



Research paper

## Analytical study of heat and mass transfer in axisymmetric unsteady flow by ADM

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### Abstract

Finding the solutions for heat and mass transfer problems is significant to reveal the main physics of engineering issues. In this work, the Adomian decomposition method is chosen as a robust analytical method for the investigation of temperature and flow features in a viscous fluid that moves between two parallel surfaces. To ensure the validation of results, the outcome of the Adomian decomposition method is compared with that of the Runge-Kutta method, and reasonable agreement is observed. The comparison confirms that the Adomian decomposition method is a robust and reliable approach for solving this problem. Then, diverse parameters such as Prandtl number and squeeze number are studied. Besides, the effect of chemical reaction parameter, Eckert number, and Schmidt number are comprehensively discussed. Findings reveal that the Sherwood number rises when the chemical reaction parameter and Schmidt number increase. Also, it declines with growths of the squeeze number. Likewise, The findings confirm that the Nusselt number enhances with the rising of the Eckert number and Prandtl number, and it declines when the squeeze number increases.

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## 1. Introduction

In recent decades, scientists applied theoretical methods for solving problems with a simple domain. In fact, the methods are reliable techniques for obtaining the on-hand solution for various scientific problems. Mathematicians then tried to apply the methods for real work

applications. In engineering problems, heat transfer studies are significant due to their effects on the performance of devices such as heat dries, food industries, and microprocessors. Since nonlinear terms exist in most engineering problems, finding reasonable results is crucial for engineering applications. Hence, analytical methods were extensively employed. Among the

semi-exact approaches that applied linearization or discretization, the Adomian decomposition method (ADM) is an efficient approach, and several types of research are done to improve its capability [1, 2]. The main gain of this method is that it can present analytical approximation for a relatively widespread type of nonlinear equations without specific considerations such as linearization, closure approximation, perturbation, or discretization approaches.

Dissimilar to the conventional approaches, i.e., small perturbation and delicate nonlinearity, which modify the nature of the problem due to adjustments, ADM presents a reliable resolution of the engineering problems without any explanation. Therefore, realistic outcomes are obtained by this approach. Various authors have used this technique for solving problems of fluid dynamics. Sheikholeslami et al. [3] used the ADM to study the impact of nanoparticle and magnetic fields on the Jeffery-Hamel flow. They displayed that growing Hartmann number will reduce backflow. Similarly, their results demonstrated that momentum boundary layer thickness rises when nanoparticle volume fraction surges. Jafari et al. [4] offered a modified ADM to resolve nonlinear equations that generated sequences of results with quicker convergence than the series gained by the normal ADM. Allen and Syam [5] examined non-homogeneous and traditional Blasius equations. Hashim [6] applied the ADM for resolving 4th order equations of boundary value problems and the Blasius equation [7]. Arslanturk [8] reviewed this method on the performance of convective straight fins when their thermal conductivity is temperature-dependent. In fact, previous studies just focused on the main theoretical aspects of the problem. Since the main purpose of the analytical studies is to present guidelines for the initial results, it is crucial to find a reasonable outcome for the engineering [9-15].

ADM similarly has been applied by numerous scholars to solve an extensive range of engineering applications such as porous media and fluid flow domain [16-21] and other nonlinear problems [22-27]. In fact, nonlinear problems are widely considered by various researchers to find reliable results [28-33]. Among various topics, heat transfer problems, due to presence of nonlinear terms, are highly considered for evaluation of these approaches [34-39].

The investigation of the transient enfolded of a viscous fluid with constant density between two parallel plates, with respect to time, has been considered as one of the most significant investigations due to its applications [40, 41], for example, hydrodynamical machines, lubrication system and injection molding. The primary study on the squeezing flow under lubrication estimation was stated by Stefan [42]. Lately, because of their requests in numerous divisions of engineering applications, analysis of flow parameters has been improved. Meanwhile, parametric flow analysis of chemical reaction plays a vigorous effect in the chemical process design. The heat transfer properties in the squeezed flow within a porous domain are widely studied by Mahmood et al. [43]. The result of transient chemical reaction on the hydrodynamic of a viscous fluid is measured by Abd-El Aziz [44]. In the other works, Domairry and Aziz [45] studied the magnetohydrodynamic squeezing flow of a viscous fluid between parallel disks. Several researchers applied these methods for non-Newtonian fluid and MHD problems. They found that these approaches present reliable results [46-51].

The key purpose of the present work is to use the ADM to obtain reasonable answers for nonlinear differential equations [51-55]. This work initially explains the ADM for the solving of the nonlinear equations [56-61]. Then, a model is presented and the definitions and boundary conditions are presented. Next, the non-dimension process of equations are briefly explained. Among various complicated problems, the problem of mass and heat transfer within the transient squeezing flow between parallel surfaces is significant. In this work, the concentration and temperature profiles are compared and the effect of the important parameters on the hydrodynamic and flow features is comprehensively explained.

## 2. Equations and units

In solving each engineering problems, finding the governing equations and recognition of the main parameters are the main step for solving the problem. In this work, the heat and mass transfer analyses of the transient 2-dimensional squeezing flow are chosen for the evaluation of this method [62-65]. In this domain, it is

assumed that the fluid is incompressible and viscous between the infinite parallel surfaces. As shown in [Fig. 1](#), the two surfaces are located at  $z = \pm \ell(1 - \alpha t)^{1/2} = \pm h(t)$ . For  $\alpha > 0$ , the two surfaces are squeezed till they reach  $t = 1/\alpha$ , and the two surfaces are disconnected in  $\alpha < 0$ . In this problem, the effects of viscous dissipation and heat production are considered because friction of shear in the flow is preserved. This outcome is significant when the fluid flowing at a high velocity or fluid is mainly viscous. This specific characteristic happens at a high Eckert number ( $>> 1$ ). The chemical reaction of the transient reaction rate in mass transfer equations is also considered. Furthermore, flow is assumed symmetric.

