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Research Paper

CoM-polynomials and indices of some drugs used against Monkeypox disease

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Abstract. With the rapidly developing technology, computer-aided, time-saving drug discovery without experimentation has become necessary to treat diseases. One of these is that in chemistry, by modeling chemicals with graph theory, we can predict the properties of new drugs with numerical values obtained using the structural properties of graphs. In this study, CoM-polynomials and some coindices of graphs of some drugs used in the treatment of monkeypox disease were calculated and compared numerically.

Keywords. graph theory, topological index, M-coindex, CoM-polynomials, Monkeypox drugs.
Mathematics Subject Classification (2020): $05C90$, $05C92$, $05C09$, $92E10$.

1 Introduction

Viral diseases negatively affect human life. These diseases are not only transmitted from person to person, but also from animals to people. One of these is Mpox (monkeypox). It is a disease caused by monkeypox virus, which is a zoonotic virus and a species of the Orthopoxvirus genus. This disease was first seen in a monkey in Denmark in 1958. It was first seen in a human in the Democratic Republic of the Congo in 1970. Today, this disease still poses a threat. There are two genetic variants of this virus. Clade II was seen in the 2022-2023 outbreak. After August 2024, Clade Ib began to be seen outside Africa. WHO declared Mpox a public health emergency of international concern in May 2022 and August 2024 [19]

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Mpox, there are two vaccines approved by the FDA to prevent. However, this disease outbreak is still ongoing/seen in many countries [8]. Some antivirals are used to treat this disease and new drug studies are also ongoing. Drugs used in treatment are tecovirimat, cidofovir, brincidofovir. In new studies, Aciclovir and similar have been found to be effective [16].

In rapidly changing living conditions, time and cost must be minimized for new drug discoveries. For this purpose, studies are being conducted to shed light on drug discoveries with graph theory. One of these is topological indices. Topological index is a quantitative representation/indicator of the representation of a chemical structure with vertices and edges. Topological indices are used in intermolecular QSPR/QSAR modeling and in comparing graphs of chemical structures (see detail [9]).

The first application of topological indices in chemistry was to determine some properties of alkanes [17]. Since then, numerous topological indices have been defined and have applications in chemistry. The calculation of topological indices can be difficult in some cases, to overcome this they used polynomials. One of these is the M-polynomials, which are the closed formula of the topological indices that depend on the degree.

Topological indices were studied starting from vertices, adjacent vertex pairs or adjacent edges, but later these indices were studied from non-adjacent vertex pairs and these indices were called coindices. Zaman et al. calculated the coindex of drugs used in the treatment of COVID-19 [20]. Kirmani and Ali studied the coindex of drugs that reduce cancer activity [11]. Sozen et al. studied the distance-dependent coindices [13]. Ozkan and Kara examined the coindices of some anticancer drugs [12]. Shanmukha et al. compared the coM-polynomials of anthracene molecules [15].

Kalaimathi and Balamurugan obtained QSPR modeling by calculating some indices of Cidofovir, Valacyclovir, Famciclovir, Acyclovir, Tecovirimat, and Brincidofovir drugs used in the treatment of Monkeypox disease [10]. Colakoglu et al. compared the M and NM-polynomials and indices of Cidofovir, Tecovirimat, and Brincidofovir drugs used in the treatment of Monkeypox disease [4]. William and Jayaraman studied the Sombor type indices of Cidofovir, Famciclovir, Acyclovir, Tecovirimat, Brincidofovir drugs and obtained QSPR models [18]. discussed .characterized .are derived with explicit formulas.

In this study, CoM-polynomials and coindexes of Cidofovir, Valacyclovir, Famciclovir, Acyclovir, Tecovirimat, and Brincidofovir drugs are calculated and compared numerically.

2 Preliminaries

Let $\Gamma(V(\Gamma), E(\Gamma))$ be a graph. Let d_τ denote the degree of a vertex τ in a graph Γ. Assume that

$$\varsigma_{ij} = \{\left|E_{(i,j)}(\Gamma)\right|, (d_\tau, d_q) = (i,j) \quad for \quad \tau q \in E(\Gamma)\},$$

$$\varsigma^*_{ij} = \{\left|E_{(i,j)}(\Gamma)\right|, (d_\tau, d_q) = (i,j) \quad for \quad \tau q \in E(\overline{\Gamma})\}.$$

Molecular Descriptor	Mathematical Expression	Derived from $M^*(\Gamma)$
$M_1^*(\Gamma)$ [7]	$\sum_{\tau q \notin E(\Gamma)}(d_\tau + d_q)$	$(d_r + d_s)CoM(\Gamma,r,s)(1,1)$
$M_2^*(\Gamma)$ [7]	$\sum_{\tau q \notin E(G)}(d_\tau d_q)$	$d_r d_s CoM(\Gamma,r,s)(1,1)$
$mM_2^*(\Gamma)$ [11]	$\sum_{\tau q \notin E(G)} \frac{1}{(d_\tau d_q)}$	$E_r E_s CoM(\Gamma,r,s)(1,1)$
$F^*(\Gamma)$ [5]	$\sum_{\tau q \notin E(G)} \left((d_\tau^2 + d_q^2)\right)$	$(d_r^2 + d_s^2)CoM(\Gamma,r,s)(1,1)$
$SDD^*(\Gamma)$ [1]	$\sum_{\tau q \notin E(G)} \left(\frac{d_\tau}{d_q} + \frac{d_q}{d_\tau}\right)$	$(d_r E_s + E_r d_s)CoM(\Gamma,r,s)(1,1)$
$ISI^*(\Gamma)$ [1]	$\sum_{\tau q \notin E(G)} \frac{(d_\tau d_q)}{(d_\tau + d_q)}$	$(E_r J d_r d_s)CoM(\Gamma,r,s)(1)$
$H^*(\Gamma)$ [11]	$\sum_{\tau q \notin E(G)} \frac{2}{(d_\tau + d_q)}$	$(2E_x J)CoM(\Gamma,r,s)(1)$
$A^*(\Gamma)$ [11]	$\sum_{\tau q \notin E(\Gamma)} \left(\frac{d_\tau d_q}{d_\tau + d_q - 2}\right)^3$	$(E_r^3 \Psi_{-2} J d_r^3 d_s^3)CoM(\Gamma,r,s)(1)$

Table 1. Various molecular descriptors derived from $M^*(\Gamma)$.

and

$$\kappa_i = \{|V_i|, d_\tau = i \quad for \quad \tau \in V(\Gamma)\}.$$

Berhe and Wang obtained the following relationship [2]:

$$\varsigma_{ij}^* = \begin{cases} \frac{\kappa_i(\kappa_i - 1)}{2} - \varsigma_{ii}, & \text{if } i = j \\ \kappa_i \kappa_j - \varsigma_{ij}, & \text{if } i \neq j \end{cases}. \quad (1)$$

