree of the system is r = 2. It is desirable that ζ_1 tracks the reference y_d.

The terminal sliding manifolds S_0, S_1 designed as,

$$\begin{cases} e = \zeta_1 - y_d \\ y_d = \sin t \end{cases} , \quad \begin{cases} S_0 = e \\ S_1 = \dot{e} + e + 3e^{\frac{7}{9}} \end{cases} \tag{30}$$

According to the theorem 2, the terminal sliding mode controller will be as follows:

$$u = -4\left(0.25x_3 + \sin t + \dot{e} + \frac{21}{9}\dot{e}e^{-\frac{2}{9}} + \varphi S_1 + \gamma S_1^{\frac{3}{5}}\right) \tag{31}$$

The external state variables are as, $(x_2, x_3) = (\zeta_1, \zeta_2)$ and $x_1 = \eta$. For simulations, the control values are $\gamma = 4, \varphi = 1.5$. Also, the external disturbance $d(t)$ is considered as a $\sin t$. The simulation results are as follow:

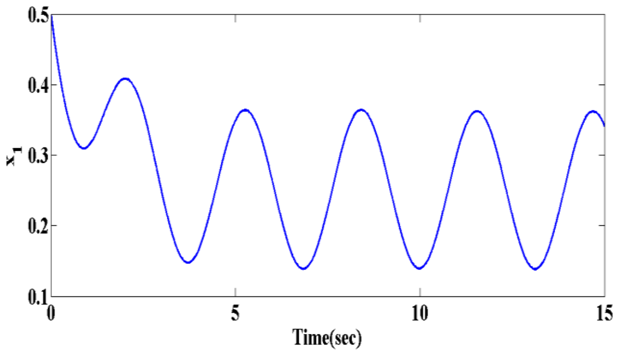

Figure 4: Time response of the internal state.

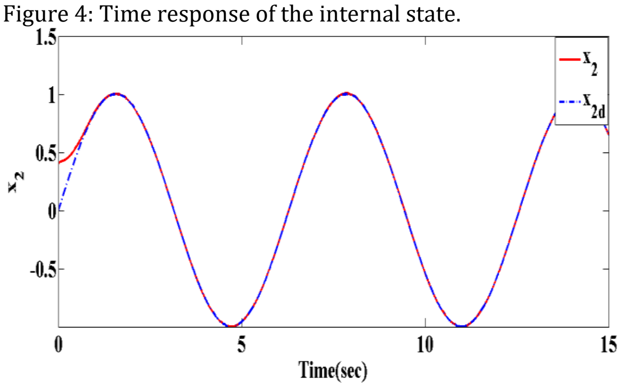

Figure 5: Time response of the external state variable (X2).

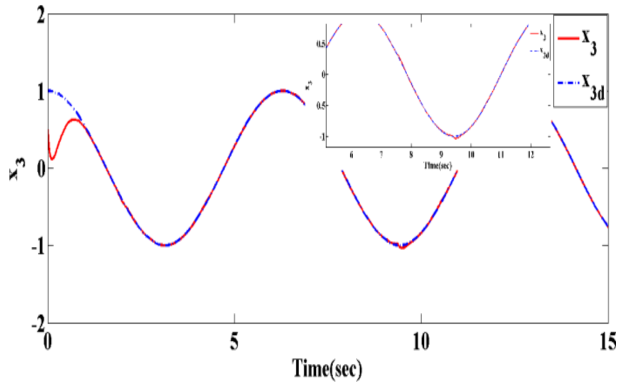

Figure 6: Time response of the external state variable (X3).

As it is shown in Figs. (4-6), the state variables converge to the origin in a finite time and the controller is robust to the uncertainty. In Fig. 7, the sliding surfaces are depicted. In this example, relative degree is 2 so that, we have 2 sliding surfaces.

As illustrated in this figure, first S_{r-1} reaches to zero and then $S_{r-2},\ldots,S_0$ converge to the origin. Also, as it is seen in Fig. 8, the control signal has a bounded value.

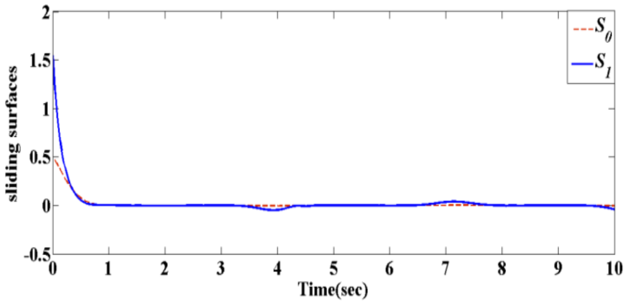

Figure 7: The time response of sliding surface.

