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Research Paper

Algebraically and geometrically closed of idempotents

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Abstract. Our aim in this article is to study algebraically and geometrically closed structures in a commutative ring with unity R. It is proved that the lattice of idempotents E of R is a algebraically closed lattice. We also show that if E is dense-in-itself, then E^* is geometrically closed in $Mod(T^*_\forall)$. Finally, the relationship between an equicharacteristic regular local ring and an algebraically closed residue field is considered.

Keywords. algebraically closed, geometrically closed, idempotents.
Mathematics Subject Classification (2020): $13L05$, $13A99$.

1 Introduction

Idempotent elements in algebras are widely researched due to their applications in fields such as theoretical physics and chemistry (see [6,7,9], as well as [22]). Determining the idempotent elements of an arbitrary ring is not an easy task in general. Idempotency holds because an algebraically closed field has no proper algebraic extension and here we study the relationship between the lattice of idempotents a commutative ring with unity and algebraically closed lattices. In the model-theoretic study of classes of commutative rings with identity, algebraically closed rings are of primary interest. In order to give a precise definition of the notion algebraically closed lattice we shall need to make use of the diction of model theory. For a basic introduction to model theory, algebraists can refer to [20] or [12]. A structure A is

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said to be algebraically closed if every finite set of equations with parameters in A which has a solution in some extension of A already has a solution in A. A structure A is geometrically closed in class if every A-algebra in the class is geometrically closed. A structure A is said to be existentially closed whenever, for every $\exists_1$-sentence with parameters from A that is true in an extension of A it is also true in A.
In this article, algebraically closed, geometrically closed, and existentially closed of the lattice of idempotents in commutative rings with unity are considered. Also, we investigate solution of a finite system of equations and inequalities with parameters in equicharacteristic regular local ring.

2 Main Results

The starting point for the results presented in this manuscript is appearing in [20]. W. Scott introduced algebraically closed structures and proved that every group could be embedded into an algebraically closed group. These concepts have been studied by many authors; restricting ourselves to lattice theory that for more information see [5, 11, 14, 16–18, 20]. In this section, first we study algebraically closed and geometrically closed lattices for idempotents in commutative rings with unity and then study algebraically closed lattices in equicharacteristic regular local rings. To continue the discussion, we start with an equation with parameters in lattice $\mathfrak{L}$, where a lattice is an algebraic structure $\mathfrak{L} = (L, \vee, \wedge)$ consisting of a set L and two binary, commutative and associative operations $\vee$ and $\wedge$ on L satisfying axiomatic identities $x \vee (x \wedge y) = x$ and $x \wedge (x \vee y) = x$ for all elements $x, y \in L$. The interested reader can see further details in the mentioned reference. A lattice polynomial is an expression involving a finite number of variables $x, y, z, \ldots$ with two symbols $\vee$, $\wedge$ and sometimes the parentheses $(,)$ in a meaningful manner. Suppose that f and h are lattice polynomials and $c_1, \ldots, c_m$ are the parameters of L. An expression of the form

$$f(c_1, \ldots, c_m, x_1, \ldots, x_n) = h(c_1, \ldots, c_m, x_1, \ldots, x_n),$$

ia called an equation with parameters in L. An inequality is obtained with replacing $=$ by $\neq$. Consider a finite system has the form $\mathcal{S} = \{f_1 = h_1, \ldots, f_m = h_m, f_{m+1} \neq h_{m+1}, \ldots, f_{m+n} \neq h_{m+n}\}$ with parameters in L.

Definition 2.1. *[20] Let $\mathfrak{L}$ be a lattice. $\mathfrak{L}$ is a algebraically closed lattice iff every finite system of equations with parameters in L which has a solution in some $\mathfrak{K} = (K, \vee, \wedge)$ ($K \supseteq L$), has already a solution in $\mathfrak{L}$.*

Recall from [8] that a bounded lattice is a lattice that additionally has a greatest element and a least element denoted by 1 and 0, respectively. If $\mathfrak{L}$ is a bounded lattice, then we say that $y \in L$ is a complement of $x \in L$ if $x \wedge y = 0$ and $x \vee y = 1$. In this case we say that x is a complemented element of L. We say that a lattice $\mathfrak{L}$ is complemented if every element of L is complemented. An element $x \in L$ is said to have a pseudocomplement if there exists a greatest element $y \in L$ with the property that $x \wedge y = 0$. We say that b covers a iff $[a, b] = \{a, b\}$. A distributive lattice is a lattice $\mathfrak{L}$ that satisfying, for any $x, y, z \in L$:

$$(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z) \quad \text{or} \quad x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z).$$

Definition 2.2. *[20] A lattice $\mathfrak{L}$ is said relatively complemented if every interval $[a, b]$ contained in L is complemented. A complement of c in the interval $[a, b]$, where c is an element of $[a, b]$, is termed a relative complement of c, $\mathfrak{L}$ is dense-in-itself if and only if L does not contain any prime intervals.*

Recall that an element r of a ring R is called idempotent if $r^2 = r$. Now we can prove the following theorem.