The calculation of topological indices can be difficult in some cases, to overcome this polynomials are used. One of them is M-polynomials. M-polynomials of a Γ are defined in ref. [6] as follows:

$$M(\Gamma,r,s) = \sum_{i \leq j} \varsigma_{ij} r^i s^j.$$

Coindices are defined and to make these indices easier to calculate, CoM-polynomials are defined as follows:

$$CoM(\Gamma,r,s) = M^*(\Gamma) = \sum_{i \leq j} \varsigma_{ij}^* r^i s^j. \quad (2)$$

In Table 1, some coindices and their formulas derived from CoM-polynomials are given. The following operators are used in Table 1:

$$d_r = r\frac{\partial CoM(\Gamma,r,s)}{\partial r}, \qquad d_s = s\frac{\partial CoM(\Gamma,r,s)}{\partial s},$$

$$E_r = \int_0^r \frac{CoM(\Gamma,t,s)}{t}\,dt, \qquad E_s = \int_0^s \frac{CoM(\Gamma,r,t)}{t}\,dt,$$

$$J(CoM(\Gamma,r,s)) = CoM(\Gamma,s,s), \qquad \Psi_k(CoM(\Gamma,r,s)) = r^k CoM(\Gamma,r,s).$$

3 Chemical Descriptions of the Selected Drugs

3.1 Tecovirimat

Tecovirimat is the most commonly used of the FDA-approved Mpox drugs. The chemical formula of Tecovirimat is $C_{19}H_{15}F_3N_2O_3$. The chemical structure of Tecovirimat is shown in Figure 1 [14].

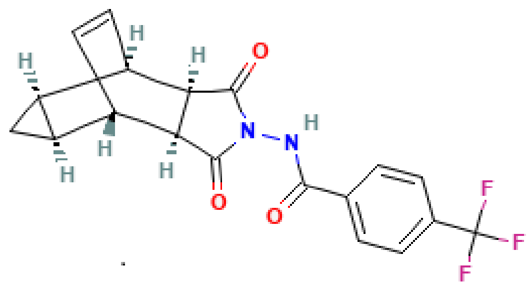

Figure 1. Chemical structure of Tecovirimat.

3.2 Cidofovir

Cidofovir has been used in the treatment of diseases caused by cytomegalovirus (CMV), herpesviruses and various DNA viruses, one of which is Mpox disease [16]. The chemical formula of Cidofovir is $C_8H_{14}N_3O_6P$. Its chemical structure is given in Figure 2 [14].

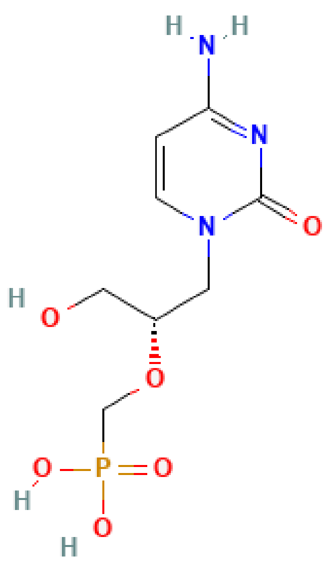

Figure 2. Chemical structure of Cidofovir.

3.3 Brincidofovir

Brincidofovir is a prodrug of cidofovir. It is more bioavailable than cidofovir. Its chemical formula is $C_{27}H_{52}N_3O_7P$. Figure 3 shows the chemical structure of Brincidofovir [14].

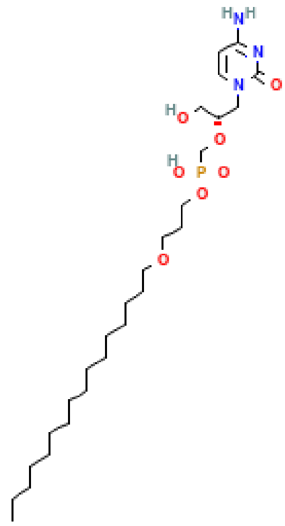

Figure 3. Chemical structure of Brincidofovir.

3.4 Valacyclovir

Valacyclovir is an antiviral drug used to treat infections caused by the herpes simplex virus (HSV), including cold sores and shingles. The chemical formula of Valacyclovir is $C_{13}H_{20}N_6O_4$. Figure 4 shows its chemical structure [14]. Valaciclovir is an antiviral that has been used to treat chickenpox and various herpes infections.

3.5 Famciclovir

Famciclovir is an antiviral medication used to treat infections caused by HSV, including genital herpes, herpes zoster, and herpes labialis. Its chemical formula is $C_{14}H_{19}N_5O_4$. Figure 5 shows the chemical structure of Famciclovir [14]. Famciclovir is a prodrug of penciclovir, an antiviral drug used to treat shingles, HIV, and some types of herpes.

3.6 Acyclovir

Acyclovir is used to treat symptoms of chickenpox, herpes labialis, and herpes zoster. While it does not cure these viral infections, it helps wounds heal faster and relieves pain. It works by preventing the spread of the herpes virus in the body. It has the formula $C_8H_{11}N_5O_3$. The chemical structure of Acyclovir is shown Figure 6 [14]. Acyclovir is an FDA-approved antiviral used to treat certain types of herpes, chickenpox, and shingles.

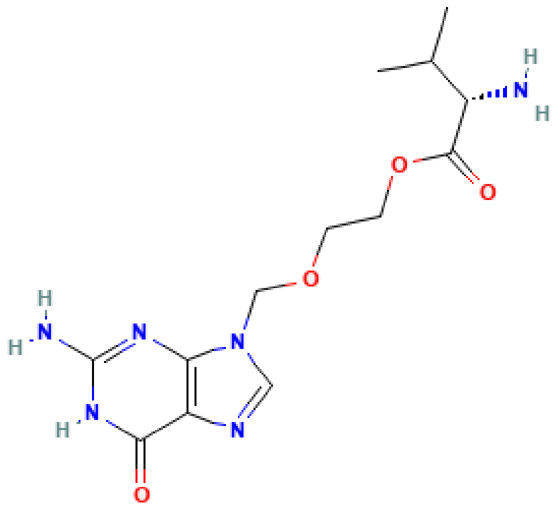

Figure 4. Chemical structure of Valacyclovir.

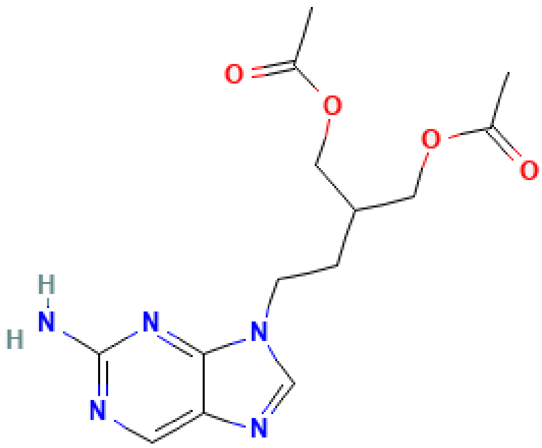

Figure 5. Chemical structure of Famciclovir.

