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Construction of solitary solution and compacton-like solution by the variational iteration method using He's polynomials

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Abstract

Variational Iteration method using He's polynomials can be used to construct solitary solution and compacton-like solution for nonlinear dispersive equatioons. The chosen initial solution can be determined in compacton-like form or in solitary form with some compacton-like or solitary forms with some unknown parameters, which can be determined in the solution procedure. The compacton-like solution and solitary solution can be converted into each other.

1. Introduction

It is well known that many important dynamics processes can be described by specific nonlinear partial differential equations. But, because of the nonlinear part that exists in most of these equations, a limited number of them have precise analytical solution and most of them do not have any analytical solutions. So, these nonlinear equations should be solved using other methods. In recent decades, numerical calculation methods have been good means of analyzing these equations. But, in the numerical method, stability and convergence should be considered in order to avoid divergent or inappropriate results. Therefor, most scientists believe that the combination of numerical and semi-exact analytical methods can also end with useful results. Some of these useful methods are: Variational Iteration Method (VIM) [1, 2, 3, 4, 5], Homotopy Perturbation Method (HPM) [6, 7, 8], Homotopy Analysis Method (HAM) [9] and Adomian Decomposition Method (ADM) [10]. One important equation that has been widely used in physics is KdV-like, K(p,q), equation which was introduced in the 1990s by Rosenau [11, 12]. This equation arises in the process of understanding the role of nonlinear dispersion and in the formation of structures like liquid drops and exhibits compactons: solitons with compact support [11]. kdv-like equation has the following form:

$$u_t \pm a(u^p)_x + (u^q)_{xxx} = 0. \qquad p, q > 1, \tag{1}$$

where p and q are integers. The (+) case is

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known as the focusing branch and the (-) case as the defocusing branch. In this paper, the nonlinear dispersive equation k(p,q) with the following form was considered

$$u_t + a(u^p)_x + (u^q)_{xxx} = 0. \quad (2)$$

Because of the ability of Eq. (2) in modeling different phenomena, especially in physics, like the formation of liquid drops and theory of water waves in shallow channels [11, 13], it has been widely studied by many authors via various methods [14, 15, 16, 17, 18]. In this paper, Variational Iteration Method using He's polynomials (VIMHP) was applied to find the solitary solution and compacton-like solution of this equation and it was demonstrated that the VIMHP could find the unknown parameters more easily than other methods.

2. Basic idea of variational iteration method using He's polynomials

To introduce VIMHP, VIM and HPM are required to be known.

2.1. Variational iteration method

The Variational Iteration Method, which provides an analytical approximate solution, has been applied to various nonlinear problems [19, 20]. In this section, an alternative approach of VIM is presented, which can be implemented in a reliable and efficient way to handle the nonlinear differential equation,

$$L[u(r)] + N[u(r)] = g(r), \quad r > 0, \quad (3)$$

where $L = \frac{d^m}{dr^m}, m \in N$ is a linear operator, N is a nonlinear operator and $g(r)$ is the source *non-homogeneous* term, subjected to the initial conditions,

$$u^{(k)}(0) = c_k,\ k = 0,1,2,\cdots,m-1. \quad (4)$$

where c_k is a real number. According to the He's Variational Iteration Method, a correction functional can be constructed for (3) as follows:

$$u_{n+1}(r) = u_n(r) + \int_0^r \lambda(\tau) \begin{Bmatrix} L u_n(\tau) + N \tilde{u}_n(\tau) \\ -g(\tau) \end{Bmatrix} d\tau,$$
$$n \geq 0,$$

where λ is a general Lagrangian multiplier which can be optimally identified via variational theory. As can be seen, because of the existence of nonlinear part in Eq. (3), it is not possible to exactly find the optimal value of Lagrange multiplier. So, it is necessary to consider a limitation on the nonlinear part causing this part to be ignored. Therefore, $\tilde{u}_n$ is allocated to show the nonlinear part which has a special property. It has restricted variation, i.e. $\delta\tilde{u}_n = 0$. making the above functional stationary considering that

$$\delta\tilde{u}_n = 0,$$
$$\delta u_{n+1}(r) = \delta u_n(r) + \delta\int_0^r \lambda(\tau)\ \{Lu_n(\tau) - g(\tau)\} d\tau,$$

yields the following Lagrange multipliers,

$$\lambda = -1 \quad for \quad m = 1,$$
$$\lambda = \tau - r, \quad for \quad m = 2,$$

and in general,

$$\lambda = \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}, \quad for \ m \geq 1.$$