Theorem 2.3. *Suppose R is a commutative ring with unity. If E is the lattice of idempotents of R, then E is algebraically closed lattice.*

Proof. To prove the theorem, we define on E an ordering $\leq$ such that for all $e, f \in E$:

$$e \leq f \quad \text{if and only if} \quad e = ef.$$

We know that E forms a distributive lattice in which

$$\inf\{e, f\} = ef \ \textit{and} \ \sup\{e, f\} = e + f - ef.$$

Let $a \leq y \leq b$. Then the relative complement $c = b \wedge (a \vee y') = a \vee (b \wedge y')$ is the pseudocomplement of y in $[a, b]$,

$$\sup\{y, c\} = b \ \textit{and} \ \inf\{y, c\} = a.$$

Consequently, E is relatively complemented. By applying theorem 6 of [20], E is a algebraically closed lattice.

□

Example 2.4. *The idempotent elements of the ring $Z_p[i]$ are of the form $z = a + bi$, where $a = a^2 - b^2$ and $b = 2ab$, if $b = 0$ then $z = 0$ or $z = 1$. Hence, the set of idempotents elements of the ring $Z_p[i]$ is*

$$E(Z_{p^k}[i]) = \{f_1^r, f_2^r, f_3^r, f_4^r\},$$

where $r = p^{k-1}$ and form a algebraically closed lattice.

By a Boolean lattice we mean a complemented distributive lattice. By a Boolean algebra we mean a boolean lattice together with the unary operation of complementation. Recall that an element b of a Boolean algebra B is said to be an atom if $b > 0$ and there is no $a \in B$ such that $0 < a < b$. Accordingly, a Boolean algebra B is said to be atomic if, for every $a \in B$, there exists an atom $b \leq a$ and a Boolean algebra is said to be atomless if it has no atoms. Also, a Boolean algebra B is free on an arbitrary cardinal number n generators if it is generated by a subset B' with cardinality n and with the property that every mapping of B' into a Boolean algebra C can be extended to a homomorphism of B into C. It is assumed that the reader

possesses a foundational understanding of universal algebra and model theory. The terminology and notation employed herein will be consistent with those defined in [10].
A first-order language $\mathcal{L}$ is a set of symbols partitioned into two subsets, $\mathcal{F}$ and $\mathcal{R}$, such that each symbol in $\mathcal{L}$ is associated with a non-negative integer, termed its arity, where $\mathcal{R}$ is the set of relation symbols and $\mathcal{F}$ is the set of function symbols in $\mathcal{L}$. We construct new language $\mathcal{L}^*$ by taking $\mathcal{L}$ and adding, a new relation symbol $\mathcal{R}^*$ with the same arity of $\mathcal{R}$. It is important to note that, in general, for A, it is not necessarily true that $(A_0)^* = A$ and this equality holds if and only if A satisfies $\emptyset^*$ (or $A \models \emptyset^*$), where A^* is the $\mathcal{L}^*$-structure that is the expansion of A by the symbols $\mathcal{R}^*$, interpreted in A as the complements of $\mathcal{R}$ and $\emptyset$ is the empty theory. Such a structure is called regular. We will say that a class $\mathcal{K}$ of $\mathcal{L}$-structures is elementary, if there exists a first order theory T, such that its models form the class $\mathcal{K}$. We recall from [21] that, for some natural number n, the class $\mathcal{K}$ is $\forall_n$-elementary, provided that T consists entirely of $\forall_n$-sentences. For example, the theory Ring of rings (commutative, with identity) is $\forall_1$ and we can conclude that the class of all rings is $\forall_1$-elementary.