4 Results and Discussion-CoM-Polynomials and Some Coindices of Mpox

In this section, some degree-dependent topological indices of the molecular graphs of drugs used in Mpox and their closed formula CoM-polynomials are obtained.

Theorem 4.1. *Let T be the chemical graph of Tecovirimat. Then,*

$$M^*(T,x,y) = 69xy^3 + 3xy^4 + 25x^2y^2 + 86x^2y^3 + 55x^3y^3 + 11x^3y^4.$$

Proof. From Figure 1, it is observed that $|V| = 27$, $|E| = 31$ and

$$\varsigma_{13} = 3,\ \varsigma_{14} = 3,\ \varsigma_{22} = 3,\ \varsigma_{23} = 10,\ \varsigma_{33} = 11, \text{ and } \varsigma_{34} = 1.$$

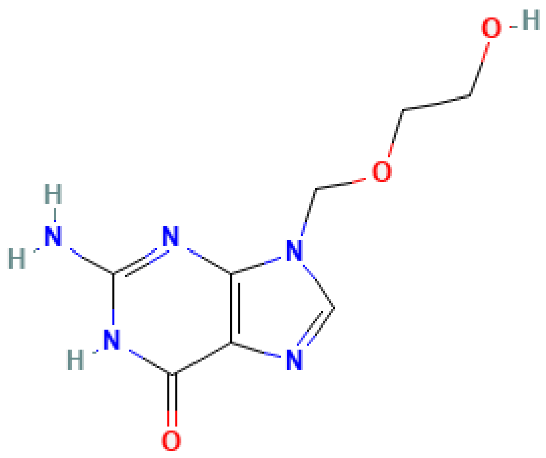

Figure 6. Chemical structure of Acyclovir.

From Equations (1) and (2):

$$M^*(T,x,y) = \sum_{i \le j} \tau_{ij} x^i y^j = 69xy^3 + 3xy^4 + 25x^2y^2 + 86x^2y^3 + 55x^3y^3 + 11x^3y^4.$$

□

The following figure shows the graphical of the CoM-polynomial of the Tecovirimat graph (see Figure 7).

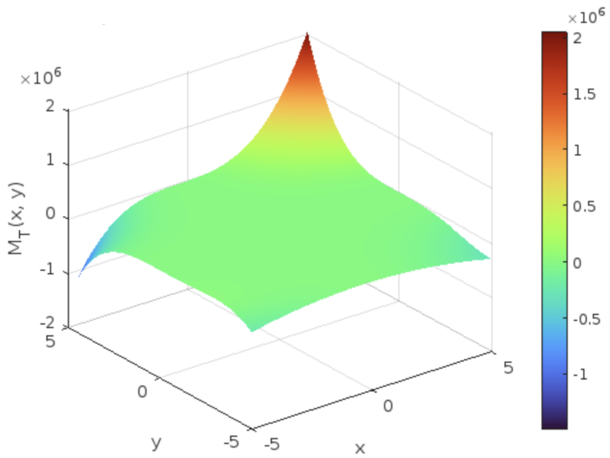

Figure 7. Chemical structure of Tecovirimat graph.

Theorem 4.2. *Let T be the chemical graph of Tecovirimat. The corresponding coindices are:*

$$M_1^*(T) = 1228, \ M_2^*(T) = 1462, \ mM_2^*(T) = 51.3611, \ F^*(T) = 3342,$$
$$SD_{deg}^*(T) = 612, \ ISI^*(T) = 283.70, \ H^*(T) = 104.0761, \ and \ A^*(T) = 1906.534.$$

Proof. By differentiating $M^*(T,r,s)$, the following equations are used:

$$d_r(M^*) = 69rs^3 + 3rs^4 + 50r^2s^2 + 172r^2s^3 + 165r^3s^3 + 33r^3s^4,$$

$$d_s(M^*) = 207rs^3 + 12rs^4 + 50r^2s^2 + 258r^2s^3 + 165r^3s^3 + 44r^3s^4,$$

$$d_rd_s(M^*) = 207rs^3 + 12rs^4 + 100r^2s^2 + 516r^2s^3 + 495r^3s^3 + 132r^3s^4,$$

$$E_rE_s(M^*) = \frac{69}{3}rs^3 + \frac{3}{4}rs^4 + \frac{25}{24}r^2s^2 + \frac{86}{6}r^2s^3 + \frac{55}{9}r^3s^3 + \frac{11}{12}r^3s^4,$$

$$d_r^2\left(M^*(T,r,s)\right) = 69rs^3 + 3rs^4 + 100r^2s^2 + 344r^2s^3 + 495r^3s^3 + 99r^3s^4,$$

$$d_s^2\left(M^*(T,r,s)\right) = 621rs^3 + 48rs^4 + 100r^2s^2 + 774r^2s^3 + 495r^3s^3 + 176r^3s^4,$$

$$\begin{aligned}(d_rE_s + E_rd_s)\left(M^*(T,r,s)\right) = &\left(207 + \frac{69}{3}\right)rs^3 + \left(12 + \frac{3}{4}\right)rs^4 + 50r^2s^2 \\ &+ \left(\frac{172}{3} + 129\right)r^2s^3 + 110r^3s^3 + \left(\frac{33}{4} + \frac{44}{3}\right)r^3s^4,\end{aligned}$$

$$(E_rJd_rd_s)\left(M^*(T,r,s)\right) = \frac{132}{7}r^7 + \frac{495}{6}r^6 + \frac{528}{5}r^5 + \frac{307}{4}r^4,$$

$$(2E_rJ)\left(M^*(T,r,s)\right) = \frac{94}{2}r^4 + \frac{178}{5}r^5 + \frac{55}{3}r^6 + \frac{22}{7}r^7,$$

$$(E_r^3\Psi_{-2}Jd_r^3d_s^3)\left(M^*(T,r,s)\right) = \frac{3468}{8}r^2 + \frac{18768}{27}r^3 + \frac{17820}{64}r^4 + \frac{8448}{125}r^5.$$