Because for $m = 1,$, the following can be written:

$$\delta u_{n+1}(r) = \delta u_n(r) + \delta\int_0^r \lambda(\tau)(u_{n\tau}(\tau) - g(\tau))d\tau,$$

which is equal to

$$\delta u_{n+1}(r) = \delta u_n(r) + \int_0^r \delta\{\lambda\ (\tau)\ (u_{n\tau}(\tau)\}\ d\tau,$$

or

$$\delta u_{n+1}(r) = \delta u_n(r) + \int_0^r \lambda(\tau)\, \delta u_{n\tau}(\tau) d\tau.$$

Using integration by parts would result in

$$\delta u_{n+1}(r) = \delta u_n(r) + \lambda(\tau)\delta u_n(\tau)|_{\tau=r} - \int_0^r \lambda'(\tau)\, \delta u_n(\tau)\, d\tau.$$

So,

$$\begin{cases} 1+\lambda(\tau) = 0\,|_{\tau=r}, \\ \lambda'(\tau) = 0\,|\, r, \end{cases}$$

which results in $\lambda(\tau) = -1$, And, for m=2, the following can be written:

$$\delta u_{n+1}(r) = \delta u_n(r) + \delta\int_0^r \lambda(\tau)\, (u_{n\tau\tau}(\tau) - g(\tau))d\tau,$$

as equal to

$$\delta u_{n+1}(r) = \delta u_n(r) + \int_0^r \delta\{\lambda(\tau)\, (u_{n\tau\tau}(\tau)\}\, d\tau,$$

or

$$\delta u_{n+1}(r) = \delta u_n(r) + \int_0^r \lambda(\tau)\, \delta u_{n\tau\tau}(\tau) d\tau.$$

Using integration by parts would result in

$$\delta u_{n+1}(r) = \delta u_n(r) + \lambda(\tau)\, \delta u_{n\tau}(\tau)|_{\tau=r} - \lambda'(\tau)\, \delta u_n(\tau)|_{\tau=r} + \int_0^r \lambda''(\tau)\, \delta u_n(\tau)\, d\tau.$$

So,

$$\begin{cases} 1-\lambda'(\tau) = 0\,|_{\tau=r}, \\ \lambda(\tau) = 0\,|_{\tau=r}, \\ \lambda''(\tau) = 0\,|_{\tau=r} \end{cases}$$

which would result in $\lambda(\tau) = \tau - r$.

The successive approximations $u_n(r), n \geq 0$ of the solution $u(r)$ would be readily obtained upon using the obtained Lagrange multiplier and by any selective function u_0. Consequently, the exact solution might be obtained using

$$u(r) = \lim_{n\to\infty} u_n(r).$$

2.2. Homotopy perturbation method

To illustrate the basic ideas of HPM, the nonlinear boundary value problem should be considered

$$L[u(r)] + N[u(r)] = g(r), \quad r \in \Omega, \tag{5}$$

with the boundary condition of

$$B(u, \frac{\partial u}{\partial n}) = 0, \qquad r \in \Gamma, \tag{6}$$

where L is a linear operator, N is a nonlinear operator and $g(r)$ is the source *non-homogeneous* term, B is a boundary operator and Γ is the boundary of the domain Ω. He's Homotopy Perturbation Technique [21, 22, 23, 24] defines a homotopy $v(r;p): R \times [0,1] \to R$ which_satisfies

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[L(v) + N(v) - g(r)] = 0, \tag{7}$$

In Eq. (7), $p \in [0,1]$ is an embedding parameter and $u_0(r)$ is the first approximation that satisfies the boundary conditions. It follows from Eq. (7) that

$$H(v,0) = L(v) - L(u_0) = 0 \;\; and \qquad H(v,1) = L(v) + N(v) - g(r) = 0, \tag{8}$$

Thus, the changing process of p from 0 to 1 is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v)-L(u_0)$ and $L(v)+N(v)-g(r)$ are called homotopic. Here, the embedding parameter p is introduced much more naturally, unaffected by artificial factors. Due to the fact that $0 \le p \le 1$, the embedding parameter can be considered as a small parameter. So, it is very natural to assume that the solution of (5) and (6) can be expressed as a series of the power of p

$$v = v_0 + p\,v_1 + p^2 v_2 + \ldots, \tag{9}$$

Substituting $p=1$ in (9) yields the approximate solution of (5) and (6) as follows