According to these assumptions, the governing equations for mass, momentum, energy, and mass transfer in the unsteady two-dimensional flow of a viscous fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\nu}{C_p} \left( 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right), \quad (4)$$

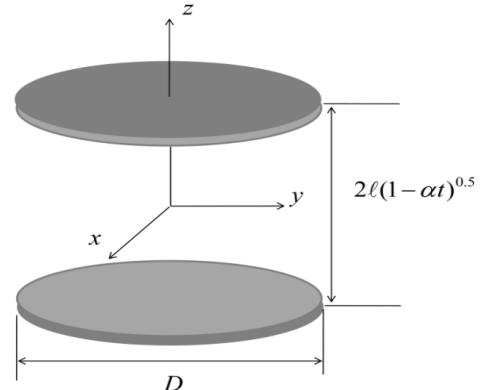
$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - K_1(t) C. \quad (5)$$

In these equations,  $u$  and  $v$  are the velocities in  $x$  and  $y$  directions, respectively,  $T$  is the temperature,  $C$  is the concentration,  $p$  is the pressure,  $\rho$  is the fluid density,  $\nu$  is the kinematic viscosity,  $k$  is the thermal conductivity,  $C_p$  is the specific heat,  $D$  is the diffusion coefficient of the diffusing species, and  $k_1(t) = k_1(1 - \alpha t)$  (see [\[16\]](#)) is the time-dependent reaction rate. The relevant boundary conditions are:

$$C = 0, \quad v = v_w = dh/dt, \quad T = T_H, \quad C = C_H \quad \text{at } y = h(t), \quad (6)$$

$$v = \partial u / \partial y = \partial T / \partial y = \partial C / \partial y = 0 \quad \text{at } y = 0.$$

The following parameters are presented:



**Fig. 1.** Geometry of problem.

$$\begin{aligned} \eta &= \frac{y}{[l(1 - \alpha t)^{1/2}]}, & u &= \frac{\alpha x}{[2(1 - \alpha t)]} f'(\eta), \\ v &= -\frac{\alpha l}{[2(1 - \alpha t)^{1/2}]} f(\eta), & \theta &= \frac{T}{T_H}, \quad \phi = \frac{C}{C_H} \end{aligned} \quad (7)$$

Replacing the above variables into [Eqs. \(2 and 3\)](#) and then disregarding the pressure gradient from the resulting equations give:

$$f^{iv} - S \left( \eta f''' + 3f'' + ff'' - ff''' \right) = 0, \quad (8)$$

Using [Eq. \(7\)](#), the [Eqs. \(4 and 5\)](#) transform to the subsequent equations:

$$\theta'' + \text{Pr} S \left( f\theta' - \eta\theta' \right) + \text{Pr} Ec \left( f''^2 + 4\delta^2 f'^2 \right) = 0, \quad (9)$$

$$\phi'' + Sc S \left( f\phi' - \eta\phi' \right) - Sc\gamma\phi = 0, \quad (10)$$

and boundary conditions are as follows:

$$\begin{aligned} f(0) &= 0, & f''(0) &= 0, & \theta'(0) &= 0, \\ \phi'(0) &= 0, & f(1) &= 1, & f'(1) &= 0, \\ \theta(1) &= \phi(1) = 1, \end{aligned} \quad (11)$$

where  $S$ ,  $\text{Pr}$ ,  $\text{Ec}$ ,  $\text{Ec}$ , and  $\gamma$  are the squeeze number, the Prandtl number, the Eckert number, the Schmidt number, and the chemical reaction parameter, respectively. These parameters are determined as:

$$\begin{aligned} S &= \frac{\alpha l^2}{2\nu}, & \text{Pr} &= \frac{\mu C_p}{k}, & \text{Ec} &= \frac{1}{C_p} \left( \frac{\alpha x}{2(1 - \alpha t)} \right)^2 \\ Sc &= \frac{\nu}{D}, & \gamma &= \frac{k l^2}{\nu}, & \delta &= \frac{l}{x}, \end{aligned} \quad (12)$$

The main parameters for the evaluation of hydrodynamic feature of the flow are the Nusselt

number, skin friction coefficient, and Sherwood number, and they are calculated as follows:

$$C_f = \frac{\mu \left( \frac{\partial u}{\partial y} \right)_{y=h(t)}}{\rho_w^2}, \quad Nu = \frac{-lk \left( \frac{\partial T}{\partial y} \right)_{y=h(t)}}{kT_H}, \quad Sh = \frac{-lk \left( \frac{\partial C}{\partial y} \right)_{y=h(t)}}{DC_H} \quad (13)$$

In terms of Eq. (7), the followings can be obtained:

$$\begin{aligned} l^2 / x^2 (1-\alpha t) \operatorname{Re}_x C_f &= f''(1), \\ \operatorname{Re}_x &= \rho v_w x / \mu \\ \sqrt{1-\alpha t} Nu &= -\theta'(1), \\ \sqrt{1-\alpha t} Sh &= -\phi'(1). \end{aligned} \quad (14)$$

### 3. Details of ADM

By considering equation  $Fu(t) = g(t)$ , where  $F$  denotes an overall nonlinear ordinary, the linear terms are decomposed into  $L + R$ , where  $L$  is simply invertible and  $R$  is the remained of the linear operator. Thus, the equation can be presented as [51]:

$$Lu + Nu + Ru = g \quad (15)$$

where  $Nu$  specifies the nonlinear terms. By resolving Eq. (15) for  $Lu$ , since  $L$  is invertible, it can be written as:

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (16)$$

$L^{-1}$  is a twofold indefinite integral. If  $L$  is a second-order operator. By solving Eq. (16), Eq. (17) is obtained:

$$u = A + Bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (17)$$

where  $A$  and  $B$  are factors of integration. These factors are determined by the boundary or initial conditions. ADM adopts the solution  $u$ , and it could be extended into infinite series as:

$$u = \sum_{n=0}^{\infty} u_n \quad (18)$$

Similarly, the  $Nu$  term will be written as bellows:

$$Nu = \sum_{n=0}^{\infty} A_n \quad (19)$$

The singular Adomian polynomials are defined by  $A_n$ . By determining  $A_n$ , next element of  $u$  can be calculated as:

$$u_{n+1} = L^{-1} \sum_{n=0}^n A_n \quad (20)$$

Lastly, after a few iterations and receiving adequate correctness, the solution can be stated by Eq. (17). In this equation, the Adomian polynomials could be produced by numerous methods. Now, the subsequent recursive formulation is applied:

$$A_n = \frac{1}{n!} \left[ \frac{d^n}{d\lambda^n} \left[ N \left( \sum_{i=0}^n \lambda^i u_i \right) \right] \right]_{\lambda=0}, \quad n = 0, 1, 2, 3, \dots \quad (21)$$

As this technique does not alter linearization, the generated solution is overall more accurate than those attained by shortening the model of the physical problem.