Substituting $r = s = 1$ yields the numerical values for the indices. The following results are obtained from the formulas in Table 1 and above equations:

$$M_1^*(T) = \left(276rs^3 + 15rs^4 + 100r^2s^2 + 430r^2s^3 + 330r^3s^3 + 77r^3s^4\right)(1,1) = 1228,$$

$$M_2^*(T) = \left(207rs^3 + 12rs^4 + 100r^2s^2 + 516r^2s^3 + 495r^3s^3 + 132r^3s^4\right)(1,1) = 1462,$$

$$mM_2^*(T) = \left(\frac{69}{3}rs^3 + \frac{3}{4}rs^4 + \frac{25}{4}r^2s^2 + \frac{86}{2}r^2s^3 + \frac{55}{9}r^3s^3 + \frac{11}{12}r^3s^4\right)(1,1) = 51.3611,$$

$$F^*(T) = \left(690rs^3 + 51rs^4 + 200r^2s^2 + 1118r^2s^3 + 990r^3s^3 + 275r^3s^4\right)(1,1) = 3324,$$

$$SD^*_{\deg}(T) = \left(\frac{690}{3}rs^3 + \frac{51}{4}rs^4 + 50r^2s^2 + \frac{559}{3}r^2s^3 + 110r^3s^3 + \frac{275}{12}r^3s^4\right)(1,1) = 612,$$

$$ISI^*(T) = \left(\frac{132}{7}r^7 + \frac{495}{6}r^6 + \frac{528}{5}r^5 + \frac{307}{4}r^4\right)(1) = 283.70,$$

$$H^*(T) = \left(\frac{22}{7}r^7 + \frac{55}{3}r^6 + \frac{178}{5}r^5 + \frac{94}{2}r^4\right)(1) = 104.0761,$$

$$A^*(T) = \left(\frac{19008}{125}r^5 + \frac{40095}{64}r^4 + \frac{6256}{9}r^3 + \frac{3463}{8}r^2\right)(1) = 1906.5348.$$

□

The following indices are computed using the formulas in Table 1 and the expressions above.

Theorem 4.3. *Let C be the chemical graph of Cidofovir. Then*

$$M^*(C,r,s) = 41rs^2 + 22rs^3 + 3rs^4 + 19r^2s^2 + 20r^2s^3 + 6r^2s^4 + 5r^3s^3.$$

Proof. From Figure 2, we have $|V| = 18$, $|E| = 18$, and

$$\varsigma_{12} = 1,\ \varsigma_{13} = 2,\ \varsigma_{14} = 3,\ \varsigma_{22} = 2,\ \varsigma_{23} = 8,\ \varsigma_{24} = 1, \text{ and } \varsigma_{33} = 1. \tag{3}$$

Thus

$$M^*(C,r,s) = 41rs^2 + 22rs^3 + 3rs^4 + 19r^2s^2 + 20r^2s^3 + 6r^2s^4 + 5r^3s^3.$$

□

Figure 8 shows the graphical of the CoM-polynomial of the cidofovir graph.

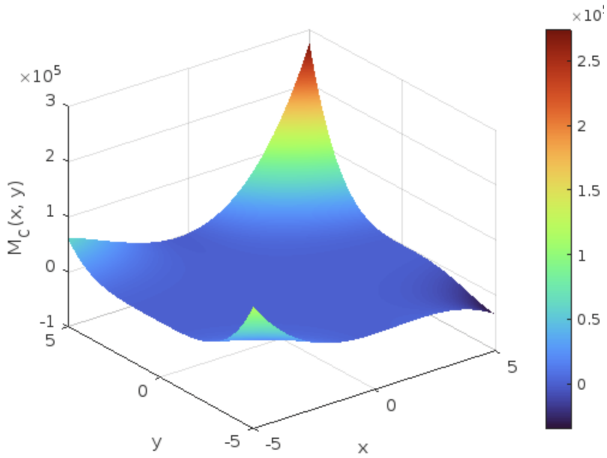

Figure 8. The graphical of CoM-polynomial of Cidofovir graph.

Theorem 4.4. *Let C be the chemical graph of Cidofovir. The corresponding coindices are:*

$$M_1^*(C) = 468,\ M_2^*(C) = 449,\ mM_2^*(C) = 37.9722,\ F^*(C) = 1098,\ SD_{deg}^*(C) = 294.9166,$$
$$ISI^*(C) = 104.7333,\ H^*(C) = 60.70, \text{ and } A^*(C) = 826.3142$$

Proof. From Theorem 4.3, according to Table 1, the following calculations are obtained:

$$d_r\left(M^*(C,r,s)\right) = 41rs^2 + 22rs^3 + 3rs^4 + 38r^2s^2 + 40r^2s^3 + 12r^2s^4 + 15r^3s^3,$$

$$d_s\left(M^*(C,r,s)\right) = 82rs^2 + 66rs^3 + 12rs^4 + 38r^2s^2 + 60r^2s^3 + 24r^2s^4 + 15r^3s^3,$$

$$d_rd_s\left(M^*(C,r,s)\right) = 82rs^2 + 66rs^3 + 12rs^4 + 76r^2s^2 + 120r^2s^3 + 48r^2s^4 + 45r^3s^3,$$

$$E_rE_s\left(M^*(C,r,s)\right) = \frac{41}{2}rs^2 + \frac{22}{3}rs^3 + \frac{3}{4}rs^4 + \frac{19}{4}r^2s^2 + \frac{20}{9}r^2s^3 + \frac{3}{4}r^2s^4 + \frac{5}{9}r^3s^3,$$

$$d_r^2\left(M^*(C,r,s)\right) = 41rs^2 + 22rs^3 + 3rs^4 + 76r^2s^2 + 80r^2s^3 + 24r^2s^4 + 45r^3s^3,$$

$$d_s^2\left(M^*(C,r,s)\right) = 164rs^2 + 198rs^3 + 48rs^4 + 76r^2s^2 + 180r^2s^3 + 96r^2s^4 + 45r^3s^3,$$

$$\left(d_rE_s + E_rd_s\right)\left(M^*(C,r,s)\right) = \left(82 + \frac{41}{2}\right)rs^2 + \left(66 + \frac{22}{3}\right)rs^3 + \left(12 + \frac{3}{4}\right)rs^4$$
$$+38r^2s^2 + 50r^2s^3 + \left(12 + \frac{3}{4}\right)r^2s^4 + 10r^3s^3,$$

$$\left(E_rJd_rd_s\right)\left(M^*(C,r,s)\right) = \frac{82}{3}r^3 + \frac{142}{4}r^4 + \frac{132}{5}r^5 + \frac{93}{6}r^6,$$

$$\left(2E_rJ\right)\left(M^*(C,r,s)\right) = \frac{82}{3}r^3 + \frac{82}{4}r^4 + \frac{46}{5}r^5 + \frac{22}{6}r^6,$$

$$\left(E_r^3\Psi_{-2}Jd_r^3d_s^3\right)\left(M^*(C,r,s)\right) = 328r + \frac{1810}{8}r^2 + \frac{4512}{27}r^3 + \frac{6717}{64}r^4.$$