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots. \tag{10}$$

The convergence of series (10) was discussed in [25]. The method considers the nonlinear term $N[v]$ as

$$N(v) = \sum_{i=0}^{+\infty} p^i H_i = H_0 + pH_1 + p^2 H_2 + \cdots$$

where H_n s are the so-called He's polynomials [26], which can be calculated using the following formula

$$H_n(v_0, v_1, \cdots, v_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left(N\left(\sum_{i=0}^{n} p^i v_i\right)\right)_{p=0},\ n=0,1,\cdots$$

2.3. Variational iteration method using He's polynomials

To illustrate the basic idea of the Variational Iteration Method using He's polynomials, the following general differential equation can be considered:

$$L[u(r)] + N[u(r)] = g(r), \tag{11}$$

where L is a linear operator, N is a nonlinear operator and $g(r)$ is the source *non-homogeneous* term. According to VIM, for $n \ge 0$, a correct functional can be constructed as follows:

$$u_{n+1}(r) = u_n(r) + \int_0^r \lambda(\tau)\left\{\frac{d^m}{d\tau^m}u_n(\tau) + N[\tilde{u}_n(\tau)] - g(\tau)\right\}d\tau, \tag{12}$$

where $\lambda(\tau) = \dfrac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}$. Now, applying a series of the power of p and then using He's polynomials, the following can be obtained:

$$\sum_{n=0}^{\infty} p^n v_n = u_0 + p\int_0^r \lambda(\tau)\begin{bmatrix}\frac{d^m}{dr^m}(\sum_{n=0}^{\infty} p^n v_n(\tau)) \\ +N(\sum_{n=0}^{\infty} p^n v_n(\tau)) - g(\tau)\end{bmatrix}d\tau$$
$$= u_0 + p\int_0^r \lambda(\tau)\begin{bmatrix}\frac{d^m}{dr^m}(\sum_{n=0}^{\infty} p^n v_n(\tau)) + \\ \sum_{n=0}^{\infty} p^n H_n - g(\tau)\end{bmatrix}d\tau, \tag{13}$$

which is the modified variational iteration method using He's polynomials [27, 28]. Now, equating coefficients of like powers of p would result in

$$p^0: v_0 = u_0,$$

$$p^1: v_1 = \int_0^r \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}\begin{bmatrix}\frac{d^m}{dr^m}(v_0(\tau)) \\ +H_0(v_0) - g(\tau)\end{bmatrix}d\tau,$$

$$p^2: v_2 = \int_0^r \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}\begin{bmatrix}\frac{d^m}{dr^m}(v_1(\tau)) \\ +H_1(v_0, v_1)\end{bmatrix}d\tau,$$

$$\vdots$$

$$p^j : v_j = \int_0^r \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)} \begin{bmatrix} \frac{d^m}{d r^m}(v_{j-1}(\tau)) \\ + H_{j-1}(v_0, v_1, \cdots ; v_{j-1}) \end{bmatrix} d\tau,$$

$$\vdots \tag{14}$$

Therefore, the approximated solutions of (11) can be obtained as follows:

$$u = v_0 + v_1 + v_2 + \dots. \tag{15}$$

The zeroth (initial) approximation $v_0 = u_0$ can be freely chosen if the initial and boundary conditions of the problem are satisfied. The success of the method depends on the proper selection of the initial approximation v_0. However, the initial values $u^{(k)}(0) = c_k, k = 0,1,2,\cdots,m-1$ are preferably used for the selective zeroth approximation v_0 as will be seen later. For later computation, the expression $u_n(x,t) = \sum_{i=0}^{n} v_i(x,t)$ is allowed to denote the n-term approximation to u(x,t). For more information about the VIMHP, refer to [29, 30].

3. Compacton-like solution and solitary solution by the VIMHP

To find the Compacton-like solution and solitary solution of Eq. (2) by the VIMHP, first, the correction functional of the desired equation should be made:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\tau)\,(u_{n\tau}(x,\tau) + a\tilde{u}_{nx}^p(x,\tau) + \tilde{u}_{nxxx}^q(x,\tau))d\tau, \tag{16}$$

where $\tilde{u}_n$ is considered as restricted variations, i.e. $\delta\tilde{u}_n = 0$.