Now, we consider geometrically closed the lattice of idempotents of a commutative ring with unity. Basic concepts of algebraic geometry is introduced by B. Plotkin in [19]. For a tuple of variables $\overline{x}$, we denote by $\varphi(\overline{x})$, an $\mathcal{L}$-formula whose free variables are among $\overline{x}$ and $|\overline{x}|$, where $|\overline{x}|$ is the length of $\overline{x}$. Recall from [21] that an $\mathcal{L}$-formula $\theta(x)$ is defined as quasi-algebraic if it is of the form $\forall \overline{z} \wedge \Phi(\overline{x}, \overline{z}) \Longrightarrow \phi(\overline{x}, \overline{z})$, where the union of Φ and ϕ constitutes a finite set of atomic formulas. The set of quasi-algebraic consequences of T is denoted T_W. Recall from [21] that A is geometrically closed in $\mathcal{L}$ if it is in $\mathcal{L}$ and satisfies the geometric form of Hilbert's Nullstellensatz (Theorem 1.2 of [13]) relatively to $\mathcal{L}$. Let A and B be $\mathcal{L}$-structures of the class $\mathcal{K}$. Then homomorphism $f : A \longrightarrow B$ is called geometrically closed if, for any quasi-algebraic sentence $\theta(\overline{a})$ with parameters in A, the condition $A \models \theta(\overline{a})$ implies that $B \models \theta(f\overline{a})$. A is geometrically closed in $\mathcal{K}$ if and only if every A-algebra in $\mathcal{K}$ is geometrically closed.

Definition 2.5. *Let Φ and Ψ be finite sets of atomic formulas. An $\mathcal{L}$-formula is universal if it has the form $\forall \overline{z}[\wedge\Phi(\overline{x}, \overline{z}) \Longrightarrow \vee\Psi(\overline{x}, \overline{z})]$. We denote the set of all universal consequences of first order theory T by $T_\forall$.*

Recall that the set of all quasi-algebraic sentences with parameters taken from the quasi-algebraic diagram of A that are satisfied by A, is denoted by $\mathrm{Th}_W(A|A)$. An elementary class $\mathcal{K}$ of $\mathcal{L}$-structures is the class of models of a first order theory T in $\mathcal{L}$ and is denoted here as $Mod(T)$ and is closed under isomorphic copies (see [1, 3, 4, 15]). Now we can prove the following theorem.

Theorem 2.6. *Suppose R is a commutative ring with unity. Let E id the lattice of idempotents of R. If E is dense-in-itself, then E^* is geometrically closed in $Mod(T^*_\forall)$.*

Proof. Suppose $\mathcal{S}$ is a set of inequalities and equations that solvable in some extension $L_1 \supseteq L$ (see [20]). We have that $BL_1 \supseteq BL$, and choose a solution $z = (z_l, \ldots, z_n) \in L_1^n \subseteq BL_1^n$ where

BL_1 and BL are free Boolean extensions of L_1 and L, respectively. Let BL be an atomless Boolean lattice and be all elements of L that is appeared in S and B_0 be the finite Boolean lattice generated by BL. We know that there is a sequence $x_l, \ldots, x_n$ of elements of BL such that the Boolean lattices generated by $B_0 \cup \{x_l, \ldots, x_n\}$ and $B_0 \cup \{z_l, \ldots, z_n\}$ are isomorphic in BL_1. Let j be the isomorphic. We have $ja = a$ for $a \in B_0$, and $jx_i, = z_i$, for $1 \leq i \leq n$. Thus $X = (x_l, \ldots, x_n)$ is a solution of $\mathcal{S}$ in BL. So, we have the existence of a solution in L and E is existentially closed. Now we can definite an L^*-homomorphism in $Mod(T^*_\forall)$. Suppose that $g : E^* \longrightarrow F$ be an L^*-homomorphism, where we have $(T^*)_\forall$ and $T^*_\forall$ are equivalent. On the other hand $E^*, F \models \emptyset^*$, so g is an embedding. Suppose $\chi = \forall \overline{x} \wedge \Phi(\overline{x}, \overline{a}) \Longrightarrow \Psi(\overline{x}, \overline{a})$ is a quasi-algebraic sentence of the diagram $\mathrm{Th}_W(E^*|E^*)$. It is clear that χ is equivalent to a universal $\mathcal{L}(E)$ sentence χ', in every regular L^*-extension of E^*. As E is existentially closed in $Mod(T_\forall)$, $g : E \hookrightarrow F_0$ is existentially closed where F_0 is the $\mathcal{L}$-reduct of F, we have that F is regular, so $(F_0, g) \models \chi'$ and then $(F, g) \models \chi$, because F is regular. Therefore, the morphism g is $\mathcal{L}^*$-geometrically closed, and consequently, E^* is geometrically closed in $Mod(T^*_\forall)$. □