The following results are obtained from the formulas in Table 1 and above equations:

$$
\begin{aligned}
M_1^*(C) &= \left(123rs^2 + 88rs^3 + 15rs^4 + 76r^2s^2 + 100r^2s^3 + 36r^2s^4 + 30r^3s^3\right)(1,1) = 468, \\
M_2^*(C) &= \left(82rs^2 + 66rs^3 + 12rs^4 + 76r^2s^2 + 120r^2s^3 + 48r^2s^4 + 45r^3s^3\right)(1,1) = 449, \\
mM_2^*(C) &= \left(\frac{41}{2}rs^2 + \frac{22}{3}rs^3 + \frac{3}{4}rs^4 + \frac{19}{4}r^2s^2 + \frac{20}{6}r^2s^3 + \frac{3}{4}r^2s^4 + \frac{5}{9}r^3s^3\right)(1,1) = 37.9722, \\
F^*(C) &= \left(205rs^2 + 220rs^3 + 51rs^4 + 152r^2s^2 + 260r^3s^3 + 120r^2s^4 + 90r^3s^3\right)(1,1) = 1098, \\
SD^*_{\text{deg}}(C) &= \left(\frac{205}{2}rs^2 + \frac{220}{3}rs^3 + \frac{51}{4}rs^4 + 38r^2s^2 + 50r^2s^3 + \frac{51}{4}r^2s^4 + 10r^3s^3\right)(1,1) = 294.9166, \\
ISI^*(C) &= \left(\frac{93}{6}r^6 + \frac{132}{5}r^5 + \frac{142}{4}r^4 + \frac{82}{3}r^3\right)(1) = 104.7333, \\
H^*(C) &= \left(\frac{11}{3}r^6 + \frac{46}{5}r^5 + \frac{82}{2}r^4 + \frac{82}{3}r^3\right)(1) = 60.7, \\
A^*(C) &= \left(\frac{6717}{64}r^4 + \frac{4512}{27}r^3 + \frac{1810}{8}r^2 + 328r\right)(1) = 826.3142.
\end{aligned}
$$

□

Theorem 4.5. *If B is the chemical graph of Birincidofovir, then*

$$M^*(B,r,s) = 160rs^2 + 22rs^3 + 4rs^4 + 330r^2s^2 + 100r^2s^3 + 25r^2s^4 + 5r^3s^3.$$

Proof. From Figure 3, it is easy to see that $|V(B)| = 38$, $|E(B)| = 38$ and also

$$\varsigma_{12} = 2,\ \varsigma_{13} = 2,\ \varsigma_{14} = 2,\ \varsigma_{22} = 21,\ \varsigma_{23} = 8,\ \varsigma_{24} = 2, \text{ and } \varsigma_{33} = 1.$$

From Equations 1 and 2 , it is obtained:

$$
\begin{aligned}
M^*(B,r,s) = {} & \textstyle\sum_{1\leq 2} \tau_{12}rs^2 + \sum_{1\leq 3} \tau_{13}rs^3 + \sum_{1\leq 4} \tau_{14}rs^4 + \sum_{2\leq 2} \tau_{22}r^2s^2 \\
& \textstyle+ \sum_{2\leq 3} \tau_{23}r^2s^3 + \sum_{2\leq 4} \tau_{24}r^2s^4 + \sum_{3\leq 3} \tau_{33}r^3s^3,
\end{aligned}
$$

or

$$M^*(B,r,s) = 160rs^2 + 22rs^3 + 4rs^4 + 330r^2s^2 + 100r^2s^3 + 25r^2s^4 + 5r^3s^3.$$

□

Figure 9 shows the graphical of the CoM-polynomial of the Birincidofovir graph.

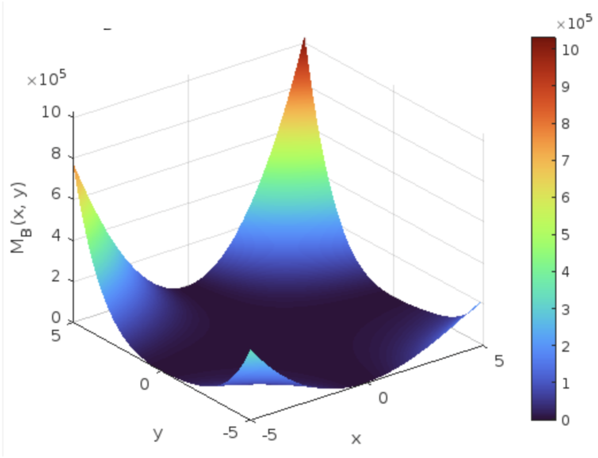

Figure 9. The graphical of CoM-polynomial of Birincidofovir graph.

Theorem 4.6. *If B be chemical graph of Birincidofovir then coindices of the graph are as follows:*

$$M_1^*(B) = 2588,\ M_2^*(B) = 2567,\ mM_2^*(B) = 191.1805,\ F^*(B) = 5618,$$
$$SD^*_{\text{deg}}(B) = 1439.5,\ ISI^*(B) = 617.2,\ H^*(B) = 334.266,\ \textit{and}\ A^*(B) = 5914.9814.$$

Proof. The following equations are obtained from Theorem 4.5

$$\begin{aligned}
d_r(M^*(B,r,s)) &= 160rs^2 + 22rs^3 + 4rs^4 + 660r^2s^2 + 200r^2s^3 + 50r^2s^4 + 15r^3s^3,\\
d_s(M^*(B,r,s)) &= 320rs^2 + 66rs^3 + 16rs^4 + 660r^2s^2 + 300r^2s^3 + 100r^2s^4 + 15r^3s^3,\\
d_rd_s(M^*(B,r,s)) &= 320rs^2 + 66rs^3 + 16rs^4 + 1320r^2s^2 + 600r^2s^3 + 200r^2s^4 + 45r^3s^3,\\
E_rE_s\left(M^*(B,r,s)\right) &= 80rs^2 + \frac{22}{3}rs^3 + rs^4 + \frac{165}{2}r^2s^2 + \frac{50}{3}r^2s^3 + \frac{25}{8}r^2s^4 + \frac{5}{9}r^3s^3,\\
d_r^2\left(M^*(B,r,s)\right) &= 160rs^2 + 22rs^3 + 4rs^4 + 1320r^2s^2 + 400r^2s^3 + 100r^2s^4 + 45r^3s^3,\\
d_s^2\left(M^*(B,r,s)\right) &= 640rs^2 + 198rs^3 + 64rs^4 + 1320r^2s^2 + 900r^2s^3 + 400r^2s^4 + 45r^3s^3,\\
d_rd_s(d_r + d_s)\left(M^*(B,r,s)\right) &= rs^3 + rs^4 + r^2s^2 + r^2s^3 + r^3s^3 + r^3s^4,\\
\left(d_rE_s + E_rd_s\right)\left(M^*(B,r,s)\right) &= 400rs^2 + \left(66 + \frac{22}{3}\right)rs^3 + 17rs^4 + 660r^2s^2\\
&\quad + \left(150 + \frac{200}{3}\right)r^2s^3 + \left(50 + \frac{25}{2}\right)r^2s^4 + 10r^3s^3,\\
\left(E_rJd_rd_s\right)\left(M^*(B,r,s)\right) &= \frac{320}{3}r^3 + \frac{1386}{4}r^4 + \frac{616}{5}r^5 + \frac{245}{6}r^6,\\
\left(2E_rJ\right)\left(M^*(B,r,s)\right) &= \frac{320}{3}r^3 + \frac{352}{2}r^4 + \frac{208}{5}r^5 + 10r^6,\\
\left(E_r^3\Psi_{-2}Jd_r^3d_s^3\right)\left(M^*(B,r,s)\right) &= 1280r + \frac{21746}{8}r^2 + \frac{21856}{27}r^3 + \frac{14420}{64}r^4.
\end{aligned}$$