To find optimal value of $\lambda(\tau)$, teh following is done

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda(\tau)(u_{n\tau}(x,\tau) + a\tilde{u}_{nx}^p(x,\tau) + \tilde{u}_{nxxx}^q(x,\tau))d\tau, \tag{17}$$

which is equal to

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \int_0^t \delta\{\lambda(\tau)\,(u_{n\tau}(x,\tau) + a\tilde{u}_{nx}^p(x,\tau) + \tilde{u}_{nxxx}^q(x,\tau))\}\,d\tau, \tag{18}$$

or

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \int_0^t \lambda(\tau)\,\delta u_{n\tau}(x,\tau)\,d\tau.$$

Using integration by parts results in

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \lambda(\tau)\,\delta u_n(x,\tau)|_{\tau=t} - \int_0^t \lambda'(\tau)\,\delta u_n(x,\tau)\,d\tau. \tag{19}$$

As is known, each approximate solution should satisfy initial conditions. So, based on the calculus of variations $\delta u_n^{(k)}(0) = 0,\ \forall k \ge 0$ and in order to obtain the optimal value of $\lambda(\tau)$, $\delta u_{n+1}(x,t) = 0$ which results in Euler equation and natural boundary condition (the stationary conditions), the following can be presented:

$$\begin{cases} 1 + \lambda(\tau) = 0\,|_{\tau=t}, \\ \lambda'(\tau) = 0\,|_{\tau=t}, \end{cases} \tag{20}$$

which results in $\lambda(\tau) = -1$.

Now, using this value of Lagrange multiplier, the VIMHP can be applied to solve Eq. (2):

$$\sum_{i=0}^{\infty} p^i\, v_i = u_0(x,t) - p\int_0^t [a\,(\sum_{i=0}^{\infty} p^i\, v_{ix})^p + (\sum_{i=0}^{\infty} p^i\, v_{ixxx})^q]\,d\tau. \tag{21}$$

Comparison of like powers of p gives:

$$p^0 : v_0(x,t) = u_0(x,t),$$

$$p^1 : v_1(x,t) = -\int_0^t (au_{0x}^p + u_{0xxx}^q) d\tau,$$
$$\vdots \quad (22)$$

And, the solution will be obtained as $u(x,t) = \sum_{i=0}^{\infty} v_i(x,t)$.

Before finding the component of the solution, it should be noted that, because of the nature of solitary and compacton-like solution, the following can be given:

$$u_{n+1}(x,t) = u_n(x,t) \quad (23)$$

and

$$\frac{\partial^k u_n}{\partial t^k} = \frac{\partial^k u_{n+1}}{\partial t^k}. \quad (24)$$

So, for $n = 0$ we obtain:

$$u_0(x,t) = u_1(x,t). \quad (25)$$

Thus we have:

$$v_0(x,t) = v_0(x,t) + v_1(x,t), \quad (26)$$

which results in $v_1(x,t) = 0.$. In this case, using VIMHP recursive formulation gives $v_i(x,t) = 0,\ \forall i \ge 1$ and $u(x,t) = \sum_{i=0}^{\infty} v_i(x,t) = u_0(x,t)$. So, to determine unknown parameter, only $u_0(x,t)$ should be substituted in Eq. (2):

$$u_{0t} + au_{0x}^p + u_{0xxx}^q = 0. \quad (27)$$

On the other hand, by having $u_0(x,t)$ and substituting it in Eq. (2), unknown parameters can be determined and the solitary and compacton-like solution can be found and there is no need to calculate their other components or derivatives. Thus, the VIMHP can overcome the difficulty arising in VIM [31] and ADM [32, 33].

As an illustrating example, $k(3,1)$: can be considered

$$u_t + u^2 u_x + u_{xxx} = 0. \quad (28)$$

To find the compacton-like solution of this equation, the initial solution can be assumed in the following form

$$u_0(x,t) = \frac{asin^2(kx + wt)}{b + csin^2(kx + wt)} \quad (29)$$

where a, b, c, k and w are unknown parameters which should be determined. Based on what stated in VIMHP, the following can be written:

$$\sum_{i=0}^{\infty} p^i v_i = u_0(x,t) + p\int_0^t \lambda\ (\tau)\ \{(\sum_{i=0}^{\infty} p^i v_i)^2 (\sum_{i=0}^{\infty} p^i v_{ix}) + (\sum_{i=0}^{\infty} p^i v_{ixxx})\}\ d\tau. \quad (30)$$

By using VIM, the optimal value of Lagrange multiplier is obtained as $\lambda(\tau) = -1$.