### 4. Implementation of ADM

Consistent with Eq. (15), Eqs. (8-10) can be rewritten as follows:

$$\begin{aligned} L_1 f &= S(\eta f''' + 3f'' + ff'' - ff'''), \\ L_2 \theta &= -\operatorname{Pr} S(f\theta' - \eta\theta') - \operatorname{Pr} Ec(f''^2 + 4\delta^2 f'^2), \\ L_3 \phi &= -Sc(f\phi' - \eta\phi') + Sc\gamma\phi. \end{aligned} \quad (22)$$

where the differential operator  $L_1$ ,  $L_2$  and  $L_3$  are given by  $L_1 = \frac{d^4}{d\eta^4}$ ,  $L_2 = \frac{d^2}{d\eta^2}$  and  $L_3 = \frac{d^2}{d\eta^2}$ , respectively. Assume the inverse of the operator  $L_i$  ( $i=1, 2, 3$ ) is exist and from 0 to  $\eta$  it can be integrated, i.e.:

$$L_1^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (\bullet) d\eta d\eta d\eta d\eta, \quad L_2^{-1} = \int_0^\eta \int_0^\eta (\bullet) d\eta d\eta, \quad L_3^{-1} = \int_0^\eta \int_0^\eta (\bullet) d\eta d\eta. \quad (23)$$

Operating with  $L_i^{-1}$  on Eq. (21), and subsequently applying boundary condition on it, the following equation is obtained:

$$\begin{aligned} f(\eta) &= f(0) + f'(0)\eta + f''(0)\frac{\eta^2}{2} + f'''(0)\frac{\eta^3}{6} + L^{-1}(N_1 u), \\ \theta(\eta) &= \theta(0) + \theta'(0)\eta + L^{-1}(N_2 u), \\ \phi(\eta) &= \phi(0) + \phi'(0)\eta + L^{-1}(N_3 u). \end{aligned} \quad (24)$$

where  $N_i u$  are introduced as:

$$\begin{aligned} N_1 u &= S(\eta f''' + 3f'' + ff'' - ff'''), \\ N_2 u &= -\Pr S(f\theta' - \eta\theta') - \Pr Ec(f''^2 + 4\delta^2 f'^2), \\ N_3 u &= -Sc(f\phi' - \eta\phi') + Sc\gamma\phi. \end{aligned} \quad (25)$$

ADM presented the following expression:

$$\begin{aligned} f(\eta) &= \sum_{m=0}^{\infty} f_m(\eta), \quad f(\eta) = \sum_{m=0}^{\infty} f_m = f_0 + L^{-1}(N_1 u) \\ \theta(\eta) &= \sum_{m=0}^{\infty} \theta_m(\eta), \quad \theta(\eta) = \sum_{m=0}^{\infty} \theta_m = \theta_0 + L^{-1}(N_2 u) \\ \phi(\eta) &= \sum_{m=0}^{\infty} \phi_m(\eta), \quad \phi(\eta) = \sum_{m=0}^{\infty} \phi_m = \phi_0 + L^{-1}(N_3 u) \end{aligned} \quad (26)$$

To limit the components of  $f_m(\eta), \theta_m(\eta)$  and  $\phi_m(\eta)$ , the  $f_0(\eta), \theta_0(\eta)$  and  $\phi_0(\eta)$  are determined by using the boundary condition of Eq. (11) :

$$\begin{aligned} f_0(\eta) &= a_1 \frac{\eta^6}{6} + a_2 \eta, \\ \theta_0(\eta) &= a_3, \\ \phi_0(\eta) &= a_4. \\ f_1(\eta) &= \frac{1}{30} S \eta^5 a_1 + \frac{1}{2520} S a_1^2 \eta^7, \\ \theta_1(\eta) &= -\frac{1}{30} \Pr Ec \delta^2 \eta^6 a_1^6 + \\ &\quad \left( -\frac{1}{12} \Pr Ec a_1^2 - \frac{1}{3} \Pr Ec \delta^2 a_1 a_2 \right) \eta^4 \\ &\quad - 2 \Pr Ec \delta^2 a_2^2 \eta^2, \\ \phi_1(\eta) &= \frac{1}{2} Sc \gamma a_4 \eta^2. \end{aligned} \quad (27)$$

$f_m(\eta), \theta_m(\eta)$  and  $\phi_m(\eta)$  for  $m \geq 2$  are

defined in the similar method from Eq. (25). Then, using

$$f(\eta) = \sum_{m=0}^{\infty} f_m(\eta), \quad \theta(\eta) = \sum_{m=0}^{\infty} \theta_m(\eta) \quad \text{and}$$

$\phi(\eta) = \sum_{m=0}^{\infty} \phi_m(\eta)$  following equations are obtained:

$$\begin{aligned} f(\eta) &= \sum_{m=0}^{\infty} f_m(\eta) = a_1 \frac{\eta^6}{6} + a_2 \eta + \frac{1}{30} S \eta^5 a_1 + \\ &\quad \frac{1}{2520} S a_1^2 \eta^7 + \dots, \\ \theta(\eta) &= \sum_{m=0}^{\infty} \theta_m(\eta) = a_3 - \frac{1}{30} \Pr Ec \delta^2 \eta^6 a_1^6 \\ &\quad + \left( -\frac{1}{12} \Pr Ec a_1^2 - \frac{1}{3} \Pr Ec \delta^2 a_1 a_2 \right) \eta^4 - \\ &\quad 2 \Pr Ec \delta^2 a_2^2 \eta^2 + \dots, \\ \phi(\eta) &= \sum_{m=0}^{\infty} \phi_m(\eta) = a_4 + \frac{1}{2} Sc \gamma a_4 \eta^2 + \dots. \end{aligned} \quad (29)$$

As mentioned in Eq. (28), the precision of the ADM solution rises by growing the number of solution terms ( $m$ ). To attain the final solution for Eq. (28),  $a_i, (i=1, 2, 3, 4)$  with a boundary condition at  $\eta=1$  must be initially calculated. E.g., constant values are attained as follow:

$$\begin{aligned} a_1 &= -2.099635946, \\ a_2 &= 1.423330239, \\ a_3 &= 1.628275971, \text{ and} \\ a_4 &= 0.6581746879 \end{aligned}$$

when:

$$S = \Pr = Ec = Sc = \gamma = 1 \text{ and } \delta = 0.1.$$

## 5. Results and discussion

After the governing equations are determined, the ADM as a reliable technique is applied to analyze hydrodynamic characteristics of viscous squeezed fluid between parallel surfaces. This method could predict reasonable results for the nonlinear equations. Now, the different aspects of the results are comprehensively studied. Fig. 2 displays errors for  $f(\eta), \theta(\eta)$ , and  $\phi(\eta)$  versus  $\eta$  when other significant parameters are fixed. This plot also displays that extreme errors

values occurs at  $\eta = 0.6$  and  $\eta = 1$ , respectively.

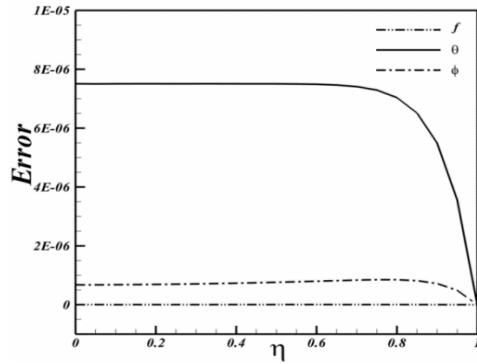
**Fig. 3, Tables 1 and 2** compare results of the numerical method with those of the ADM when diverse values of significant parameter is demonstrated.