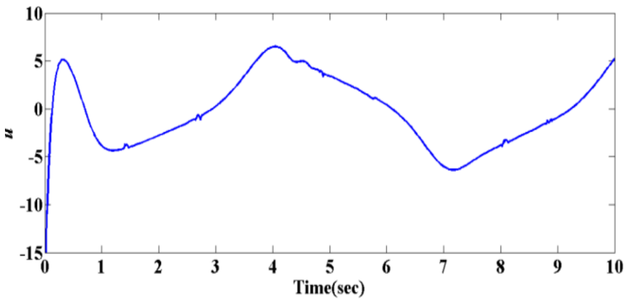

Figure 8: The time response of the control input.

4.3 Fighter Aircraft

A simplified model of a fighter aircraft is extracted from the nonlinear model of six degrees of freedom which may be considered as follows [19]:

$$\dot{v} = 1.8254\cos\left(0.0175\left(\alpha + 11.3404\right)\right) - 1.9821 \times 10^{-3}\left(0.0886 + 1.75\times 10^{-2}\alpha\right)v^{2},$$

$$\dot{\alpha} = -0.5923\alpha + 50.7296q - 0.1145u$$

$$\dot{q} = -0.0178\alpha - 0.3636q - 0.0676u$$

where the airspeed v (m/s), angle of attack α (deg), and pitch angular rate q (rad/s) are state variables and the deflection of elevator u (deg) is the control input of the model. The main limitations of this control problem are $|u| \le 10$ and the limited overshoot.

The control object is set the angle of attack to a reference attitude 5°. According to the dynamical model, the relative degree is equal to 1 .

It is clear from remark 2 that the sliding surface is $s = \left(\alpha - 5\right)$. The control signal is obtained as follow:

$$\dot{s} = \dot{\alpha} + \left(-\varphi\left(\alpha - 5\right) - \gamma\left(\alpha - 5\right)^{\frac{q}{p}}\right)$$

$$u = \frac{1}{0.1145}\left(-0.5923\alpha + 50.7296q\right.$$

$$\left. +0.5\left(\alpha\text{-}5\right) + 1.5\left(\alpha\text{-}5\right)^{5/27}\right) \tag{32}$$

As shown in Fig. 10, the output signal is tracked the desired angle in a finite time. Also, the control signal is bounded.

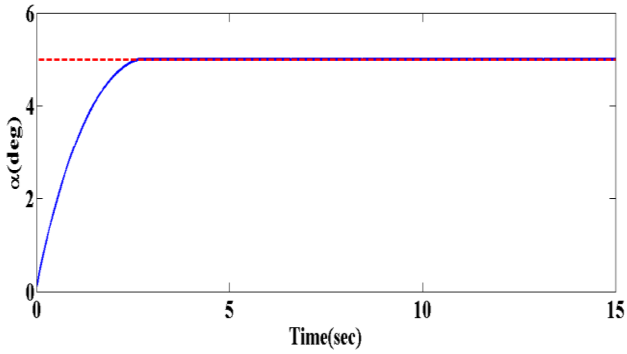

Figure 9: The time response of the output.

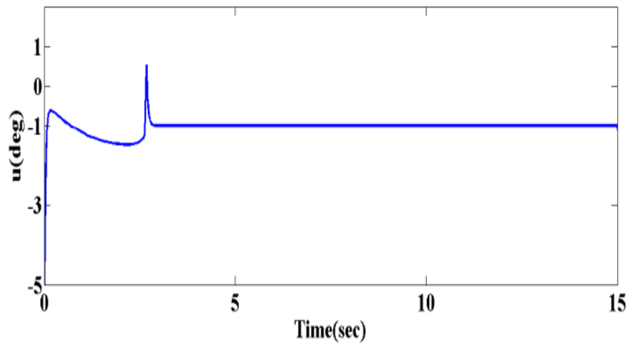

Figure 10: The time response of the control input.

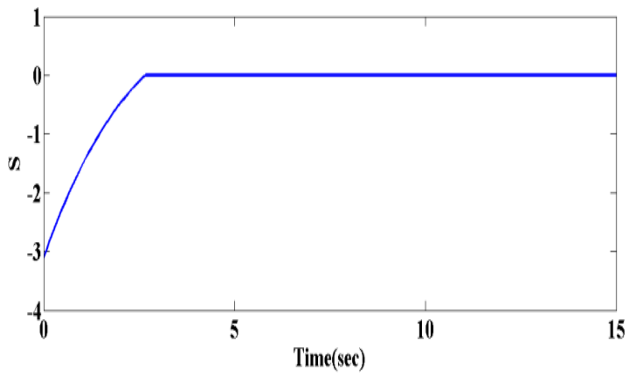

Figure 11: The time response of the surface.

5. Conclusion

In this paper, a new class of FTSM controllers is developed in order to stabilize nonlinear systems in a finite-time. Firstly, the model of the system was transformed to the normal form. In this new form, the state variables of internal dynamic were supposed as disturbances in the external dynamics. Then, based on input-output feedback linearization, the FTSM controller was designed to realize finite-time stability of the output variables. An important benefit of this method is that all the state variables are not necessary to be measured in designing of the control law. Finally, efficacy of the proposed method in the finite-time stabilization of output variables and robustness of control law against the external disturbances were shown using computer simulations.

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