Consider a graph $G = (V, E)$, denoted by $V = V(G)$, is the set of vertices, and $E = E(G)$, denoted by $E(G)$, is the set of edges. We denote the complement, diameter, and girth of by $\overline{G}$, $\mathrm{diam}(G)$, and $\mathrm{gr}(G)$, respectively. We denote the degree of a vertex v belonging to the set of vertices V by $\deg(v)$, and the maximum degree within a graph G by $\Delta(G)$. We consider a graph $H = (V_0, E_0)$ to be a subgraph of G if V_0 is a subset of V and E_0 is a subset of E. Additionally, a subgraph H is said to be induced by V_0, when V_0 is a subset of V and E_0 is the set of all edges in E that connect vertices in E_0. A graph G is totally disconnected, meaning that no two vertices are connected by an edge. The annihilator ideal graph of a commutative ring with unity R, denoted by $\Gamma_{Ann}(R)$, is a graph whose vertices are all non-trivial ideals of R and two distinct vertices I and J are adjacent if and only if $I \cap Ann(J) \neq 0$ or $J \cap Ann(I) \neq 0$. Also, recall that a graph G is said to be totally disconnected if it has no edge.
Recall that a local ring R is a ring with a unique maximal ideal, typically denoted by m, and its residue field is typically denoted by $\kappa = \frac{R}{\boldsymbol{m}}$. To emphasize this property, we might write (R, m) or (R, m, κ). We define R to be a Cohen-Macaulay ring when $\mathrm{depht}(R)$ equals $\dim(R)$, where $\dim(R)$ and $\mathrm{depth}(R)$ represent the dimension and depth of R, respectively. The condition $\mathrm{depth}(R) = 0$ holds if and only if every non-unit element of a ring R is a zero-divisor. When R is a Noetherian ring, R is Cohen-Macaulay if and only if $R_{\boldsymbol{m}}$ is Cohen-Macaulay for all maximal ideals m.

In this part of the paper, the relationship between an equicharacteristic regular local ring and algebraically closed is considered, and we deal with equicharacteristics or the case when residue field is embeddable. Bouchiba and Kabbaj proved in [2] that given κ-algebras R and S, the tensor product $R \otimes_k S$ being Noetherian implies that$R \otimes_k S$ is a Cohen-Macaulay ring if and only if R and S are Cohen-Macaulay rings. A local ring is equicharacteristic iff it contains a subfield. A local complete intersection ring is a Noetherian local ring whose completion is the quotient of a regular local ring by an ideal generated by a regular sequence. Also, a Noetherian local ring R is called Gorenstein if R is Cohen-Macaulay and $\dim_{\frac{R}{m}}(soc(R)) = 1$, where m is the unique maximal ideal of R, where $Soc(R)$ is the sum of all minimal ideals of R.

Theorem 2.7. *Suppose $(R, \boldsymbol{m})$ is an equicharacteristic regular local ring length at most l with an algebraically closed residue field κ and $\alpha(\Gamma_{Ann}(R)) < \infty$. If K is an extension field of κ and set $S = R \otimes_{\kappa} K$, then every finite system of equations and inequalities with parameters in R which has a solution in some S^n, has already a, solution in R^n.*

Proof. Notice that among local rings there is a well known chain:

$$\text{Regular} \Longrightarrow \text{Complete intersection} \Longrightarrow \text{Gorenstein} \Longrightarrow \text{Cohen-Macaulay}.$$

So, regular local ring is a Cohen-Macaulay ring. We will now prove that the ring R is Artinian. Assume that $\alpha(\Gamma_{Ann(R)}) < \infty$. To show that $\operatorname{depth}(R) = 0$, we proceed as follows. Assume, for the sake of contradiction, that $\operatorname{depth}(R) \neq 0$. If y is a regular element in R, then the subgraph induced by the vertex set $\{Ry^i\}_{i \in N}$ is totally disconnected and this is a contradiction. Therefore, $\operatorname{depth}(R) = 0$, and the graph $\Gamma_{Ann}(R)$ is complete. Furthermore, $\forall I, J \lhd R, I \neq J \Longrightarrow Ann(I) \neq Ann(J)$. We can define on τ-tuples N^τ an ordering $\leq$ such that $a \leq b \Longleftrightarrow a + c \leq b + c$, for all $a, b, c \in N^\tau$.