The influence of the squeeze number on the velocity profile is also revealed in **Fig. 4**. One of the main crucial parameters in this problem is squeeze number. It is significant to mention that the squeeze number ( $S$ ) pronounces the displacement of the surfaces ( $S > 0$  belongs to the plates moving apart, while  $S < 0$  belongs to the surfaces moving together (the so-called squeezing flow)). As the surfaces move apart, velocity rises with growth in the squeeze number when  $\eta > 0.5$ , while it declines when  $\eta < 0.5$ .

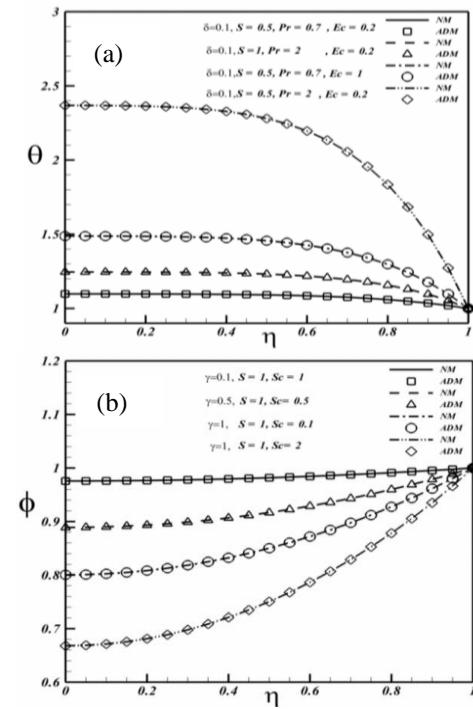
Reverse patterns are detected when the surfaces close together. Consistent with **Eq. (14)**,  $f''(1)$  presents the skin friction coefficient. As perceived from **Fig. 4**, skin friction coefficient declines as the squeeze number intensifies. According to the industrial viewpoint, the power outflow elaborates in the production of motion of surfaces, and it is diminished when  $S$  is negative.

As explained in the text, there are significant results that should also be investigated to reveal the main effects of various parameters. **Fig. 5** displays the outcome of the Prandtl number, Eckert number and squeeze number on the profile of temperature. It is found that increasing the squeeze number can be associated with some changes such as a decrease in the kinematic viscosity, a rise in the spacing between the surfaces, and a growth in the speed at which the surfaces move.

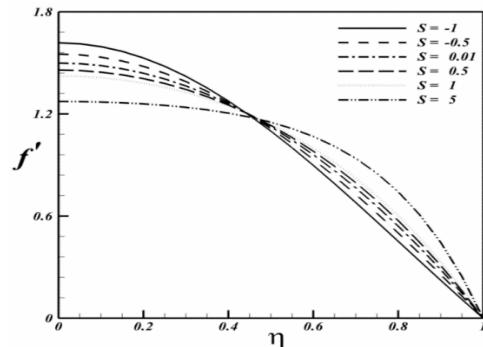
The obtained results also show that when  $S < 0$ , thermal boundary layer thickness intensifies as the absolute degree of the squeeze number rises. In addition, thermal boundary layer thickness declines with the rise in  $|S|$  when  $S > 0$ . It is apparent that the thickness of the temperature boundary layer is relatively high when the surfaces become close. It is also found that the rise of squeeze number can be relevant to growth in the distance between the surfaces, the reduction of the kinematic viscosity, and an increase in the velocity at which the surfaces move.



**Fig. 2.** Error for  $f(\eta)$ ,  $\theta(\eta)$  and  $\phi(\eta)$  versus  $\eta$  when  $S = Ec = 1$ ,  $\delta = 0.1$ ,  $\gamma = 1$ ,  $Sc = 1$  and  $Pr = 1$ .



**Fig. 3.** Comparison between (a) numerical and, (b) ADM solution results.



**Fig. 4.** Effect of the squeeze number on the velocity profile.

**Table 1.** Comparison between the numerical results and ADM solution for  $f(\eta), \theta(\eta)$  and  $\phi(\eta)$  when  $S=0.5, Ec=0.2, \delta=0.1, \gamma=1, Sc=1$  and  $Pr=0.7$ .

$\eta$	$f(\eta)$			$\theta(\eta)$			$\phi(\eta)$		
	NM	ADM	Error	NM	ADM	Error	NM	ADM	Error
0	0	0	0	1.097532	1.097524	0	0.653436	0.653437	0
0.1	0.145385	0.145385	2.43E-10	1.097465	1.097458	7.51E-06	0.656705	0.656705	6.8E-07
0.2	0.288260	0.288260	1.86E-09	1.097179	1.097172	7.51E-06	0.666529	0.666529	6.9E-07
0.3	0.426066	0.426066	2.83E-09	1.096409	1.096401	7.51E-06	0.682963	0.682964	7.07E-07
0.4	0.556143	0.556143	3.86E-09	1.094689	1.094682	7.51E-06	0.706108	0.706108	7.31E-07
0.5	0.675682	0.675682	4.56E-09	1.091321	1.091313	7.51E-06	0.736111	0.736112	7.62E-07
0.6	0.781671	0.781671	4.54E-09	1.085316	1.085308	7.49E-06	0.773188	0.773189	7.99E-07
0.7	0.870846	0.870846	4.05E-09	1.075322	1.075315	7.41E-06	0.817636	0.817637	8.37E-07
0.8	0.939640	0.939640	3.53E-09	1.059519	1.059512	7.04E-06	0.869860	0.869860	8.5E-07
0.9	0.984134	0.984134	1.61E-09	1.035483	1.035478	5.49E-06	0.930404	0.930405	7.18E-07
1	1	1	0	1	1	0	1	1	0

**Table 2.** Comparison between the numerical results and ADM solution for  $f(\eta), \theta(\eta)$  and  $\phi(\eta)$  when  $S=1, Ec=1, \delta=0.1, \gamma=1, Sc=1$  and  $Pr=1$ .