The following results are obtained from the formulas in Table 1 and above equations:

$$M_1^*(B) = \left(480rs^2 + 88rs^3 + 20rs^4 + 1320r^2s^2 + 500r^2s^3 + 150r^2s^4 + 30r^3s^3\right)(1,1) = 2588,$$

$$M_2^*(B) = \left(320rs^2 + 66rs^3 + 16rs^4 + 1320r^2s^2 + 600r^2s^3 + 200r^2s^4 + 45r^3s^3\right)(1,1) = 2567,$$

$$mM_2^*(B) = \left(80rs^2 + \frac{22}{3}rs^3 + rs^4 + \frac{165}{2}r^2s^2 + \frac{50}{3}r^2s^3 + \frac{25}{8}r^2s^4 + \frac{5}{9}r^3s^3\right)(1,1) = 191.1805,$$

$$F^*(B) = \left(800rs^2 + 220rs^3 + 68rs^4 + 2640r^2s^2 + 2418r^2s^3 + 500r^2s^4 + 90r^3s^3\right)(1,1) = 5618,$$

$$SD_{deg}^*(B) = \left(400rs^2 + \frac{220}{3}rs^3 + 17rs^4 + 660r^2s^2 + \frac{650}{3}r^2s^3 + \frac{125}{2}r^2s^4 + 10r^3s^3\right)(1,1) = 1439.5,$$

$$ISI^*(B) = \left(\frac{245}{6}r^6 + \frac{616}{5}r^5 + \frac{1386}{4}r^4 + \frac{320}{3}r^3\right)(1) = 617.2,$$

$$H^*(B) = \left(10r^6 + \frac{208}{5}r^5 + \frac{352}{2}r^4 + \frac{320}{3}r^3\right)(1) = 334.2666,$$

$$A^*(B) = \left(\frac{3710}{64}r^4 + \frac{256}{27}r^3 + \frac{21746}{8}r^2 + 1280r\right)(1) = 5914.9814.$$

□

Theorem 4.7. *If V is the chemical graph of Valacyclovir then*

$$M^*(V,r,s) = 42rs^3 + 31r^2s^2 + 64r^2s^3 + 23r^3s^3,$$

Proof. From Figure 4, it's easy to see that $|V| = 23$, $|E| = 24$ and also $\varsigma_{13} = 6$, $\varsigma_{22} = 5$, $\varsigma_{23} = 8$ and $\varsigma_{33} = 5$. From Equations 1 and 2, it is obtained

$$M^*(V,r,s) = \sum_{1\leq 3} \tau_{13}rs^3 + \sum_{2\leq 2} \tau_{22}r^2s^2 + \sum_{2\leq 3} \tau_{23}r^2s^3 + \sum_{3\leq 3} \tau_{33}r^3s^3,$$

or

$$M^*(V,r,s) = 42rs^3 + 31r^2s^2 + 64r^2s^3 + 23r^3s^3.$$

□

Figure 10 shows the graphical of the CoM-polynomial of the Valacyclovir graph.

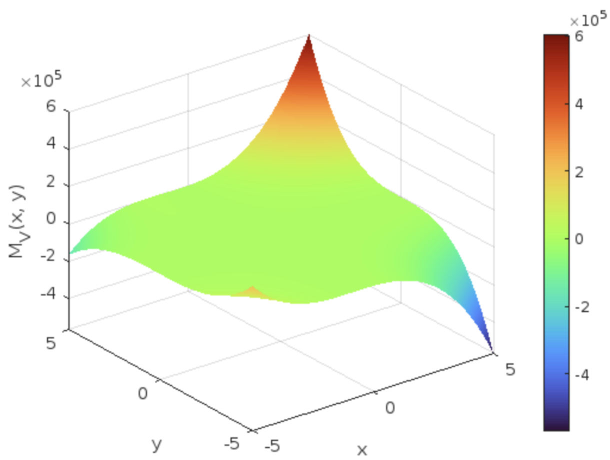

Figure 10. The graphical of CoM-polynomial of Valacyclovir graph.

Theorem 4.8. *Let V be the chemical graph of Valacyclovir. The various coindices of the V graph are as follows:*

$$M_1^*(V) = 750,\ M_2^*(V) = 841,\ mM_2^*(V) = 34.9722,\ F^*(V) = 1914,$$
$$SD_{\deg}^*(V) = 386.6666,\ ISI^*(V) = 173.8,\ H^*(V) = 69.7666,\ \textit{and}\ A^*(V) = 1163.7343.$$

Proof. According to Theorem 4.7 and Table 1, the following calculations are obtained:

$$d_r\left(M^*(V,r,s)\right) = 42rs^3 + 62r^2s^2 + 128r^2s^3 + 69r^3s^3,$$

$$d_s\left(M^*(V,r,s)\right) = 126rs^3 + 62r^2s^2 + 192r^2s^3 + 69r^3s^3,$$

$$d_rd_s\left(M^*(V,r,s)\right) = 126rs^3 + 124r^2s^2 + 384r^2s^3 + 207r^3s^3,$$

$$E_rE_s\left(M^*(V,r,s)\right) = \frac{42}{3}rs^3 + \frac{31}{4}r^2s^2 + \frac{32}{3}r^2s^3 + \frac{23}{9}r^3s^3,$$

$$d_r^2\left(M^*(V,r,s)\right) = 42rs^3 + 124r^2s^2 + 256r^2s^3 + 207r^3s^3,$$

$$d_s^2\left(M^*(V,r,s)\right) = 378rs^3 + 124r^2s^2 + 576r^2s^3 + 207r^3s^3,$$

$$\left(d_rE_s + E_rd_s\right)\left(M^*(V,r,s)\right) = \left(126 + \frac{42}{3}\right)rs^3 + 62r^2s^2 + \left(96 + \frac{128}{3,}\right)r^2s^3 + 46r^3s^3$$

$$\left(E_rJd_rd_s\right)\left(M^*(V,r,s)\right) = \frac{250}{4}r^4 + \frac{384}{5}r^5 + \frac{207}{6}r^6,$$

$$\left(2E_rJ\right)\left(M^*(V,r,s)\right) = \frac{73}{2}r^4 + \frac{128}{5}r^5 + \frac{23}{3}r^6,$$

$$\left(E_r^3\Psi_{-2}Jd_r^3d_s^3\right)\left(M^*(V,r,s)\right) = \frac{3118}{8}r^2 + \frac{13824}{27}r^3 + \frac{16767}{64}r^4.$$