Let $x = (x_1, \ldots, x_\tau)$ be a set of minimal generators $\boldsymbol{m}$ and $\alpha = (\alpha_1, \ldots, \alpha_\tau) \in N^\tau$ and $x^\alpha = x_1^{\alpha_1} \ldots x_\mu^{\alpha_\mu}$. We have the support $\operatorname{Supp}(R)$ as the set of all α, where $x^\alpha \neq 0$ in R where it is a finite set. Also, for any $\alpha \in \operatorname{Supp}(R)$, the ideal $\mathbf{a}_R(\alpha)$ (or $\mathbf{a}(\alpha)$) as the ideal generated by all x^β, with $\beta > \alpha$. The collection of all α is denoted by Δ_R, for which x^α not in $\mathbf{a}_R(\alpha)$. Given that R is equicharacteristic, it follows that R has the form $\kappa[X]/I$, where X denotes a collection of variables and I is a (X)-primary ideal. Here, the liftings of the P_i to $\kappa[T, X]$ is denoted by $P_i(T, X)$. let $P_i(T) \in R[T = (T_1, \ldots, T_n)]$ be polynomials, for $i = 1, \ldots, v$. Suppose , for $i = 1, \ldots, u$, we have that $\exists s, s \in S^n; P_i(s) = 0$ and $P_i(s) \neq 0$, for $i = u + 1, \ldots, v$. It is clear that the same monomials form a basis for S over K. Let

$$g_j(B, X) = \Sigma_{\delta \in \Delta_R} B_{j\delta} X^\delta,$$

where B is the set of all variables $B_{j\delta}$. Get $g = (g_1, \ldots, g_n)$. We can find $p_{i\delta}(B) \in \kappa[B]$, the condition,

$$P_i(g, X) = \Sigma_{\delta \in \Delta_R} p_{i\delta}(A) X^\delta$$

over R, for each $i = 1, \ldots v$. Finally, we obtain $b_{j\delta} \in \kappa$, for $j = 1, \ldots, n$ and $\delta \in \Delta_R$ such that

$$g(b, X) = s$$

in S, where $b = (b_{j\delta})$. Finally, for all $\delta \in \Delta_R$ and all $i = 1, \ldots, u$, $p_{i\delta}(b) = 0$. Also, for each $i = u + 1, \ldots, v$, there exists an $\delta_i \in \Delta_R$, such that $p_{i\delta_i}(b) \neq 0$. We know that κ is algebraically closed and is possible to find elements $a_{i\delta}$ of κ, such that satisfies the given inequalities and equalities. For all $\delta \in \Delta_R$ and $a = (a_{i\delta})$, we have

$$p_{i\delta}(a) = \begin{cases} 0 & i \in \{1, \ldots, u\} \\ \neq 0 & i \in \{u + 1, \ldots, v\}. \end{cases}$$

Therefore, to complete the proof, it suffices to assume $g(a, X) = \bar{r}$ until n-tuple over R satisfies our theorem. □

3 Conclusion

This study has delved into the intricate relationship between the lattice of idempotents in commutative rings with unity and various notions of algebraic closure within the framework of model theory. We have meticulously examined the properties of algebraically closed, geometrically closed, and existentially closed lattices of idempotents, shedding light on how these model-theoretic concepts manifest in the algebraic structure of commutative rings. Our investigations have provided a deeper understanding of the conditions under which the lattice of idempotents exhibits these crucial closure properties, extending the classical understanding of idempotency beyond its inherent algebraic definition.

Furthermore, the article has presented a detailed analysis of the solutions to finite systems of equations and inequalities with parameters in equicharacteristic regular local rings. This exploration has not only provided concrete results regarding the solvability of such systems but has also illuminated the interplay between the local ring structure and the existence of solutions within these specific algebraic contexts. The insights gained from this analysis contribute significantly to the ongoing discourse in commutative algebra and its connections to model theory.

In sum, this work underscores the profound utility of model-theoretic tools in elucidating complex algebraic structures. By applying the precise definitions and methodologies of model theory, we have gained novel perspectives on the lattice of idempotents and the solvability of systems of equations within commutative rings. The findings presented herein pave the way for further research into the model theory of rings, particularly in exploring other classes of rings and their associated algebraic and logical properties. This interdisciplinary approach remains crucial for advancing our understanding of fundamental algebraic structures and their broader mathematical implications.

Finding

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Data is contained within the article.

Conflicts of Interests

The author declare that I have no competing of interest.

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