So,

$$\sum_{i=0}^{\infty} p^i\ v_i = u_0(x,t) - p\int_0^t \{(\sum_{i=0}^{\infty} p^i\ v_i)^2 (\sum_{i=0}^{\infty} p^i\ v_{ix}) + (\sum_{i=0}^{\infty} p^i v_{ixxx})\}\ d\tau. \quad (31)$$

By comparison of like powers of p, the solution will be obtained but, based on the result of using above-obtined VIMHP, there is no need to do time consuming calculations for finding the components of solution and unknown parameters. Hence, a majority of calculation is reduced.

By substituting $u_0(x,t)$ in Eq. (28) and using MATLAB, the followng can be presented:

$$\frac{[Acos^4(\eta) + Bcos^2(\eta) + c][2abcos(\eta)sin(\eta)]}{D} = 0 \quad (32)$$

where

$$\eta = kx + \omega t$$

$$A = wc^2 - 4k^3c^2 + ka^2$$
$$B = -16bk^3c - 2bwc - 2ka^2 - 2wc^2 - 4k^3c^2$$
$$C = -4b^2k^3 + b^2w + 8k^3c^2 + 4bk^3c + ka^2 + 2bwc + wc^2$$
$$D = c^4cos^8(kx+wt) - (4bc^3 + 4c^4)cos^6(kx+wt)$$
$$+ (6c^4 + 12bc^3 + 6b^2c^2)cos^4(kx+wt)$$
$$-(4b^3c + 12b^2c^2 + 12bc^3 + 4c^4)cos^2(kx+wt)$$
$$+ (4b^3c + 6b^2c^2 + 4bc^3 + b^4 + c^4) \quad (33)$$

For solving Eq. (32), the following is set

$$A = 0 \quad , B = 0, \quad C = 0 \quad (34)$$

Solving (34) results in obtaining

$$\begin{cases} a = \pm 2\sqrt{2}ck, \\ b = -1.5c, \\ w = -4k^3. \end{cases} \quad (35)$$

Substituting these values in the compacton-like solution lead to reading

$$u(x,t) = \frac{\pm 2\sqrt{2}kcsin^2(kx-4k^3t)}{-1.5c + csin^2(kx-4k^3t)} \quad (36)$$

or

$$u(x,t) = \frac{\pm 4\sqrt{2}ksin^2(kx-4k^3t)}{-3 + 2sin^2(kx-4k^3t)}. \quad (37)$$

Also, it can be started by a more general initial solution in the form of

$$u_0(x,t) = a + \frac{1}{c + dcos(kx+wt)}, \quad (38)$$

where a, c, d, k and w are unknown constants. Setting $\frac{\partial u_0}{\partial t} = \frac{\partial u_1}{\partial t}$ and using MATLAB result in:

$$\frac{[Ecos^4(kx+wt) + Fcos(kx+wt) + G][dsin(kx+wt)]}{H} = 0, \quad (39)$$

where

$$E = ka^2d^2 - k^3d^2 + wd^2$$
$$F = 4ck^3d + 2cwd + 2cka^2d + 2kad$$
$$G = c^4 + 4c^3dcos(kx+wt) + 6c^2d^2cos^2(kx+wt) + 4cd^3cos^3(kx+wt) + d^4cos^4(kx+wt)$$
$$H = d^4cos^4(kx+wt) + 4cd^3cos^3(kx+wt) + 6c^2d^2cos^2(kx+wt) + 4c^3dcos(kx+wt) + c^4. \quad (40)$$

The following can be given for solving Eq. (39):

$$E = 0, \qquad F = 0, \qquad G = 0. \quad (41)$$

And solving (41) yields

$$\begin{cases} w = k^3 - ka^2, \\ c = \frac{-a}{3k^2}, \\ d = \pm \frac{\sqrt{4a^2 - 6k^2}}{6k^2}, \end{cases} \quad (42)$$

Therefore, the following new compacton-like solution is obtained:

$$u(x,t) = a + \frac{1}{\frac{-a}{3k^2} \pm \frac{\sqrt{4a^2 - 6k^2}}{6k^2} cos(kx + (k^3 - ka^2)t)}, \quad (43)$$

which satisfies Eq. (28) and a is an arbitrary coefficient. If $a = -\sqrt{2}k$, is chosen, (43) can be reduced to (37). As was stated before, compacton-like solution and solitary solution can be converted into each other if $k = iK$, is chosen where K is a constant; then, (43) becomes

$$u(x,t) = a + \frac{1}{\frac{a}{3k^2} \pm \frac{\sqrt{4a^2+6k^2}}{6k^2} cosh(kx + (-k^3 - ka^2)t)}$$

$$= a + \frac{1}{I}, \quad (44)$$

where

$$I = \frac{a}{3K^2} \pm \frac{\sqrt{4a^2+6K^2}}{12K^2}[\mathbf{exp}(Kx + (-K^3 - Ka^2)t) + \mathbf{exp}(-Kx - (-K^3 - Ka^2)t)]$$

which is a solitary solution of Eq. (28).