$\eta$	$f(\eta)$			$\theta(\eta)$			$\phi(\eta)$		
	NM	ADM	Error	NM	ADM	Error	NM	ADM	Error
0	0	0	0	1.628962	1.628276	0.000686	0.658171	0.658175	3.33E-06
0.1	0.141982	0.141982	2.67245E-08	1.628521	1.627835	0.000686	0.661463	0.661466	3.35E-06
0.2	0.281844	0.281844	5.16693E-08	1.626762	1.626077	0.000686	0.671342	0.671345	3.4E-06
0.3	0.417380	0.417380	7.25832E-08	1.622311	1.621625	0.000686	0.687829	0.687832	3.48E-06
0.4	0.546214	0.546214	8.87184E-08	1.612646	1.611960	0.000686	0.710964	0.710968	3.6E-06
0.5	0.665708	0.665708	9.81879E-08	1.593714	1.593028	0.000686	0.740827	0.740831	3.75E-06
0.6	0.772872	0.772872	9.85931E-08	1.559320	1.558635	0.000685	0.777554	0.777558	3.93E-06
0.7	0.864254	0.864254	8.72818E-08	1.500179	1.499501	0.000678	0.821372	0.821376	4.12E-06
0.8	0.935839	0.935839	6.25974E-08	1.402502	1.401857	0.000645	0.87264	0.872644	4.21E-06
0.9	0.982926	0.982926	2.58639E-08	1.245862	1.245355	0.000507	0.931908	0.931911	3.61E-06
1	1	1	0	1	1	0	1	1	0

Besides, the thickness of the thermal boundary layer decreases when the Prandtl number intensifies. In low Prandtl numbers ( $Pr \ll 1$ ), the substance is liquid, and thermal diffusivity is high while viscosity is low. On the other side, when the Prandtl number ( $Pr \gg 1$ ) is high, the substance similar to high-viscosity oils. The existence of viscous dissipation effects meaningfully raises the temperature. The influence of the Eckert number on the thickness

of thermal boundary layer is close to the Prandtl number.

In order to analyze the heat transfer in this problem, the Nusselt number as the main non-dimensional number should be investigated. [Fig. 6](#) compares the Nusselt number profile for different values of the squeeze number, Eckert number, and Prandtl number.

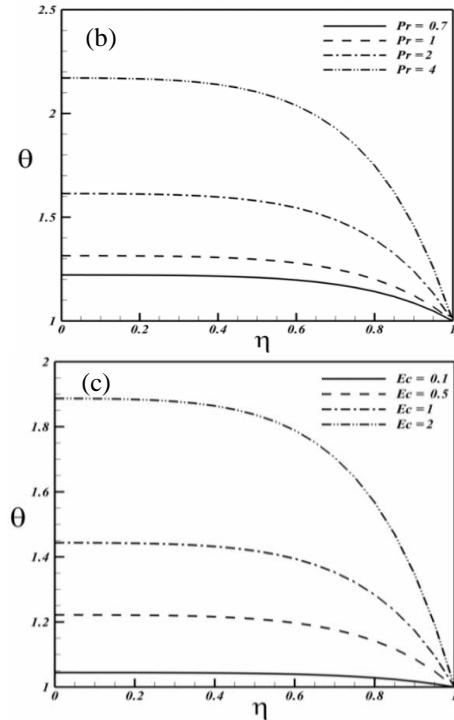
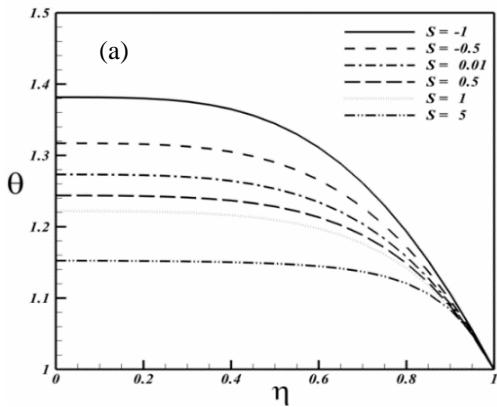
As depicted in [Fig. 6](#), the variation of temperature gradient in the vicinity of the wall significantly varies as the Prandtl number

changes. In addition, the effect of squeeze number pronounces more as the Prandtl number is raised. As shown in the plot, the effect of squeeze number is not noticeable in low Prandtl number. Meanwhile, the effect of the Eckert number is also significant on the results. In contradict to the effect of the Prandtl number, the effect of the squeeze number is not substantial as the squeeze number increases.

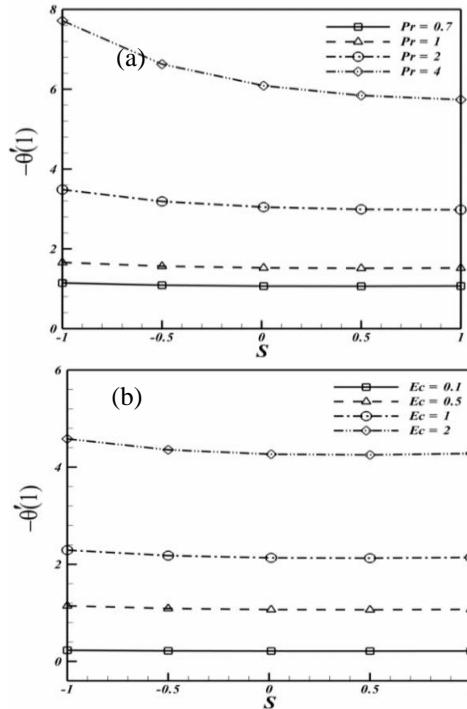
In order to evaluate real-time momentum and mass diffusion convection procedures, the Schmidt number ( $Sc$ ), as the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity is analyzed. This non-dimensional number clearly characterizes fluid features.

**Fig. 7** displays the impact of chemical reaction parameter and the Schmidt number on the concentration profiles. When  $Sc > 1$ , growing Schmidt number reduces the concentration in the centerline but reverse characteristic is detected when  $Sc < 1$ . It is worthy to note that  $\gamma > 0$  signifies the destructive chemical reaction and  $\gamma < 0$  exemplifies the generative chemical reaction. Besides, concentration reduces as destructive chemical reaction parameter is raised while it increases with increasing of chemical reaction parameter.

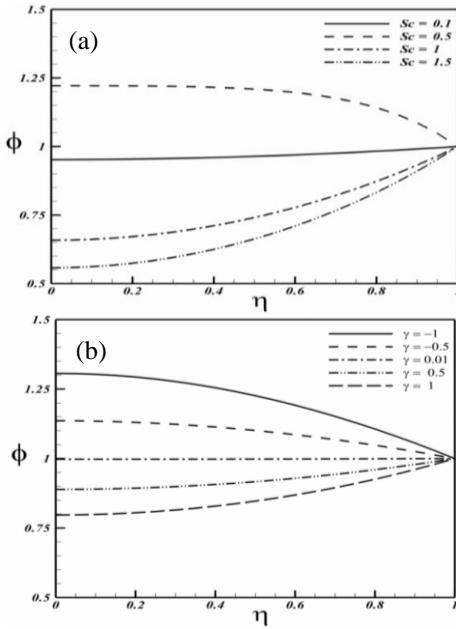
Moreover, the fragile molecular diffusivity and the stripper boundary layer thickness are the main reasons for the slow rise of  $Sc$  (**Fig. 8**). Consequently, the Sherwood number rises with the growth of the Schmidt number. Obtained results also show that the impact of chemical reaction parameter on the Sherwood number is the same as the Schmidt number.