If $r = s = 1$ is written to the equations above, the following coindices are obtained:

$$\begin{aligned}
M_1^*(V) &= \left(168rs^3 + 124r^2s^2 + 320r^2s^3 + 138r^3s^3\right)(1,1) = 750,\\
M_2^*(V) &= \left(126rs^3 + 124r^2s^2 + 384r^2s^3 + 207r^3s^3\right)(1,1) = 841,\\
mM_2^*(V) &= \left(\frac{42}{3}rs^3 + 62r^2s^2 + \frac{31}{4}r^2s^3 + \frac{23}{9}r^3s^3\right)(1,1) = 34.972,\\
F^*(V) &= \left(420rs^3 + 248r^2s^2 + 832r^2s^3 + 414r^3s^3\right)(1,1) = 1914,\\
SD_{\deg}^*(V) &= \left(\frac{420}{3}rs^3 + 62r^2s^2 + \frac{416}{3}r^2s^3 + 46r^3s^3\right)(1,1) = 386.66,\\
ISI^*(V) &= \left(\frac{207}{6}r^6 + \frac{384}{5}r^5 + \frac{250}{4}r^4\right)(1) = 173.8,\\
H^*(V) &= \left(\frac{23}{3}r^6 + \frac{128}{5}r^5 + \frac{73}{2}r^4\right)(1) = 69.76,\\
A^*(V) &= \left(\frac{16767}{64}r^4 + \frac{13824}{27}r^3 + \frac{3118}{8}r^2\right)(1) = 1163.7343.
\end{aligned}$$

□

Theorem 4.9. *If F is the chemical graph of Famciclovir then*

$$M(F,x,y) = 30xy^3 + 50x^2y^2 + 65x^2y^3 + 19x^3y^3.$$

Proof. From Figure 5, we known that $|V| = 23$, $|E| = 24$ and also $\varsigma_{13} = 6$, $\varsigma_{22} = 5$, $\varsigma_{23} = 8$, and $\varsigma_{33} = 5$.

From Equations 1 and 2, it is obtained

$$M^*(F,x,y) = \sum_{1\leq 3}\tau_{13}xy^3 + \sum_{2\leq 2}\tau_{22}x^2y^2 + \sum_{2\leq 3}\tau_{23}x^2y^3 + \sum_{3\leq 3}\tau_{33}x^3y^3,$$

or

$$M^*(F,x,y) = 30xy^3 + 50x^2y^2 + 65x^2y^3 + 19x^3y^3.$$

□

Figure 11 shows the graphs of the CoM-polynomial of the Famciclovir graph.

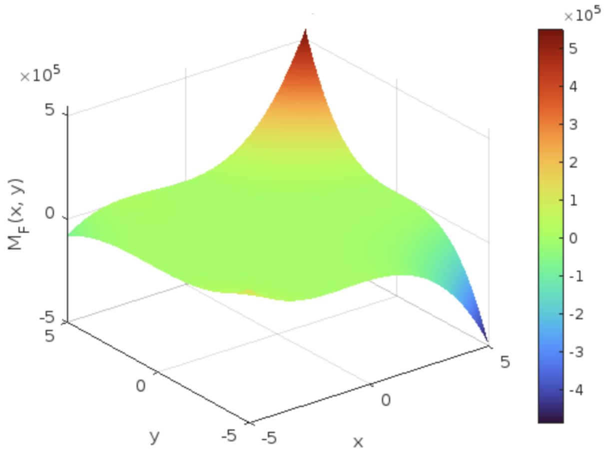

Figure 11. The graphical of CoM-polynomial of Famciclovir graph.

Theorem 4.10. *Let F be the chemical graph of Famciclovir. The various coindices of the graph :*

$$M_1^*(F) = 759,\ M_2^*(F) = 851,\ mM_2^*(F) = 35.444,\ F^*(F) = 1887,$$
$$SD_{\text{deg}}^*(F) = 378.8333,\ ISI^*(F) = 179,\ H^*(F) = 72.33, \text{ and } A^*(F) = 1237.67.$$

Proof. By differentiating $M^*(F,r,s)$ and evaluating the results at $(r,s) = (1,1)$, the following equations are used:

$$M_1^*(F) = \left(120rs^3 + 200r^2s^2 + 305r^2s^3 + 114r^3s^3\right)(1,1) = 759,$$
$$M_2^*(F) = \left(90rs^3 + 200r^2s^2 + 390r^2s^3 + 171r^3s^3\right)(1,1) = 851,$$
$$mM_2^*(F) = \left(10rs^3 + \frac{25}{2}r^2s^2 + \frac{65}{6}r^2s^3 + \frac{19}{9}r^3s^3\right)(1,1) = 35.444,$$
$$F^*(F) = \left(300rs^3 + 400r^2s^2 + 845r^2s^3 + 342r^3s^3\right)(1,1) = 1887,$$
$$\text{SD}_{\text{deg}}^*(F) = \left(100rs^3 + 100r^2s^2 + \frac{845}{6}r^2s^3 + 38r^3s^3\right)(1,1) = 378.8333,$$
$$\text{ISI}^*(F) = \left(\frac{171}{6}r^6 + \frac{390}{5}r^5 + \frac{290}{4}r^4\right)(1) = 179,$$
$$H^*(F) = \left(\frac{19}{3}r^6 + 26r^5 + 40r^4\right)(1) = 72.33,$$
$$A^*(F) = \left(\frac{13851}{64}r^4 + \frac{14040}{27}r^3 + \frac{4010}{8}r^2\right)(1) = 1237.67.$$

□

Theorem 4.11. *If A is the chemical graph of Acylovir, then*

$$M^*(A,r,s) = 23rs^2 + 13rs^3 + 24r^2s^2 + 33r^2s^3 + 7r^3s^3.$$

Proof. From Figure 6, it is seen that $|V| = 23$, $|E| = 24$ and also $\varsigma_{12} = 1$, $\varsigma_{13} = 2$, $\varsigma_{22} = 4$, $\varsigma_{23} = 7$, and $\varsigma_{33} = 3$.