As another case, the initial solution can be constructed in a solitary form. So, the following can be used to start with

$$u_0(x,t) = a + \frac{1}{b + c\exp(kx+wt) + d\exp(-kx-wt)}. \quad (45)$$

By the same technique that is illustrated above, $\frac{\partial u_0}{\partial t} = \frac{\partial u_1}{\partial t}$. is set. As is known and based on the obtained results using the VIMHP, there is no need to calculate $u_1(x,t)$.. Substituting $u_0(x,t)$ in Eq. (28) results in:

$$\frac{J}{[b + c\exp(kx+wt) + d\exp(-kx-wt)]^4} = 0, \quad (46)$$

where

$$J = [-c\exp(kx+wt) + d\exp(-kx-wt)][L\exp(kx+wt) + M\exp(2kx+2wt) + N + O\exp(-2kx-2wt) + P\exp(-kx-wt)]$$

$$L = 2c(wb + ka^2b + ka - 2k^3b),$$

$$M = c^2(w + ka^2 + k^3),$$

$$N = b^2(w + k^3 + ka^2) + 2cd(w + ka^2 - 11k^3) + 2kab + k,$$

$$O = d^2(w + ka^2 + k^3),$$

$$P = 2d(bw + ka^2b + ka - 2k^3b). \quad (47)$$

From (46),

$$L = 0, \quad M = 0, \quad N = 0, \quad O = 0, \quad P = 0 \quad (48)$$

Solving (48) yields

$$\begin{cases} w = -k(a^2 + k^2), \\ b = \frac{a}{3k^2}, \\ d = \frac{3k^2 + 2a^2}{72k^4 c}. \end{cases} \quad (49)$$

So, the solitary solution is

$$u(x,t) = a + \frac{1}{Q}, \quad (50)$$

where

$$Q = \frac{a}{3k^2} + c\exp(kx + (-k^3 - ka^2)t) + \frac{3k^2 + 2a^2}{72k^4}\frac{1}{c}\exp(-kx - (-k^3 - ka^2)t),$$

Hereby, a and c are arbitrary parameters when $d = c$, Eq. (50) reduces to (44). It is interesting to note that solitary solution can be converted into a compacton-like solution. Choosing $k = iK$, where K is a constant, causes (50) to become

$$u(x,t) = a + \frac{1}{R}, \quad (51)$$

where

$$R = \frac{-a}{3k^2} + (c + \frac{-3k^2 + 2a^2}{72k^4 c})cos(kx + (k^3 - ka^2)t) + (c - \frac{-3k^2 + 2a^2}{72k^4 c})isin(kx + (k^3 - ka^2)t). \quad (52)$$

In order to convert (50) into a compact form, the last term should be equal to zero. So, teh following can be written

$$c - \frac{-3k^2 + 2a^2}{72k^4 c} = 0, \quad (53)$$

Solving (53) for c and substituting it in (52) and then (51) yield the compacton-like solution

$$u(x,t) = a + \frac{1}{\frac{-a}{3k^2} \pm \frac{\sqrt{4a^2 - 6k^2} cos(kx + (k^3 - ka^2)t)}{6k^2}}$$

4. Conclusions

In this paper, the effectiveness and convenience of the VIMHP were demonstrated for solitary solution and compacton-like solution of nonlinear dispersive equations. The advantages of the suggested method included:

1-By ignoring the linear part of equation in the recursive formulation, a large amount of calculation was reduced.

2-Unknown constants could be rapidly determined in comparison with VIM and ADM.

3-Using VIMHP, there was no need to calculate $u_i(x,t), \forall i \geq 1$ for finding unknown parameters because they were equal to zero and it was only required to substitute the initial solution into original equation. So, finding the solitary solution and compacton-like solution would not be time consuming and a hard work.

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