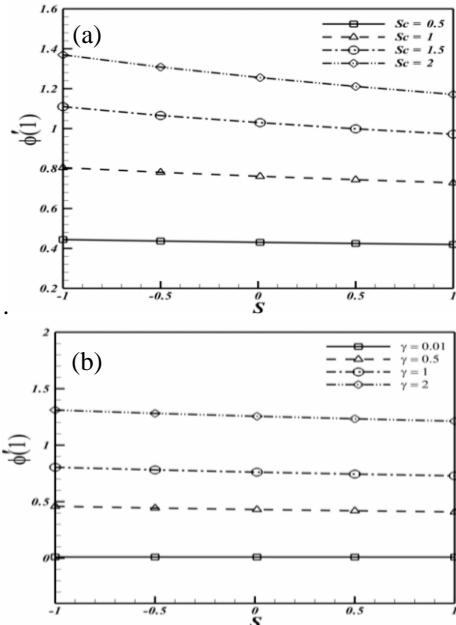
**Fig. 5.** Effect of the squeeze number, Prandtl number, and Eckert number on the temperature profile when (a)  $Ec = 0.5, Pr = 0.7$ ; (b)  $Ec = 0.5, S = 1$ ; and (c)  $Pr = 0.7, S = 1$ , and  $\delta = 0.1$ .



**Fig. 6.** Effects of the squeeze number, Prandtl number and Eckert number on the Nusselt number when (a)  $Ec = 0.5$ ; (b)  $Pr = 0.7$ .



**Fig. 7.** Effects of Schmidt number and chemical reaction parameter on concentration profiles when (a)  $S=1, \gamma=1$ .;(b)  $S=1, Sc=1$ .



**Fig. 8.** Effects of the squeeze number, Schmidt number and chemical reaction parameter on the Sherwood number when (a)  $\gamma=1$ .;(b)  $Sc=1$ .

## 6. Conclusions

In this research, ADM is used to study and analyze mass and heat transfer of the unsteady squeezing flow through parallel surfaces. A

comprehensive comparison between the obtained results with those of the numerical method (4<sup>th</sup> order Runge-Kutta technique) is done, and it is found that there is a good agreement. The findings confirm that the Nusselt number directly varies with change of the Eckert number and the Prandtl number, but it has an opposite relationship with the squeeze number. Furthermore, the obtained results reveal that the chemical reaction parameter and the Schmidt number are raised when the Sherwood number increases. However, the Sherwood number declines as the squeeze number increases.

## References

- [1] G. Adomian, "A review of the decomposition method in applied mathematics", *J. Math. Anal. Appl.* Vol. 135, No. 2, pp. 501-544, (1988).
- [2] S. Ghosh, A. Roy, D. Roy, An adaptation of adomian decomposition for numeric-analytic integration of strongly nonlinear and chaotic oscillators, *Comput. Meth. Appl. Mech. Eng.*, Vol. 196, No. 4-6, pp.1133–1153, (2007).
- [3] A. Hassanvand, M. Barzegar Gerdoobary, R. Moradi, and Y. Amini, "Application of Knudsen thermal force for detection of inert gases." *Results in Phys.*, Vol. 9, pp. 351-358, (2018).
- [4] H. Jafari, V. Daftardar-Gejji, "Revised Adomian decomposition method for solving a system of non-linear equations", *Appl. Math. Comput.*, Vol. 175, No. 1, pp.1–7, (2006).
- [5] F. M. Allan and M. I. Syam, "On the analytic solutions of the nonhomogeneous Blasius problem", *J. Comput. Appl. Math.*, Vol. 182, No. 2, pp. 362-371, (2005).
- [6] I. Hashim, Adomian decomposition method for solving BVPs for fourth-order integro-differential quations, *J. Comput. Appl. Math.* 193, p658–64, (2006).
- [7] I. Hashim, "Comments on A new algorithm for solving classical Blasius equation", *J. Com. Appl.Math.*, Vol. 182, pp. 362-71, (2005).
- [8] C. Arslanturk, "A decomposition method for fins efficiency of convective straight

fins with temperature-dependent thermal conductivity", *Int. Commun. Heat Mass Transfer*, Vol. 32, pp. 831-41, (2005).

[9] S. Pamuk , "Solution of the porous media equation by Adomian's decomposition method", *Phys. Lett. A*. Vol. 344, pp.184-8, (2005).

[10] V. Daftardar-Gejji, H. Jafari, "An iterative method for solving nonlinear functional equations", *J. Math. Anal. Appl.*, Vol. 316, No. 2, pp.753-63, (2006).

[11] D. Lesnic, "Decomposition methods for non-linear non-characteristic Cauchy heat problems", *Commun. Nonlinear Sci. Numer. Simulat.*, Vol. 10, No. 6, pp. 581-96, (2005).

[12] S. M. Imani, A. M. Goudarzi, P. Valipour, M. Barzegar, J. Mahdinejad, and Seiyed E. Ghasemi. "Application of finite element method to comparing the NIR stent with the multi-link stent for narrowings in coronary arteries." *Acta Mech. Solida Sin.*, Vol. 28, No. 5, pp. 605-612, (2015).

[13] M. Sheikholeslami, and A. Zeeshan, "Analysis of flow and heat transfer in water based nanofluid due to magnetic field in a porous enclosure with constant heat flux using CVFEM." *Comput. Methods Appl. Mech. Eng.*, Vol. 320, pp. 68-81, (2017).

[14] Z. Li, M. Barzegar Gerdroodbar, P. Valipour, R. Moradi, H. Babazadeh, "The optimization via response surface method for micro hydrogen gas actuator", *Int. J. Hydrogen Energy*, Vol. 44, No. 59, pp. 31633-31643, (2019).

[15] X. Y. Shen, M. Barzegar Gerdroodbar, A. M. Abazari, and R. Moradi. "Computational study of blood flow characteristics on formation of the aneurysm in internal carotid artery." *The European Phys. J. Plus*, Vol. 136, No. 5 pp. 1-14, (2021).

[16] T. D. Manh, A. M. Abazari, M. Barzegar Gerdroodbar, N. D. Nam, R. Moradi, and H. Babazadeh, "Computational simulation of variable magnetic force on heat characteristics of backward-facing step flow." *J. Therm. Anal. Calorim.*, Vol. 144, No. 4, pp. 1585-1596, (2021).

[17] S. Sepasgozar, M. Faraji, and P. Valipour. "Application of differential transformation method (DTM) for heat and mass transfer in a porous channel." *Propul. power Res.*, Vol. 6, No. 1, pp. 41-48, (2017).