From Equations 1 and 2, it is obtained:

$$M^*(A,r,s) = \sum_{1\leq 2} \tau_{12} rs^2 + \sum_{1\leq 3} \tau_{13} rs^3 + \sum_{2\leq 2} \tau_{22} r^2s^2 + \sum_{2\leq 3} \tau_{23} r^2s^3 + \sum_{3\leq 3} \tau_{33} r^3s^3,$$

or

$$M^*(A,r,s) = 23rs^2 + 13rs^3 + 24r^2s^2 + 33r^2s^3 + 7r^3s^3.$$

□

Figure 12 shows the graphical of the CoM-polynomial of the Acylovir graph.

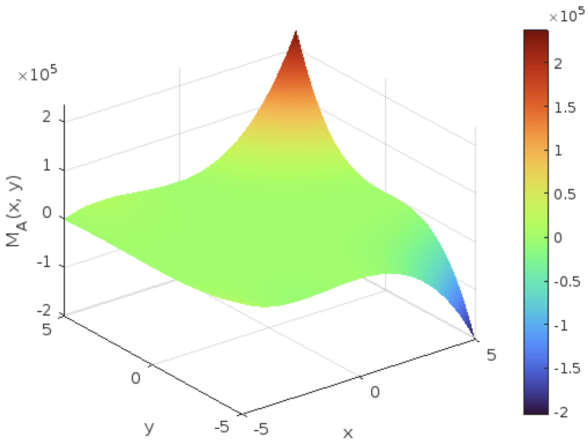

Figure 12. The graphical of CoM-polynomial of Acylovir graph.

Theorem 4.12. *Let A be the chemical graph of Acylovir. The various TIs of the A graph are as follows:*

$$M_1^*(A) = 424,\ M_2^*(A) = 442,\ mM_2^*(A) = 28.1111,\ F^*(A) = 992,$$
$$SD_{\deg}^*(A) = 234.3333,\ ISI^*(A) = 99.183,\ H^*(A) = 49.3666,\ \text{and}\ A^*(A) = 763.6093.$$

Proof. From Table 1 and Theorem 4.11 the following conclusions are written:

$$d_r(M^*(A,r,s)) = 23rs^2 + 13rs^3 + 48r^2s^2 + 66r^2s^3 + 21r^3s^3,$$
$$d_s(M^*(A,r,s)) = 46rs^2 + 39rs^3 + 48r^2s^2 + 99r^2s^3 + 21r^3s^3,$$
$$d_rd_s(M^*(A,r,s)) = 46rs^2 + 39rs^3 + 96r^2s^2 + 198r^2s^3 + 63r^3s^3,$$
$$E_rE_s(M^*(A,r,s)) = \frac{23}{2}rs^2 + \frac{13}{3}rs^3 + 6r^2s^2 + \frac{33}{6}r^2s^3 + \frac{7}{9}r^3s^3,$$
$$d_r^2(M^*(A,r,s)) = 23rs^2 + 13rs^3 + 96r^2s^2 + 132r^2s^3 + 63r^3s^3,$$
$$d_s^2(M^*(A,r,s)) = 92rs^2 + 117rs^3 + 96r^2s^2 + 297r^2s^3 + 63r^3s^3,$$

$$(d_rE_s+E_rd_s)(M(A,r,s)) = \left(\frac{23}{2}+46\right)rs^2+\left(39+\frac{13}{3}\right)rs^3+48r^2s^2 + \left(\frac{99}{2}+22\right)r^2s^3+14r^3s^3,$$

$$(E_rJd_rd_s)(M^*(A,r,s)) = \frac{46}{3}r^3+\frac{135}{4}r^4+\frac{198}{5}r^5+\frac{63}{6}r^6,$$

$$(2E_rJ)(M^*(A,r,s)) = \frac{46}{3}r^3+\frac{74}{4}r^4+\frac{66}{5}r^5+\frac{14}{6}r^6,$$

$$(E_r^3\Psi_{-2}Jd_r^3d_s^3)(M^*(A,r,s)) = 184r+\frac{1887}{8}r^2+\frac{7128}{27}r^3+\frac{5103}{64}r^4.$$

From the formulas in Table 1 and above equations, the following coindices is obtained:

$$M_1^*(A) = \left(69rs^2+52rs^3+96r^2s^2+165r^2s^3+42r^3s^3\right)(1,1) = 424,$$

$$M_2^*(A) = \left(46rs^2+39rs^3+96r^2s^2+198r^2s^3+63r^3s^3\right)(1,1) = 442,$$

$$mM_2^*(A) = \left(\frac{23}{2}rs^2+\frac{13}{3}rs^3+6r^2s^2+\frac{33}{6}r^2s^3+\frac{7}{9}r^3s^3\right)(1,1) = 28.1111,$$

$$F^*(A) = \left(115rs^2+130rs^3+192r^2s^2+429r^2s^3+126r^3s^3\right)(1,1) = 992,$$

$$SD_{\deg}^*(A) = \left(\frac{115}{2}rs^2+\frac{130}{3}rs^3+48r^2s^2+\frac{143}{2}r^2s^3+14r^3s^3\right)(1,1) = 234.3333,$$

$$ISI^*(A) = \left(\frac{63}{6}r^6+\frac{198}{5}r^5+\frac{135}{4}r^4+\frac{46}{3}r^3\right)(1) = 99.18,$$

$$H^*(A) = \left(\frac{7}{3}r^6+\frac{66}{5}r^5+\frac{37}{2}r^4+\frac{46}{3}r^3\right)(1) = 49.36,$$

$$A^*(A) = \left(\frac{5103}{64}r^4+\frac{7128}{27}r^3+\frac{1887}{8}r^2+184r\right)(1) = 763.6093.$$

□

5 Conclusions

Chemical graph theory is a field that allows us to make predictions about chemicals by modeling them with graphs and using graph techniques. In this way, information is obtained without experimenting about the chemical, saving time.

In this study, 8 indices depending on the degrees of non-adjacent peaks of the graphs of Cidofovir, Valacyclovir, Famciclovir, Acyclovir, Tecovirimat, and Brincidofovir drugs used in the treatment of monkeypox virus were studied. In the molecular graphs of all drugs except Brincidofovir, the largest value is in the forgotten coindex, while the smallest value is in mM_2^* (second modified Zagreb coindex). In Brincidofovir drug, the largest value is in the A^*(Augmented Zagreb) coindex, and the smallest value is in mM_2^*.

These results will be used to predict the physicochemical properties of drugs planned to be obtained by using these drugs, and will guide drug designs.

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Data Availability Statement

Data sharing is not applicable to this article.

Conflicts of Interests

The authors declare that they have no conflicts of interest regarding the publication of this article.

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