[18] P. Valipour, E. Ekrami, and A. Shams Nateri. "Colorimetric Properties of wool dyed with cochineal: Effect of dye-Bath pH." *Progress Color, Colorants Coat.*, Vol. 7, No. 2, pp. 129-138, (2013).

[19] T. D. Manh, M. Bahramkhoo, M. Barzegar Gerdroodbar, Nguyen Dang Nam, and Iskander Tlili. "Investigation of nanomaterial flow through non-parallel plates." *J. Therm. Anal. Calorim.*, Vol. 143, No. 5, pp. 3867-3875, (2021).

[20] M. Sheikholeslami, K. Vajravelu, "Nanofluid flow and heat transfer in a cavity with variable magnetic field", *Appl. Math. Comput.* Vol. 298, pp. 272-282, (2017).

[21] M. Sheikholeslami, M. Darzi, M. K. Sadoughi, "Heat transfer improvement and Pressure Drop during condensation of refrigerant-based Nanofluid; An Experimental Procedure", *Int. J. Heat Mass Transfer*, Vol. 122, pp. 643-650, (2018).

[22] T. T. Tsotsis, F. Egolfopoulos, N. Nair, R. Prosser, J. Y. Ren, A. Divsalar, M. M. Y. Motamedhashemi, M. Monji, "Catalytic removal of gas phase contaminants". US Patent 9700747 B2, (2017).

[23] M. Barzegar Gerdroodbar, R. Moradi, and H. Babazadeh. "Computational investigation of multi hydrogen jets at inclined supersonic flow." *Int. J. Energy Res.*, Vol. 45, No. 2, pp. 1661-1672, (2021).

[24] M. Sheikholeslami, and M. M. Rashidi. "Effect of space dependent magnetic field on free convection of Fe<sub>3</sub>O<sub>4</sub>-water nanofluid." *J. Taiwan Inst. Chem. Eng.*, Vol. 56, pp. 6-15, (2015).

[25] X. Liu, M. Barzegar Gerdroodbar, M. Sheikholeslami, R. Moradi, A. Shafee, and Z. Li. "Effect of strut angle on

performance of hydrogen multi-jets inside the cavity at combustion chamber." *Int. J. Hydrogen Energy*, Vol. 45, No. 55, pp. 31179-31187, (2020).

[26] M. Sheikholeslami, "Numerical modeling of Nano enhanced PCM solidification in an enclosure with metallic fin", *J. Mol. Liq.* Vol. 259, pp. 424-438, (2018).

[27] Y. Jiang, A. Poozesh, S. M. Marashi, R. Moradi, M. Barzegar Gerdroodbar, A. Shafee, Z. Li, and H. Babazadeh. "Effect of cavity back height on mixing efficiency of hydrogen multi-jets at supersonic combustion chamber." *International J. Hydrogen Energy*, Vol. 45, No. 51 pp. 27828-27836, (2020).

[28] F Rezaei, L. Maleknia, P. Valipour, G. Chizari Fard, "Improvement properties of nylon fabric by corona pre-treatment and nano coating", *J. Text. Inst.*, Vol. 107. No. 10, pp. 1223-1231, (2016).

[29] K. Nasouri, and P. Valipour. "Fabrication of polyamide 6/carbon nanotubes composite electrospun nanofibers for microwave absorption application." *Polym. Sci. Ser. A*, Vol. 57, No. 3, pp. 359-364, (2015).

[30] M. Mokhtari, M. Barzegar Gerdroodbar, R. Yeganeh, and K. Fallah. "Numerical study of mixed convection heat transfer of various fin arrangements in a horizontal channel." *Eng. Sci. . Technol. Int. J.* Vol. 20, No. 3, pp. 1106-1114, (2017).

[31] M. Mokhtari, S. Hariri, M. Barzegar Gerdroodbar, and R. Yeganeh. "Effect of non-uniform magnetic field on heat transfer of swirling ferrofluid flow inside tube with twisted tapes." *Chem. Eng. Process.: Process Intensif.*, Vol. 117, pp. 70-79, (2017).

[32] A. Hassanvand, M. Barzegar Gerdroodbar, and Amir Musa Abazari. "Injection of hydrogen sonic multi-jet on inclined surface at supersonic flow." *Int. J. Mod. Phys. C (IJMPC)*, Vol. 32, no. No. 3, pp. 1-14, (2021).

[33] M. Rahimi Takami, M. Barzegar Gerdroodbar, and D. D. Ganji. "Thermal analysis of mechanical face seal using analytical approach." *Therm. Sci. Eng. Prog.*, Vol. 5, pp. 60-68, (2018).

[34] M. Sheikholeslami, M. Barzegar Gerdroodbar, A. Shafee, and I. Tlili. "Hybrid nanoparticles dispersion into water inside a porous wavy tank involving magnetic force." *J. Ther. Anal. Calorim.*, Vol. 141, No. 5, pp. 1993-1999, (2020).

[35] M. Mosavat, R. Moradi, M. Rahimi Takami, M. Barzegar Gerdroodbar, and D. D. Ganji. "Heat transfer study of mechanical face seal and fin by analytical method." *Eng. Sci. Technol., Int. J.*, Vol. 21, No. 3, pp. 380-388, (2018).

[36] Y. Amini, M. Barzegar Gerdroodbar, M. R. Pishvaie, R. Moradi, and S. M. M. "Optimal control of batch cooling crystallizers by using genetic algorithm." *Case Stud. Therm. Eng.*, Vol. 8, pp. 300-310, (2016).

[37] W. Huawei, R. Moradi, M. Barzegar Gerdroodbar, and M. M. Shahbazi. "Development of micro gas actuator for analyzing gas mixture." *Sci. Iran.*, Vol. 27, No. 6, pp. 2945-2953, (2020).

[38] Y. Peng, M. Barzegar Gerdroodbar, M. Sheikholeslami, A. Shafee, Houman Babazadeh, and R. Moradi. "Mixing enhancement of the multi hydrogen fuel jets by the backward step." *Energy*, Vol. 203, p. 117859, (2020).

[39] M. Sheikholeslami, R. Ellahi, K. Vafai. "Study of Fe<sub>3</sub>O<sub>4</sub>-water nanofluid with convective heat transfer in the presence of magnetic source. " *Alexandria Eng. J.*, Vol. 57, No. 2, pp. 565-575, (2017).

[40] M. Sheikholeslami, Q. M. Zaigham Zia, R. Ellahi. "Influence of Induced Magnetic Field on Free Convection of Nanofluid Considering Koo-Kleinstreuer-Li (KKL) Correlation." *Appl. Sci.* Vol. 6, No. 11, p. 324, (2016).

[41] A. Khan Ambreen, F. Masood, R. Ellahi, and M. M. Bhatti. "Mass transport on chemicalized fourth-grade fluid propagating peristaltically through a curved channel with magnetic effects." *J. Mol. Liq.*, Vol. 258, pp. 186-195, (2018).

[42] M. J. Stefan, "Versuch Über die scheinbare adhesion", *Akad Wissensch Wien Math Natur.* Vol. 69, pp. 713–721, (1874).

[43] M. Mahmood, S. Asghar, M. A. Hossain "Squeezed flow and heat transfer over a porous surface for viscous fluid", *Heat Mass Transf.*, Vol. 44, No. 2, pp. 165–173, (2007).

[44] M. Abd-El Aziz, "Unsteady fluid and heat flow induced by a stretching sheet with mass transfer and chemical reaction". *Chem. Eng. Commun.* Vol. 197, No. 10, pp. 1261–1272, (2010).

[45] G. Domairry, A. Aziz, ""Approximate analysis of MHD squeeze flow between two parallel disks with suction or injection by homotopy perturbation method". *Math. Probl. Eng.* Vol. 2009, pp. 1-19, (2009).

[46] M. S. Kumar, N. Sandeep, B. Rushi Kumar, and S. Saleem. "Effect of Aligned Magnetic Field on MHD Squeezing Flow of Casson Fluid between Parallel Plates." *Defect Diffus. Forum*, Vol. 384, pp. 1-11, (2018).

[47] Y. Amini, and M. Nasr Esfahany. "CFD simulation of the structured packings: A review." *Sep. Sci. Technol.* Vol. 54, No. 15, pp. 1-19, (2018).

[48] J. V. R. Reddy, V. Sugunamma, and N. Sandeep, "Effect of nonlinear thermal radiation on MHD flow between rotating plates with homogeneous-heterogeneous reactions." *Int. J. Eng. Res. Afr.*, Vol. 20, pp. 130-143. (2016).

[49] K. Pushpalatha, J. V. Ramana Reddy, V. Sugunamma, and N. Sandeep. "Numerical study of chemically reacting unsteady Casson fluid flow past a stretching surface with cross diffusion and thermal radiation." *Open Eng.*, Vol. 7, No. 1, pp. 69-76, (2017).

[50] T. Hayat, M. Qasim, Z. Abbas, "Radiation and mass transfer effects on magnetohydrodynamic unsteady flow induced by a shrinking sheet". *Zeitschrift für Naturforschung A*, Vol. 65, No. 3, pp. 231–239, (2010).

[51] G. Adomian, "A review of the decomposition method in applied mathematics", *J. Math. Anal. Appl.*, Vol. 135, No. 2, pp. 501–544, (1988).

[52] M. Sheikholeslami, "Numerical investigation of nanofluid free convection under the influence of electric field in a porous enclosure. " *J. Mol. Liq.*, Vol. 249, pp. 1212–1221, (2018).

[53] Y. Li, A. M. Abazari, M. Barzegar Gerdroodbar, T. Dinh Manh, N. Dang Nam, P. Valipour, R. Moradi, and H. Babazadeh. "Three-dimensional DSMC simulation of thermal Knudsen force in micro gas actuator for mass analysis of gas mixture." *Meas.*, Vol. 160, 107848, (2020).

[54] F. Pish, T. D. Manh, M. Barzegar Gerdroodbar, N. Dang Nam, R. Moradi, and H. Babazadeh. "Computational study of the cavity flow over sharp nose cone in supersonic flow." *Int. J. Mod. Phys.* Vol. 31, No. 6, 2050079, (2020).

[55] A. Sadeghi, Y. Amini, M. H. Saidi, and H. Yavari. "Shear-rate-dependent rheology effects on mass transport and surface reactions in biomicrofluidic devices." *AIChE J.* Vol. 61, No. 6, pp. 1912-1924, (2015).

[56] M. Sheikholeslami, " Solidification of NEPCM under the effect of magnetic field in a porous thermal energy storage enclosure using CuO nanoparticles. " *J. Mol. Liq.* Vol. 263, pp. 303–315, (2018).

[57] M. Sheikholeslami, " Finite element method for PCM solidification in existence of CuO nanoparticles. " *J. Mol. Liq.* Vol. 265, pp. 347–355, (2018).

[58] A. Sadeghi, Y. Amini, M. H. Saidi, and S. Chakraborty, "Numerical modeling of surface reaction kinetics in electrokinetically actuated microfluidic devices." *Anal. Chim. Acta*. Vol. 838, pp.64-75, (2014).

[59] M. Sheikholeslami, M. Barzegar Gerdroodbar, R. Moradi, Ahmad Shafee, and Zhixiong Li. "Application of Neural Network for estimation of heat transfer treatment of Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O nanofluid through a channel." *Comput. Methods Appl. Mech. Eng.* Vol. 344 pp. 1-12, (2019).

[60] X. Liu, M. Sheikholeslami, M. Barzegar Gerdroodbary, Amin Poozesh, Ahmad Shafee, Rasoul Moradi, and Zhixiong Li. "Numerical simulation of the hydrogen mixing in downstream of lobe strut at supersonic flow." *International J. Hydrogen Energy*, Vol. 45, No. 46, pp. 25438-25451, (2020).

[61] M. Barzegar Gerdroodbary, M. Sheikholeslami, S. Valiallah Mousavi, A. Anazadehsayed, Rasoul Moradi. "The influence of non-uniform magnetic field on heat transfer intensification of ferrofluid inside a T - junction" *Chem. Eng. Process.: Process Intensif.*, Vol. 123, pp. 58–66, (2018).

[62] M. Sheikholeslami, M. Barzegar Gerdroodbary, D. D. Ganji. "Numerical investigation of forced convective heat transfer of Fe<sub>3</sub>O<sub>4</sub>-water nanofluid in presence of external magnetic source " *Comput. Methods Appl. Mech. Eng.* Vol. 315, pp. 831–845, (2017).

[63] M. Sheikholeslami. "CVFEM for magnetic nanofluid convective heat transfer in a porous curved enclosure." *Eur. Phys. J. Plus*, Vol. 131, No. 11, pp. 1-14, (2016).

[64] R. Moradi, M. Mosavat, M. Barzegar Gerdroodbary, A. Abdollahi, and Younes Amini. "The influence of coolant jet direction on heat reduction on the nose cone with Aerodome at supersonic flow." *Acta astronautica*, Vol. 151, pp. 487-493, (2018).

[65] A. Edalatpour, A. Hassanvand, M. Barzegar Gerdroodbary, Rasoul Moradi, and Y. Amini. "Injection of multi hydrogen jets within cavity flameholder at supersonic flow." *Int. J. Hydrogen Energy*, Vol. 44, No. 26, pp. 13923-13931, (2019).

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